

DIVISION OF BUILDING TECHNOLOGY
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POST-BUCKLING BEHAVIOUR AND LOAD-
CARRYING CAPACITY OF THIN-WALLED
BOX AND HAT-SECTION BEAMS

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BOX AND HAT-SECTION BEAMS

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PREFACE

The present work has been carried out at the Department of Structural Engineering in the Lund Institute of Technology under the direction of Professor Lars Östlund, head of the Department.

The work has been sponsored by the Swedish Council for Building Research.

The material for the test beams has been manufactured by Domnarvets Jernverk.

The experimental part of the investigation has been carried out in the laboratory of the Department with the assistance of Mr. Lars Lejonberg, Mr. Bengt Götesson and Mr. Jan-Åke Larsson.

The figures have been drawn by Miss Ingbritt Liljekvist, and the manuscript has been typed by Mrs. Mary Lindqvist and Miss Monika Johnsson.

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To the above mentioned persons and to all others involved in the accomplishment of this work I express my sincere thanks.

Lund, August 1977

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ABSTRACT

The main object of this work has been to investigate the post-buckling behaviour and load-carrying capacity of thin-walled box beams subjected to different combinations of bending and torsional moments. The work also included an investigation of hat-section beams subjected to pure bending moments.

A theoretical bar model for the analysis of displacements and stresses in a box beam subjected to pure bending in the post-critical range is presented.

Results of experimental studies of 35 box beams and 6 hat-section beams are given. For the determination of the load-carrying capacity of a box beam subjected to a pure bending moment, it is proposed that effective widths be introduced for the compressed flange as well as for the webs. To determine the load-carrying capacity of a box beam in pure torsion, an empirical expression derived from these experiments is proposed. The load-carrying capacity of a box beam subjected to simultaneous bending and torsion is determined by a simple relation where the influence of the shear forces is also included. Experiments with statically indeterminate box beams show that there is only a limited possibility for a redistribution of bending and torsional moments to occur and consequently that plastic design theory cannot be used uncritically.

SUMMARY

Background

Developments within steel building technology in recent years have been towards an ever increasing use of thin-walled structures manufactured from high strength material. This has created a need for an extensive inquiry into various stability problems such as e.g. local buckling.

When the overall buckling load of a column subjected to an axial compressive force has been attained, the load-carrying capacity of the column has also been reached. For a simply supported plate subjected to uniaxial compression, however, the local buckling load can be reached without collapse of the structure. In this case, the load can often be increased still further, making use of the so called post-critical range.

With hot-rolled beams, the I-section shape is frequently used. This I-section does not, however, turn out to be so favourable for beams made of thin-walled material. Thus in bending the strength of the compressed flange may be considerably reduced due to local buckling. In this connection, the box-section beam is an interesting alternative with its better resistance to local buckling of the compression flange. In Sweden, thin-walled beams with C- and Z-section shapes are frequently used. The disadvantage of these profiles is the asymmetrical cross section giving rise to torsion under transverse loading. Further, the torsional rigidity is rather low, which often leads to transportation and handling problems. In this respect too, the box beam section is favourable because of the high torsional rigidity of closed beam sections.

Purpose

The main object of this work has been to investigate the post-buckling behaviour of thin-walled box beams subjected to different combinations of bending and torsional moments. The investigation sought in particular

- to explain the influence of buckling phenomena and their interaction in webs and flanges
- to compare the test results with theories of post-buckling
- to investigate the relation between load and different deformations up to failure
- to determine the nature of failure
- to elucidate the influence of simultaneous bending and torsional moments
- to determine the possibility of achieving a redistribution of bending and torsional moments in statically indeterminate box beams.

The work was also extended to include an investigation of the post-buckling behaviour of hat-section beams subjected to pure bending.

Scope

This report gives both results of experiments performed and theories in connection with the experiments.

The investigation included the testing of 41 beams, 35 of which were box beams and 6 were hat-section beams. A summary of the experiments performed is given in Fig. 1, where in addition to the load positions the notations of the beams are also presented. At supports and at positions where concentrated external loads acted, web stiffeners were used in the experiments. The dimensions of the test beams can easily be described by the width to thickness ratios of the flanges and the webs. Thus for the flanges of the box beams these ratios were within the range 80-109 and for the webs within the range 120-260.

In connection with the experiments, a bar model was developed for the analysis of the post-buckling behaviour of a box beam subjected to a pure bending moment.

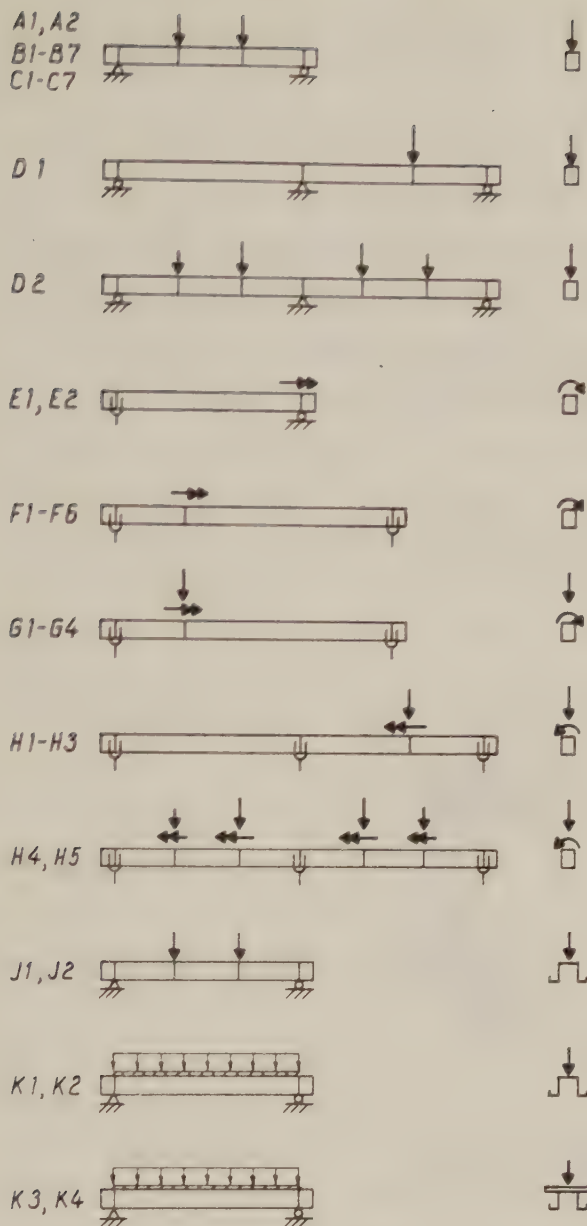


Fig. 1. Summary of experiments performed.

Bar model for the analysis of a box beam subjected to pure bending

The main purpose has been to create a calculation model which in a direct and easily comprehensible way makes it possible to study deformations and stresses in a box beam subjected to pure bending. The plate fields are assumed to be built up of different types of members which are only connected in particular joints. Three types of members are defined:

- continuous beam member
- shear member
- torque member

The problem is solved by the stiffness method and under the assumption of a linear elastic material. Since both the axial forces in the members and the geometry of the members influence the stiffness matrix of the structure, the problem has been solved by gradually increasing the displacements in small increments considering the stiffness matrix as constant at every step. A part of a box beam in bending and the corresponding part of the bar model is shown in Fig. 2.

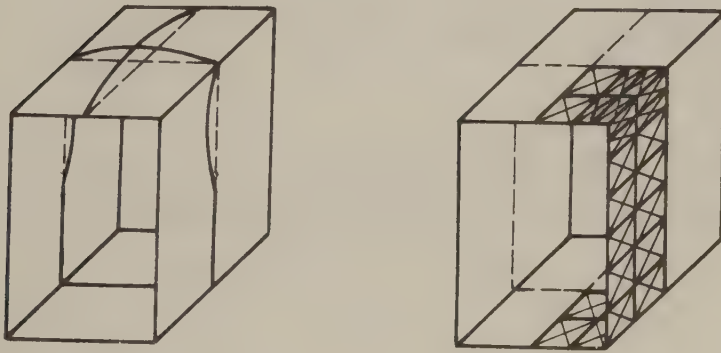


Fig. 2. Part of a box beam subjected to pure bending and the corresponding part of the bar model.

The bar model has been used to determine the influence of initial deflections and stresses. These turn out to be of great importance for the behaviour of box beams in the vicinity of the buckling load but do not seem to be of decisive importance for the load-carrying capacity.

Further, the influence of the assumed wavelength on the local buckles has been studied. The calculations show that the wavelength of the buckles tends to become shorter in the post-critical range.

Results from calculations carried out using the bar model are in good agreement with results obtained experimentally.

Box beam subjected to pure bending

The experiments comprised 18 beams, 16 of which were single-span beams (see test beams A1, A2, B1-B7, C1-C7, D1 and D2 in Fig. 1). Deflections and ultimate loads have been calculated by means of a simplified method based on the assumption that longitudinal hinges are introduced between the different plate fields and that effective widths are introduced both in the compression flange and in the webs.

In the testing of 2 two-span beams the possibility of achieving a redistribution of the bending moments has been studied. The beams show only a limited moment redistribution ability. Thus the moment cannot be maintained at a high level in the section where the first local failure occurs during the period until a second failure develops in another section. Consequently the plastic design theory cannot be used uncritically in the design of box beams subjected to pure bending.

Box beam subjected to pure torsion

The experiments comprised 8 beams, 2 of which had one end restrained from twisting and 6 had both ends restrained from twisting (see test beams E1, E2 and F1-F6 in Fig. 1). An empirical expression derived from the investigation is proposed for the determination of the maximum torsional moment. The expression contains the yield shear stress and the critical shear stress of the webs and the flanges calculated under the assumption of longitudinal hinges in the corners of the beam section.

The experiments indicate that the possibilities of achieving a redistribution of the torsional moments are limited but nevertheless worthwhile considering. Consequently the plastic design theory cannot be applied uncritically.

Box beam subjected to bending and torsion

The experiments comprised 9 beams loaded with different combinations of bending and torsional moments. The primary load in 4 of

the tests has been a torsional moment (test beams G1-G4 in Fig. 1) while in 5 of the tests it has been a bending moment (see test beams H1-H5 in Fig. 1).

In those sections where local failure developed, the internal forces in the beams were determined at the moment of failure. The failure condition can be represented by a circle in a rectangular coordinate system where the bending stress divided by the ultimate bending stress is set off along one of the axes and the sum of the average shear stresses from torsional moment and shear force divided by the ultimate shear stress is set off along the other axis.

Hat-section beam subjected to pure bending

The experiments comprised 6 beams, 4 of which had the compression flange more or less free from external restraints (see test beams J1, J2, K1 and K2 in Fig. 1). In two of the experiments the load was applied to the beams via a trapezoidal corrugated steel sheeting which was riveted to the upper flange (see test beams K3 and K4 in Fig. 1). The corrugated sheeting contributed to counteract the local buckling of the compressed flange and to reduce the transverse displacements of the beam section. The load-carrying capacity of the beams turned out to be rather sensitive to the way in which the external loads were applied.

To calculate the ultimate bending moment of a hat-section beam, effective widths have been introduced for the compression flange and the compressed parts of the webs (cf. the box beam). In addition the lower free flanges have also been reduced in the calculations to compensate for the displacements appearing as a bending of the free flanges in the transverse direction of the beams.

Development prospects

In this investigation, the post-buckling behaviour of beams built up of flat plates has been studied. The development of local buckles in the beams means a reduced bending and torsional stiff-

ness and a lowered load-carrying capacity. On account of their appearance, the buckles may also not be acceptable in certain visible structures. Some different ways of preventing the development of local buckles include to roll in longitudinal folds in webs and flanges, to give the beam some other type of initial deflection pattern, or to make the flat plate fields interact with some type of stiffening structural element. The proper way of development requires a consideration not only of the question of the load-carrying capacity of the beams but also the question of the appropriate method of production and suitable techniques of erection.

NOTATION

A	area
A_w	web area
$[A_m]_i$	action matrix for member i given in the member-oriented coordinate system
$[A_s]_i$	action matrix for member i given in the structure-oriented coordinate system
$[A_s]$	action matrix for the total structure
a	1. length of shear and torque members 2. half the wave-length of a buckle 3. width of "lip" for hat-section beam (centre dimension)
a_o	width of "lip" for hat-section beam (outer dimension)
B	longitudinal bimoment
b	width of box and hat-section beam (centre dimension)
b_e	effective width
b_o	width of box and hat-section beam (outer dimension)
c	width of free flange for hat-section beam (centre dimension)
c_A, c_B, c_C	elastic constants of supports with reference to torsion
c_o	width of free flange for hat-section beam (outer dimension)
c_1, c_2	constants
D	flexural rigidity of a plate
$[D_m]_i$	displacement matrix for member i given in the member-oriented coordinate system
$[D_s]_i$	displacement matrix for member i given in the structure-oriented coordinate system
$[D_s]$	displacement matrix for the total structure
E	modulus of elasticity
e	force eccentricity
F	stress function
f	frequency
f_o	inner distance between free flanges of hat-section beam
G	shear modulus
h	height (depth) of box and hat-section beam (centre dimension)
h_c	height of compression zone
h_{ce}	effective height of compression zone
h_o	height of box and hat-section beam (outer dimension)

I	moment of inertia
I_T	torsional constant
I_0	moment of inertia when the whole beam section is effective
i	index
k	buckling coefficient
k_A, k_B, k_C	elastic constants of supports with reference to vertical translation
k_f	buckling coefficient of flange
k_w	buckling coefficient of web
L	length
L_1, L_2	lengths
M	bending moment
M_U^{exp}	ultimate bending moment obtained experimentally
M_U^{th}	ultimate bending moment obtained theoretically
m	mass
N	normal force, axial load
N_{cr}	critical load
P	load
P_U^{exp}	ultimate load obtained experimentally
P_U^{th}	ultimate load obtained theoretically
$P_{ocr, f}$	load corresponding to the stress $\sigma_{cr, f}$
Q	1. total vertical load ($= q \cdot L$) 2. transverse bimoment
Q_U^{exp}	ultimate load obtained experimentally
$\{Q\}_i$	complete angular transformation matrix for member i
q	load intensity
r	inside corner radius
$\{R\}_i$	angular transformation matrix for member i
$\{S_m\}_i$	stiffness matrix for member i given in the member-oriented coordinate system
$\{S_s\}_i$	stiffness matrix for member i given in the structure-oriented coordinate system
$\{S_s\}$	stiffness matrix for the total structure
T	torsional moment
T_U^{exp}	ultimate torsional moment obtained experimentally
T_U^{th}	ultimate torsional moment obtained theoretically
t	plate thickness
u	1. horizontal deflection 2. generalized warping

V	shear force
v	vertical deflection
w	normal deflection
w_0	initial normal deflection
x	axis in structure-oriented coordinate system
x_m	axis in member-oriented coordinate system
y	axis in structure-oriented coordinate system
y_m	axis in member-oriented coordinate system
z	axis in structure-oriented coordinate system
z_m	axis in member-oriented coordinate system
α	$\sqrt{\frac{\tau_y}{\tau_{cr}}}$
γ	shear strain
Δ	difference
δ_1, δ_2	length reductions
δ_5	elongation of measuring length ($L_0 = 5 \cdot \text{diamater}$ for round tensile test piece) in tensile test
ϵ	strain
θ	angle of twist
κ	generalized contour deformation
$\lambda = a/b$	length to width ratio for a buckle
ν	Poisson's ratio
ρ	radius of curvature
σ	stress
σ_e	edge stress on the compression side of a box beam
σ_{cr}	critical stress
$\sigma_{cr,f}$	critical stress for the flange
$\sigma_{cr,w}$	critical stress for the web
σ_i	residual (initial) stress
σ_u	ultimate stress
σ_w	warp stress
σ_y	yield stress (determined by 0.2 per cent offset method)
σ_{yc}	corner yield stress
τ	shear stress
τ_{cr}	critical shear stress
$\tau_{cr,f}$	critical shear stress for the flange
$\tau_{cr,w}$	critical shear stress for the web
τ_s	shear stress according to Saint Venant
τ_T	shear stress from torsional moment T
τ_V	shear stress from shear force V

τ_y	yield shear stress
ϕ	principal strain direction

Other symbols are defined in the text.

1 INTRODUCTION

Developments within steel building technology in recent years have been towards an ever increasing use of thin-walled structures manufactured from high strength material. This has created a need for an extensive inquiry into various stability problems such as e.g. local buckling.

When the overall buckling load of a column subjected to an axial compressive force has been attained, the load carrying capacity of the column has also been reached. For a simply supported plate subjected to uniaxial compression, however, the local buckling load can be reached without collapse of the structure. In this case, the load can often be increased further, making use of the so-called post-critical range.

Within steel building technology, the shape most frequently used for hot rolled beams seems to be the I-section. However, the I-section does not turn out to be so favourable for beams made of thin-walled material. Thus, at bending, the strength of the compressed flange may be considerably reduced due to local buckling. In this connection the box-section beam is an interesting alternative with its better resistance to local buckling of the compression flange. In Sweden, thin walled beams with C- and Z-section shapes are often used. The disadvantage of these profiles is the asymmetrical cross section giving rise to torsional moments even when the transverse load acts through the centre of gravity. Further, the torsional rigidity of these beams is rather low, which often leads to transportation and handling problems. Also in this respect the box beam section is favourable because of the high torsional rigidity of closed beam sections.

The main object of this work was to investigate the post-buckling behaviour of thin walled box beams subjected to different combinations of bending and torsional moments. The work also includes an investigation of the post-buckling behaviour of hat-section beams subjected to pure bending.

The work starts with a literature survey in Chapter 2.

In Chapter 3 the case of pure bending of a box beam is treated. A bar model for the analysis of a simply supported plate in compression and of a box beam subjected to pure bending is presented. A simplified model for the determination of the ultimate bending moment of box beams is proposed. The experiments in Chapter 3 include single-span as well as two-span beams. The object of the experiments was to study the influence of buckling phenomena and their interaction in webs and flanges, to compare the results with theories concerning post-buckling, to investigate the relation between load and different displacements up to failure, to determine the nature of the failure and to investigate the possibility of achieving a redistribution of the bending moments.

Chapter 4 deals with an experimental investigation of box beams subjected to torsion. The chapter begins with a theoretical section where warp stresses in box beams subjected to torsion are discussed, and an empirical method for the determination of the ultimate torsional moment is proposed. The experiments reported in this chapter relate to box beams with one or two ends restrained against torsion. The object of the experiments was to elucidate the post-buckling behaviour of thin walled box beams with special reference to problems concerning torsional moment redistribution.

Chapter 5 comprises results from a number of experiments with box beams subjected to different combinations of bending and torsional moments. The object of the experiments was to elucidate the post-buckling behaviour of the box beams, to study the possibility of moment redistribution and to investigate the interaction between local failures caused by bending and torsion.

If one of the flanges of a box beam is cut and unfolded, a hat-section beam will be obtained. In Chapter 6, results from a series of experiments with hat-section beams subjected to bending are summarized. Two different cases have been studied, free compression flange and compression flange restrained by a corrugated steel sheeting. The main object of the experiments was to compare the results with those obtained from experiments with box beams in pure bending.

2 LITERATURE SURVEY

2.1 General.

The purpose by this survey is not to give a complete survey of all the literature in this field but rather to give the reader an appropriate background for studying this report and further to point out literature which treats interesting problems in more detail.

The literature survey is divided into three sections with respect to the geometry of the structures and the loading cases concerned. Thus in Sections 2.2 and 2.3 individual plates are studied under the actions of normal forces and shear forces respectively. Section 2.4 comprises an inventory of literature concerning box beams subjected to bending moments. All three sections have been divided into one part dealing with "critical load" and one part dealing with "post-buckling behaviour and maximum load".

2.2 Rectangular flat plate subjected to compression.

2.2.1 Critical load.

If an originally flat plate is subjected to compressive forces in its plane, the stability of the plate will be lost when the compressive forces reach a critical value. The slightest effect perpendicular to the surface will cause the plate to adopt a deflected equilibrium position.

At small deflections w , the following differential equation for an elastic plate subjected to compressive and shear forces in the xz -plane is used:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial z^2} + \frac{\partial^4 w}{\partial z^4} = \frac{t}{D} \left(\sigma_x \frac{\partial^2 w}{\partial x^2} + \sigma_z \frac{\partial^2 w}{\partial z^2} + 2\tau \frac{\partial^2 w}{\partial x \partial z} \right) \quad (2.2.1)$$

where σ_x = normal stress in x -direction

σ_z = normal stress in z -direction

τ = shear stress

$$D = \frac{Et^3}{12(1 - \nu^2)} = \text{flexural rigidity of the plate}$$

t = plate thickness

Solutions of the differential equation considering actual boundary conditions and elastic material have been presented by e.g. Bleich (1952), Kollbrunner (1958), Timoshenko (1961) and Gerard (1962). For more complicated problems where no exact solution can be found, methods based on energy considerations are often used as by Bleich (1952), Kollbrunner (1958), Timoshenko (1961) and Pflüger (1964).

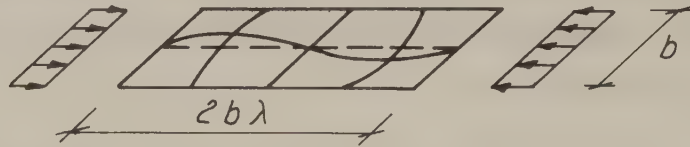
The critical stress σ_{cr} of a flat elastic plate under normal forces can be expressed as

$$\sigma_{cr} = k_\sigma \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b} \right)^2 \quad (2.2.2)$$

where k_σ = buckling coefficient

b = plate width

The buckling coefficient k_σ is dependent on the geometry of the plate, the boundary conditions and the loading case. Values of k_σ and λ ($2b\lambda$ is equal to the wave length of one buckle, see Fig. 2.2.1) for long plates subjected to uniaxial compression are given in Fig. 2.2.1 for two different boundary conditions.



Boundary condition						
	k	λ	k	λ	k	λ
	4.00	1.00	7.81	0.98	23.9	0.67
	6.97	0.67	13.56	0.65	39.6	0.47

Fig. 2.2.1. Values of k and λ for uniaxial compression of long plates.

In practice, all plates display some degree of irregular initial deflection in the unloaded state. For this reason, the local buckles will not appear instantaneously at the critical load but a continuous increase in normal deflections will occur. Provided the initial deflections are relatively small ($< 0.1 t$), a distinct acceleration in the course of buckling will be observed in the vicinity of the critical load.

For the experimental determination of the critical load, strain gauges are usually employed, two gauges being fastened opposite to each other in the vicinity of a buckle crest. In the literature, different methods have been presented including those by Hu et al (1946) and Edlund (1973). As an example, the method of "strain reversal" is illustrated in Fig. 2.2.2, where the critical load is defined by the point where the compressive strain on the convex side of the plate starts to decrease.

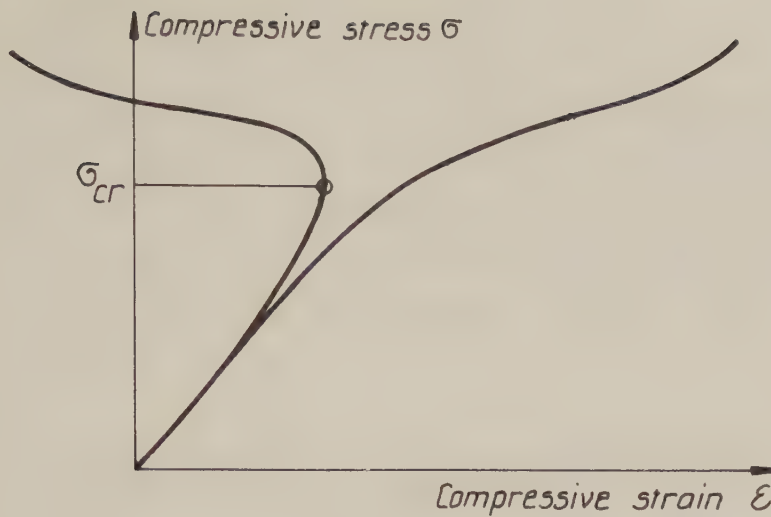


Fig. 2.2.2. Determination of the critical load of a compressed plate from strain measurements by means of the method of "strain reversal".

In the case of an initial deflection surface of constant curvature $1/R$ in the transverse direction of the plate, the problem can be analysed by regarding the plate as a shallow cylindrical panel. From theory, see e.g. Volmir (1962), we know that the classical critical load of a cylindrical panel is greater than the critical load of the corresponding flat panel. For a more profound discussion of initial deflections in the form of cylindrical deflection surfaces, see Edlund (1973).

2.2.2 Post-buckling behaviour. Maximum load.

Consider a plate subjected to stresses in the plane of the middle surface. The fundamental non-linear differential equations which treat the problem for "large deflections" were derived by von Karman. By introducing a stress function $F(x, z)$, defined by

$$\sigma_x = \frac{\partial^2 F}{\partial z^2}, \quad \sigma_z = \frac{\partial^2 F}{\partial x^2}, \quad \tau = -\frac{\partial^2 F}{\partial x \partial z} \quad (2.2.3)$$

the equations governing the problem can be written as

$$\nabla^4 w = \frac{t}{D} \left(\frac{\partial^2 F}{\partial z^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial z} \frac{\partial^2 w}{\partial x \partial z} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial z^2} \right) \quad (2.2.4)$$

and

$$\nabla^4 F = E \left[\left(\frac{\partial^2 w}{\partial x \partial z} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial z^2} \right] \quad (2.2.5)$$

where ∇^2 is the partial differential operator

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \quad (2.2.6)$$

The equations were extended to include plates with small initial curvature by Marguerre (1938). Using these equations, the post-buckling behaviour of rectangular plates under edge compression has been investigated by several workers. For example, approximate solutions based on energy methods have been obtained by Timoshenko (1936) and Marguerre (1937). Levy (1942) has given an exact solution of von Karman's equation which employs a double Fourier series for each of three functions, deflection, stress and normal pressure. Levy's solution has been extended by Coan (1951), Yamaki (1959) and Walker (1969) to include plates which are subjected to non-uniform edge displacements.

Other methods of treating the non-linear analysis of plates include finite differences, dynamic relaxation, perturbation analysis. The most popular method today, however, seems to be the finite element method, due to its great flexibility. Several papers reviewing work in this area have been published. As an example a work by Crisfield (1973) can be mentioned.

Another model for the analysis of a plate in compression is described in a paper by Nylander (1972), where the behaviour of a compressed welded box-column is studied. The four sides of the column are assumed to have the same distribution of initial stresses and initial deflections. For reasons of symmetry, only one plate field is studied. The plate field is assumed to be represented by two systems. One of the systems consists of a plate which in the elastic range can be regarded as homogeneous and isotropic. The other system consists of a number of crossing bars which are joined

to the plate in certain points. The plate is assumed to transmit bending and torsional moments as well as shear forces perpendicular to its plane. The bars are assumed to transmit normal forces and to suffer the same bending deformations as the plate. The additional normal deflections are assumed to retain the same form as the initial deflections. The deflections of the plate are determined from the condition that the total change in potential energy is a minimum.

The influence of initial deflections on the post-buckling behaviour of thin-walled plates has been studied by many workers. Nylander (1951) deals with thin plates subjected to an initial deflection which is of the same order of magnitude as the thickness of the plate, and is affine to the additional deflection. In the papers by Coan (1951) and Yamaki (1959), simply supported rectangular plates with initial deflections in the form of sinusoidal waves are studied.

Experiments and calculations described in the literature show that the total load-carrying capacity of compressed plates is fairly independent of the magnitude of the initial deflections. Even in the case of a cylindrical initial deflection where the influence on the critical load may be fairly large, the maximum load is not greatly influenced.

The theoretical differential equation for large deflections has little application in practical design because of its complexity. Instead, the concept of "effective width" is often used, in which the real stress distribution over the entire plate width b is replaced by the edge stress σ_e over the effective width b_e according to Fig. 2.2.3. Denoting the average stress by σ_a we thus have

$$\sigma_a \cdot b = \sigma_e \cdot b_e \quad (2.2.7)$$

On the basis of extensive experiments, von Karman et al (1932) derived a theoretical expression for the ultimate compressive load of a simply supported plate. The corresponding effective width b_e was derived to

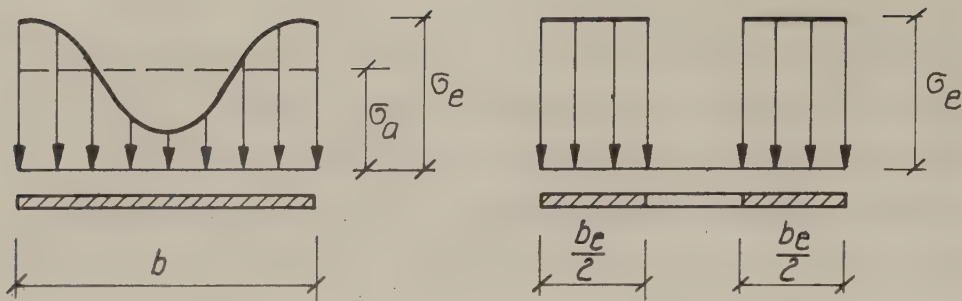


Fig. 2.2.3. a) Real stress distribution over the entire plate width.
 b) Assumed stress distribution when using the effective width concept.

$$b_e = \frac{2\pi}{\sqrt{12(1 - \nu^2)}} t \sqrt{\frac{E}{\sigma_y}} \quad (2.2.8)$$

where σ_y = yield stress.

If the buckling coefficient k_σ is assigned the value of four (i.e. hinged boundaries), Eqs. (2.2.2) and (2.2.8) give

$$b_e = b \sqrt{\frac{\sigma_{cr}}{\sigma_y}} \quad (2.2.9)$$

On the basis of experimental evidence, Winter (1950) has proposed that a correction term be introduced into Eq. (2.2.9), which gives

$$b_e = b \sqrt{\frac{\sigma_{cr}}{\sigma_y}} (1 - 0.25 \sqrt{\frac{\sigma_{cr}}{\sigma_y}}) \quad (2.2.10)$$

In the literature, many other expressions for the effective width are proposed. Thus Kenedi (1960) proposes

$$b_e = 0.65b \left(\frac{\sigma_{cr}}{\sigma_y} \right)^{0.333} \quad (2.2.11)$$

while Gerard (1957) proposes

$$b_e = 0.823b \left(\frac{\sigma_{cr}}{\sigma_y} \right)^{0.425} \quad (2.2.12)$$

For a more detailed description of the post-buckling behaviour of a rectangular plate in compression, the reader is recommended to study Edlund (1973).

2.3 Rectangular flat plate subjected to shear.

2.3.1 Critical load.

The critical stress τ_{cr} of a rectangular flat plate loaded in pure shear can be derived by means of Eq. (2.2.1) to

$$\tau_{cr} = k_{\tau} \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (2.3.1)$$

Depending on the boundary conditions of the plate, the buckling coefficient k_{τ} can be approximated by the expressions

$$k_{\tau} = 5.35 + 4.00 \left(\frac{b}{a}\right)^2 \quad (\text{simply supported edges}) \quad (2.3.2)$$

or

$$k_{\tau} = 8.98 + 5.60 \left(\frac{b}{a}\right)^2 \quad (\text{built-in edges}) \quad (2.3.3)$$

where a = plate length.

The wave length of the buckles ($2b\lambda$) for the two cases are given by

$$\lambda = 1.33 \quad (\text{simply supported edges})$$

or

$$\lambda = 0.80 \quad (\text{built-in edges})$$

Mixed boundary conditions are treated by Kollbrunner (1958) and Pflüger (1964).

The influence of the initial deflection on the critical load is very similar to that of the compressed plate described in Section 2.2.1.

2.3.2 Post-buckling behaviour. Maximum load.

The post-critical range of flat plates loaded in shear has been known for a long time. Thus Wagner (1929) developed the theory of "pure diagonal tension", where the principal compressive stress is

assumed to be equal to zero. A condition for the applicability of the theory is that the surrounding structure can sustain the principal tensile stresses. The theory is appropriate for the calculation of the maximum load of certain parts of aeroplane structures i.e. for extremely thin plates where the critical load is only a fraction of the maximum load.

Kuhn et al (1952) developed a theory for so-called "incomplete diagonal tension" in which the principal compressive stress is assumed to be larger than the critical shear stress and to be dependent on the extent to which the post-critical range is utilized. The theory was developed for the design of aeroplane structures.

In the theory of "complete diagonal tension", the principal compressive stress is assumed to be equal to the critical shear stress, as in the work of Schapitz (1951).

A fundamental analysis of girders with slender webs subjected to shear forces was carried out by Bergman (1948). The calculations were based on von Karman's equations for plates with large deflections. Bergman showed that the post-buckling behaviour of infinitely long plates was greatly affected by the boundary conditions. Bergman's opinion was that the final limiting load had been attained when the deflections caused such large bending stresses that yielding occurred. It must be pointed out that this limiting load does not correspond to collapse but only to a limit where deflections start to develop rapidly.

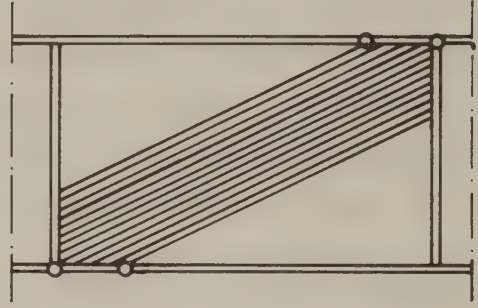
Basler (1961) presented an ultimate strength method for the design of webs provided with vertical stiffeners where the flanges of the girder were considered to be very flexible. Basler assumed that due to the flexibility of the flanges the tension fields adopt the form shown in Fig. 2.3.1 and that failure occurs when the shaded area yields. The derivation of the formulae for design calculation purposes, however, includes a calculation error which was pointed out by Fujii (1968).

A number of authors have proposed different improvements of Basler's method. As representatives of these, Rockey and Skaloud

(1968) and (1972) can be mentioned. These authors established that failure occurs when the girder develops a collapse mechanism according to Fig. 2.3.1 constituting yielding throughout the diagonal strip together with the development of plastic hinges in the flanges. They also noted that the position of these plastic hinges varies with the flexibility of the flanges.



Basler



Rockey and Skaloud

Fig. 2.3.1. Different assumptions concerning the tension field.

By means of calculations with a bar model, Höglund (1971) shows how the maximum load can be approximately determined for a long girder where the flanges are free to approach each other according to the theory for "complete diagonal tension". Some of the results applicable to practical design are given in Section 4.1.2.

2.4 Pure bending of a box beam section.

2.4.1 Critical load.

In structures composed of plate elements it is often satisfactory to calculate the critical stress of each plate with the assumption of hinged longitudinal edges. The lowest critical stress thus obtained will then give a critical load for the composite structure that is always lower than or equal to the real critical load.

Using the Cross method of moment distribution developed for the analysis of frame structures, Lundquist et al (1943) developed a method for calculating the critical load of an infinitely long plate under a longitudinal load. This method has been used in connection with box beams subjected to pure bending by Schuette et al (1947) and Eggwertz (1950).

The result of a rigorous calculation of the critical load for a box beam in bending is shown in Fig. 2.4.1. The critical load σ_{cr} is given in the form

$$\sigma_{cr} = k \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (2.4.1)$$

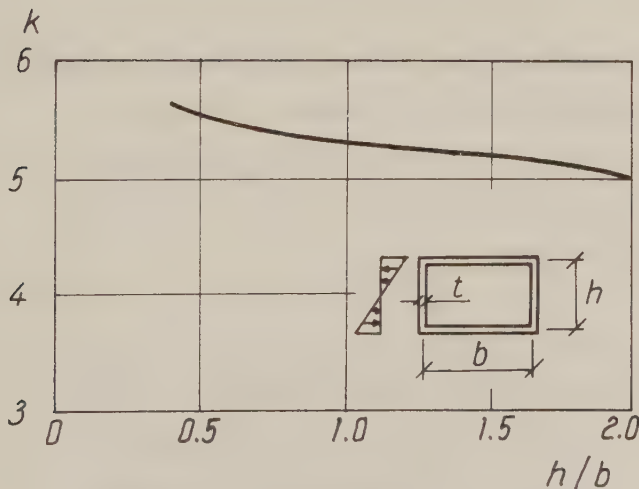


Fig. 2.4.1. Diagram for the determination of the critical load of a box beam subjected to bending.

2.4.2 Post-buckling behaviour. Maximum load.

A comparatively rigorous analysis of thin-walled box beams subjected to a pure bending moment is presented by Graves Smith (1972). The author has analysed the elasto-plastic behaviour of a square-section beam by minimizing the strain energy under a consideration of compatible displacements. The manner in which the beam develops plastic regions is demonstrated. Collapse is not considered because collapse occurs as a sudden local folding of the structure and an analysis would require the initiation of additional modes of distortion. The author points out that a complete theory of the collapse of a thin-walled beam will involve the construction of the rigid-plastic unloading curves corresponding to possible folding collapse mechanisms. The intersection of these curves with the present loading curves will then give upper limits for the collapse moment of the beam.

Klöppel et al (1971) gives both theoretical and experimental results concerning centrically and eccentrically compressed thin-walled box-shaped steel columns. The theoretical investigation is based on the non-linear plate theory where both the interaction between different plate elements and the influence of initial normal deflections and residual welding stresses have been considered. The material is assumed to have linear elastic properties. For the solution of the problem an energy method is used.

During the last ten years in Sweden, some investigations have been carried out on thin-walled trapezoidal steel sheets in bending. Since the local post-buckling behaviour of these is very reminiscent of that of box beams subjected to bending, a few of the investigations will be referred to here.

In a publication by Larsson (1972), results are given of a test series of 21 trapezoidal steel sheets in bending. One of the results of the investigation was that the AISI-Specification ought to be complemented with regard to thin-walled structures with very slender webs. This was particularly important for the determination of the maximum load, where the safety factor was found to be insufficient.

A test series including 28 trapezoidal steel sheets in bending is presented by Thomasson (1973). In this investigation also, the AISI-Specification was found to overestimate the maximum load for slender webs.

In a report by Bergfelt et al (1975), the influence of local web and flange buckling on the behaviour of trapezoidal thin-walled sheets and other light gauge beams subjected to bending is discussed. The report refers to the test series carried out by Larsson (1972) and Thomasson (1973). Some of the tests described in the present work (originally outlined in a paper by Källsner (1973)) are also mentioned. Different methods of determining the maximum bending moment are studied. Some stress diagrams of the webs used for this purpose are given in Fig. 2.4.2.

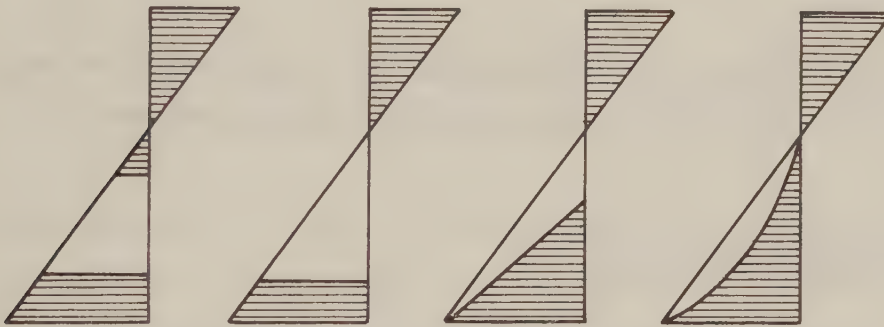


Fig. 2.4.2. Different stress distributions in the webs used for the determination of the ultimate bending moment (Bergfelt et al (1975)).

3 PURE BENDING OF BOX BEAMS

3.1 Theory

3.1.1 Introduction.

A bar model for the post-buckling analysis of plates in compression and of box beams subjected to bending has been developed. The main purpose has been to create a model which in a direct and easily comprehensible way makes it possible to study deformations and stresses. For the analysis of actions and displacements in the bar system, a computer programme based on the stiffness method has been written. The calculation agrees broadly with the theory given in Willems et al (1968). To limit the scope of this report, the author assumes that the reader is familiar with the principles of the stiffness method.

Under certain conditions, the bar model can also be used to analyse other thin-walled structures subjected to compression and bending moments.

3.1.2 Brief summary of theory.

The bar model, designed for the analysis of structures such as plates loaded in compression and box beams subjected to bending, is based on three different types of members, namely:

- 1) The continuous beam member which can only resist axial forces and bending moments (more precisely described in Section 3.1.4).
- 2) The shear member which can only resist shear forces (Section 3.1.5).
- 3) The torque member which can only resist torque (Section 3.1.6).

The members are only connected in particular joints. For the determination of member actions and displacements the stiffness method is used.

One of the problems in the analysis of plate buckling in the post-critical range is that the displacements change the geometry of the structure, implying a variation in the stiffness matrix. The axial forces will likewise have an influence on the stiffness matrix. In this case the problem has been solved by gradually increasing the displacements in small increments, considering the stiffness matrix as constant at every step. The accuracy of the method is dependent on the magnitude of the assumed displacement increment and normally requires a rather large number of calculation cycles. The course of the calculation can be summed up as follows:

The stiffness matrices of the different members $(S_m)_i$ are formulated in their respective member-oriented coordinate systems x_m, y_m, z_m by the relation:

$$\{A_m\}_i = (S_m)_i \{D_m\}_i \quad (3.1.1)$$

where $\{A_m\}_i$ = member action matrix

$\{D_m\}_i$ = member displacement matrix

The subscript m refers to the member-oriented coordinate system while the subscript i refers to the number of the member.

To transform the member stiffness matrices $(S_m)_i$ to the structure-oriented coordinate system x, y, z , a general three-dimensional transformation matrix $(R)_i$ is introduced. The matrix $(R)_i$ can be expressed as

$$(R)_i = \begin{bmatrix} \ell_1 & m_1 & n_1 \\ \ell_2 & m_2 & n_2 \\ \ell_3 & m_3 & n_3 \end{bmatrix} \quad (3.1.2)$$

where ℓ_1, m_1 and n_1 are the direction cosines for the x_m -axis, ℓ_2, m_2 and n_2 are the direction cosines for the y_m -axis, and ℓ_3, m_3 and n_3 are direction cosines for the z -axis, cf. Fig. 3.1.1. These direction cosines may be determined geometrically from the physical orientation of the member. Depending on the particular member type

considered, a complete angular transformation matrix $(Q)_i$ is built up from the matrix $(R)_i$. Thus, the relation between the member displacement matrix given in the two coordinate systems can be expressed as

$$\{D_m\}_i = (Q)_i \{D_s\}_i \quad (3.1.3)$$

where the subscript s refers to the structure-oriented coordinate system.

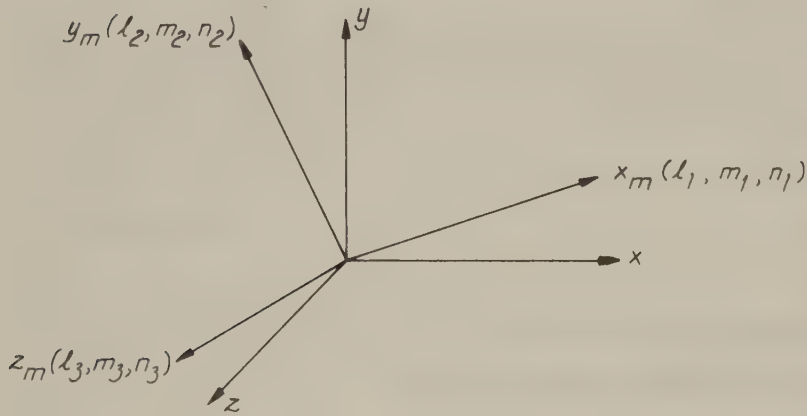


Fig. 3.1.1. Rotation of coordinate axes.

A similar expression may be written for the corresponding actions at the ends of the member, which gives

$$\{A_m\}_i = (Q)_i \{A_s\}_i \quad (3.1.4)$$

The member stiffness matrix $(S_s)_i$ defined by the relation

$$\{A_s\}_i = (S_s)_i \{D_s\}_i \quad (3.1.5)$$

can now be derived from Eqs. (3.1.1), (3.1.3), (3.1.4) and (3.1.5) and expressed in the form:

$$(S_s)_i = (Q)_i^{-1} (S_m)_i (Q)_i \quad (3.1.6)$$

We may now form the joint stiffness matrix for the complete system (S_s) by simply superposing the stiffness influence coefficients

associated with each deformation at all the joints. The influence of previous loading history is included in the joint stiffness matrix. The relation between joint actions $\{A_s\}$ and joint displacements $\{D_s\}$ can thus be written:

$$\{S_s\} \{D_s\} = \{A_s\} \quad (3.1.7)$$

which is the fundamental equation of the problem. Certain elements in the matrices $\{A_s\}$ and $\{D_s\}$ are known from the beginning and can be obtained from the boundary conditions of the problem. The method used for solving the equation system is described in Section 3.1.11.

It is often very important to obtain the member actions. They are given by the expression:

$$\{A_m\}_i = \{S_m\}_i \{D_m\}_i = \{S_m\}_i (Q)_i \{D_s\}_i \quad (3.1.8)$$

The calculated actions and displacements are added to the corresponding quantities obtained during previous calculation cycles.

3.1.3 General assumptions.

The bar model is based on the following fundamental assumptions:

- 1) The material has linear elastic properties.
- 2) The structure is built up by three types of member:
 - a) Continuous beam member. (Section 3.1.4)
 - b) Shear member. (Section 3.1.5)
 - c) Torque member. (Section 3.1.6)
 Assumptions in connection with the different members are defined in Sections 3.1.4-3.1.6.
- 3) The members are connected only in certain joints. In all other respects the members are completely independent of each other.
- 4) Large deflections in the whole system, i.e. between different members, are considered. Large deflections within a member are neglected.

- 5) Shear deformations due to shear forces acting perpendicular to the deflection surface are neglected.

3.1.4 Continuous beam member:

The continuous beam member has been developed by replacing a plate strip by a bar which has the same axial rigidity as the plate strip and the same flexural rigidity as the plate displays in the weak direction, see Fig. 3.1.2. In addition to the basic assumptions given in Section 3.1.3, the following assumptions are made:

- 1) When the axial rigidity of the bar is calculated Poisson's ratio ν is assumed to be zero.
- 2) The flexural rigidity of the bar is the same as the rigidity of a plate strip subjected to bending to a cylindrical surface i.e. a fictitious modulus of elasticity

$$E_1 = \frac{E}{1 - \nu^2} \text{ is assumed in bending.}$$

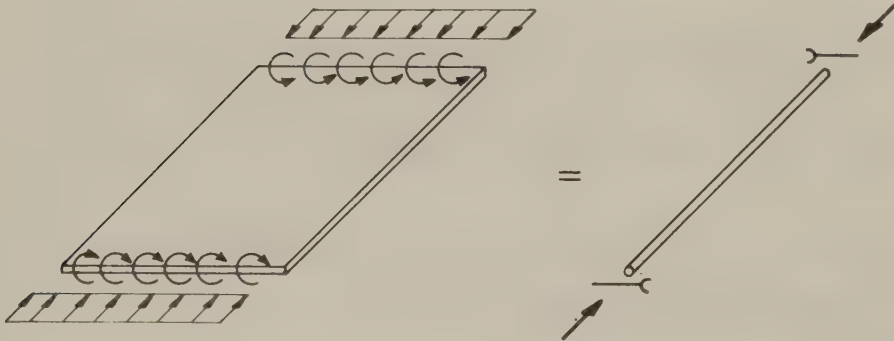


Fig. 3.1.2. Continuous beam member.

With these assumptions, the member stiffness matrix $(S_m)_i$, described in the member-oriented coordinate system x_m, y_m, z_m and with the elements ranged according to Fig. 3.1.3, can be expressed as:

$$[S_m]_i = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} & 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & \frac{6EI_z}{L^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} \\ -\frac{EA}{L} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{L} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} & 0 & \frac{12EI_z}{L^3} & 0 & 0 & 0 & -\frac{6EI_z}{L^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{2EI_z}{L} & 0 & -\frac{6EI_z}{L^2} & 0 & 0 & 0 & \frac{4EI_z}{L} \end{bmatrix}$$

(3.1.9)

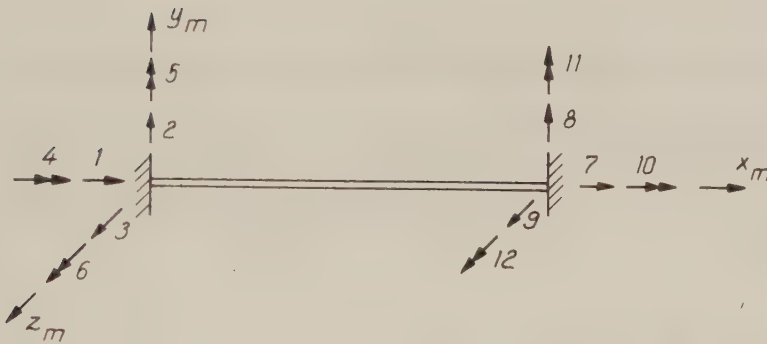


Fig. 3.1.3. Assumed local coordinate system and degrees of freedom for the continuous beam member.

In this case no consideration has been given to the elongation of the member arising as a result of a displacement of the ends in the y_m -direction. This simplification is justified as long as the displacements can be regarded as small in relation to the member length. The error can be calculated from the theorem of Pythagoras to be (see Fig. 3.1.4):

$$\Delta L = \sqrt{L^2 + \Delta w^2} - L \approx \frac{1}{2L} \Delta w^2 \quad (3.1.10)$$

The element EA/L in the member stiffness matrix would thus be replaced by the expression

$$\frac{EA}{L} \left(1 - \frac{1}{2L} \frac{\Delta w^2}{\Delta x} \right)$$

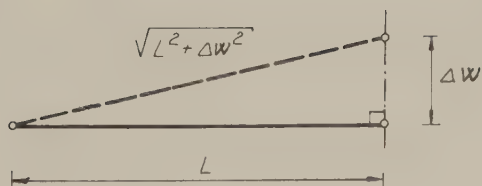


Fig. 3.1.4. Elongation of the continuous beam member resulting from a displacement of one of the ends in the y_m -direction.

This would require an iterative method, but if the value of Δw is kept small, the second term in the parentheses can be neglected (see Section 3.1.10).

In the derivation of the member stiffness matrix $(S_m)_i$ given by Eq. (3.1.9), the influence of normal forces from previous loading history has been omitted.

These forces can be derived by studying the bar described in the structure-oriented coordinate system of Fig. 3.1.5. With the notations given in the figure, the additional contribution to the forces in the y -direction resulting from a displacement increment can be calculated to be:

$$\Delta V = \Delta N \frac{w}{L} + N \frac{\Delta w}{L} + \Delta N \frac{\Delta w}{L} \quad (3.1.11)$$

The first term on the right hand side is the contribution due to the change in the normal force and is included in the member stiffness matrix given in Eq. (3.1.9).

The second term is the contribution due to displacement in the y -direction, which can be expressed by the following member stiffness matrix

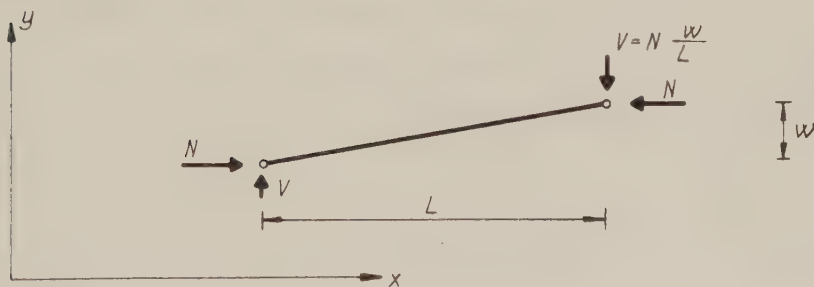
$$[S_5]_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{N}{L} & 0 & 0 & 0 & 0 & 0 & \frac{N}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{N}{L} & 0 & 0 & 0 & 0 & 0 & -\frac{N}{L} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(3.1.12)

The matrix $(S_5)_i$ can be directly used when the plate strip is located in the xz -plane. In other cases the matrix must be transformed.

The third term in Eq. (3.1.11) has been neglected. This is discussed further in Section 3.1.10.

a)



b)

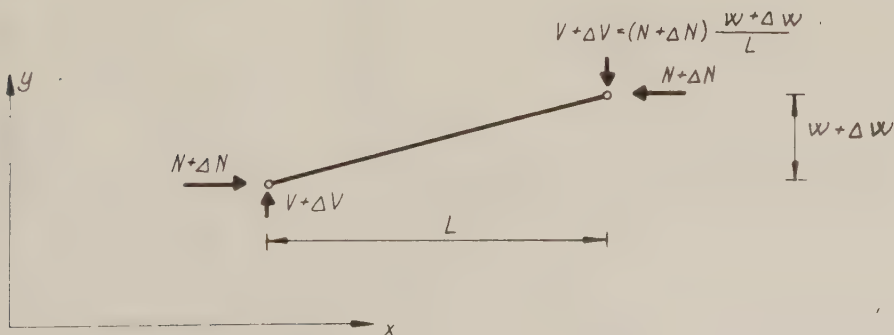


Fig. 3.1.5. Forces acting on a compressed bar after two successive displacement increments.

The transformation matrix $(Q)_i$ for the transformation of the member stiffness matrix $(S_m)_i$ to the structure-oriented coordinate system can be expressed as:

$$(Q)_i = \begin{bmatrix} (R)_i & (0) & (0) & (0) \\ (0) & (R)_i & (0) & (0) \\ (0) & (0) & (R)_i & (0) \\ (0) & (0) & (0) & (R)_i \end{bmatrix} \quad (3.1.13)$$

where $(R)_i$ is given by Eq. (3.1.2).

3.1.5 Shear member.

To introduce the shear stiffness of a plate field into the calculation model, a shear member has been developed. The member is characterized by a crossing pair of bars giving the same shear displacements as the real plate when subjected to a constant shear stress τ along the boundaries, see Fig. 3.1.6. The distributed shear forces in this case have been replaced by point loads acting at the corners of the plate field. To prevent loading of the diagonal bars when the plate field is subjected to normal stresses, it is assumed that forces can only arise in the diagonal bars when the bars attain different lengths. Further it is assumed that the forces in the diagonals are always of the same magnitude.

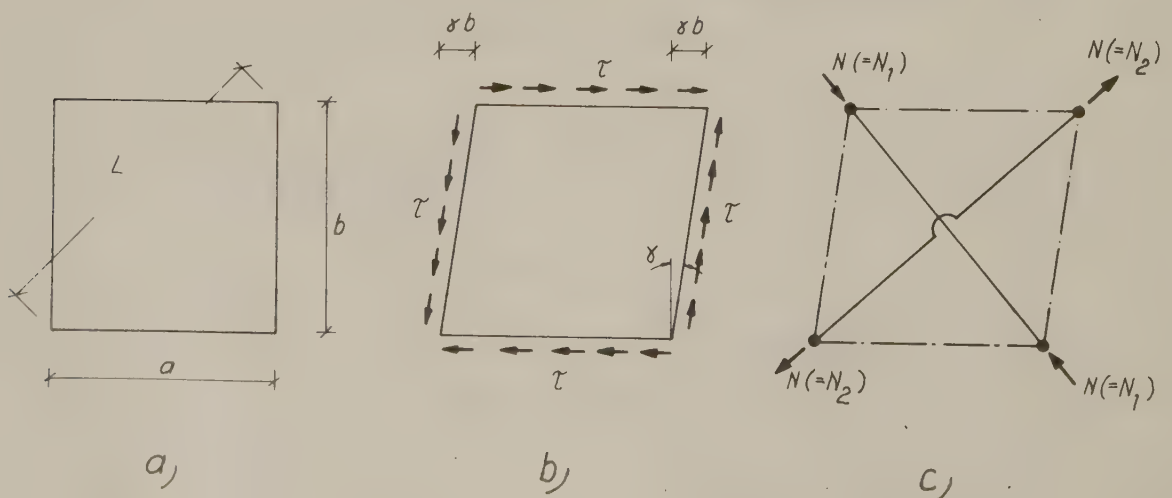


Fig. 3.1.6. a) Assumed geometry of the plate.
b) Shear forces acting along the edges.
c) Equivalent point forces acting on the shear member.

The expression to describe the fictitious axial rigidity of the bars can be derived as below:

From the condition that the external forces are equal we obtain (cf. Fig. 3.1.6)

$$N = \tau \frac{\sqrt{a^2 + b^2}}{2} t \quad (3.1.14)$$

The axial deformation ΔL of the diagonals is obtained from the theorem of Pythagoras:

$$\Delta L = \sqrt{(a + \gamma b)^2 + b^2} - \sqrt{a^2 + b^2} \approx \frac{ab\gamma}{\sqrt{a^2 + b^2}} \quad (3.1.15)$$

The definition of shear modulus gives

$$\tau = G\gamma \quad (3.1.16)$$

Substitution of the expressions from Eqs. (3.1.15) and (3.1.16) in Eq. (3.1.14) yields

$$N = \frac{a^2 + b^2}{2ab} Gt \Delta L \quad (3.1.17)$$

Eq. (3.1.17) is derived under the assumption that both diagonals have experienced an axial deformation ΔL . If the length of only one of the diagonals is altered a distance ΔL , the forces in the diagonals will be half as large. Thus the forces of the bars can be written as

$$N = \frac{EA}{L} \Delta L \quad (3.1.18)$$

where

$$\frac{EA}{L} = \frac{a^2 + b^2}{4ab} Gt \quad (3.1.19)$$

is a fictitious stiffness. The local member stiffness matrix $(S_m)_i$ expressed in the two member-oriented coordinate systems x_{m1}, y_{m1}, z_{m1} and x_{m2}, y_{m2}, z_{m2} with the axis defined and the elements ranged

where $N_1 = -N_2 = N$, see Fig. 3.1.6 c.

In the computer programme, the matrices (3.1.20) and (3.1.21) are combined into a total member stiffness matrix $(S_m)_i$.

The transformation matrix $(Q)_i$ can be expressed as

$$(Q)_i = \begin{bmatrix} (R_1)_i & (0) & (0) & (0) \\ (0) & (R_1)_i & (0) & (0) \\ (0) & (0) & (R_2)_i & (0) \\ (0) & (0) & (0) & (R_2)_i \end{bmatrix} \quad (3.1.22)$$

where $(R_1)_i$ and $(R_2)_i$ describe the relationship between the member-oriented coordinate systems and the structure-oriented system according to Eq. (3.1.2).

3.1.6 Torque member.

To consider the torsional rigidity of the plate, a torque member has been introduced into the calculation model. The torsional rigidity is essential for the determination of the local buckling load.

The torque member (Fig. 3.1.8) is characterized by four point loads of equal size acting at the corners of the member perpendicular to the plane of the plate. These point loads replace the torsional moments distributed along the boundaries. The relationship between the point load V and the deformation Δw can be derived in the following way:

According to plate theory, the distributed torsional moment can be written:

$$T_{xz} = T_{zx} = \frac{1}{6} Gt^3 \frac{\partial^2 w}{\partial x \partial z} \quad (3.1.23)$$

If the normal deflection of the plate member is given by the relation

$$w = \frac{xz}{ab} \Delta w \quad (3.1.24)$$

substitution of Eq. (3.1.24) in Eq. (3.1.23) will give

$$T_{xz} = T_{zx} = \frac{1}{6} G \frac{t^3}{ab} \Delta w \quad (3.1.25)$$

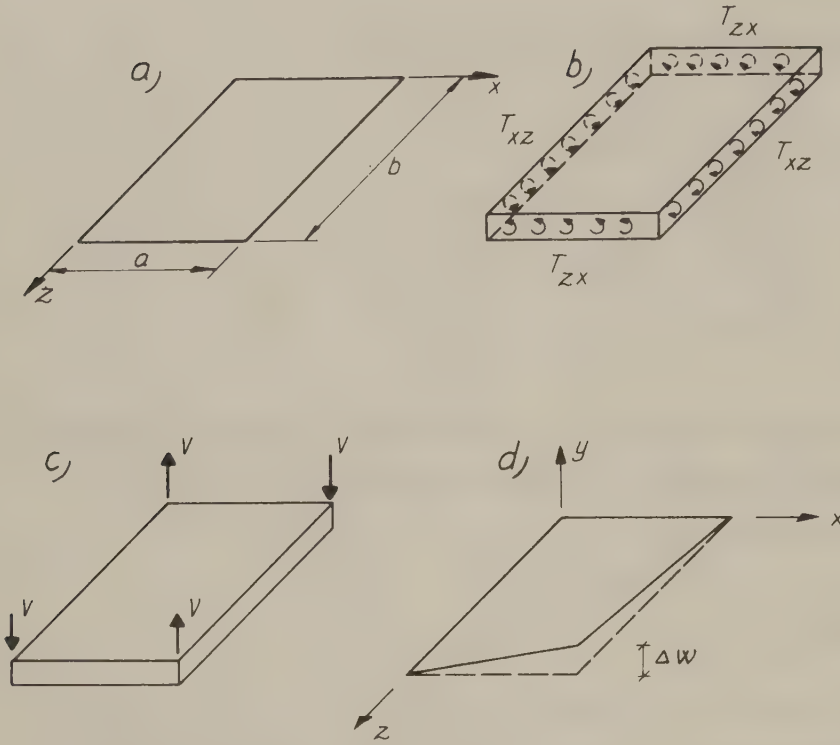


Fig. 3.1.8. a) Assumed geometry of the plate.
 b) A plate subjected to a torsional moment along its edges.
 c) Equivalent point forces acting on the torque member.
 d) Assumed deformation of the plate.

From equilibrium equations we obtain directly:

$$V = \frac{1}{3} G \frac{t^3}{ab} \Delta w \quad (3.1.26)$$

The member stiffness matrix expressed in the member-oriented coordinate system x_m, y_m, z_m and with the elements ranged as in Fig. 3.1.9 can now be written:

$$(S_m)_i = C \cdot \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (3.1.27)$$

where

$$C = \frac{1}{2} G \frac{t^3}{ab} \quad (3.1.28)$$

In the transformation of the member stiffness matrix $(S_m)_i$ to the structure-oriented coordinate system, only the force components perpendicular to the initially plane plate field have been included. The other components can to a very good approximation be neglected.

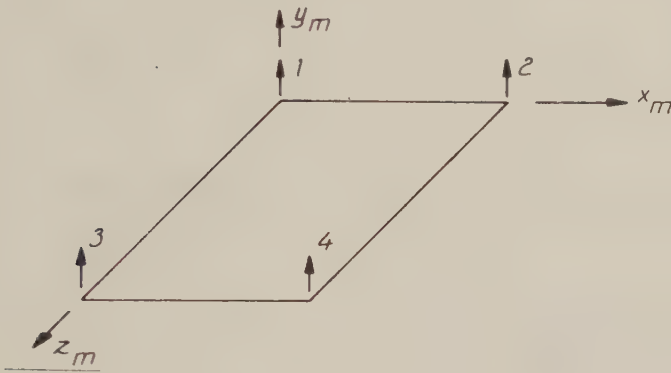


Fig. 3.1.9. Assumed local coordinate system and degrees of freedom for the torque member.

3.1.7 Solution from bar model versus solution from differential equation.

In this section, the solution obtained from the bar model is compared with the solution obtained from the fundamental differential equation describing the deflection surface of a plate at small deflections.

According to Timoshenko et al (1961), the principle course for the derivation of the differential equation is: Considering the force equilibrium of a plate segment in the xz -plane we obtain:

$$\frac{\partial^2 M_x}{\partial x^2} - 2 \frac{\partial^2 T_{xz}}{\partial x \partial z} + \frac{\partial^2 M_z}{\partial z^2} = - \left(q + N_x \frac{\partial^2 w}{\partial x^2} + N_z \frac{\partial^2 w}{\partial z^2} + 2N_{xz} \frac{\partial^2 w}{\partial x \partial z} \right) \quad (3.1.29)$$

where M_x and M_z are bending moments per unit length,

T_{xz} is the torsional moment per unit length,

N_x and N_z are normal forces per unit length,

N_{xz} is the membrane shearing force per unit length and

q is the load intensity.

Using deformation relations, the moments can be expressed as:

$$M_x = -D \left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial z^2} \right) \quad (3.1.30)$$

$$M_z = -D \left(\frac{\partial^2 w}{\partial z^2} + \nu \frac{\partial^2 w}{\partial x^2} \right) \quad (3.1.31)$$

$$T_{xz} = D(1 - \nu) \frac{\partial^2 w}{\partial x \partial z} \quad (3.1.32)$$

where $D = \frac{Et^3}{12(1 - \nu^2)}$ is the flexural rigidity.

Substituting these expressions in Eq. (3.1.29), we obtain the final equation:

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial z^2} + \frac{\partial^4 w}{\partial z^4} = \frac{1}{D} \left(q + N_x \frac{\partial^2 w}{\partial x^2} + N_z \frac{\partial^2 w}{\partial z^2} + 2N_{xz} \frac{\partial^2 w}{\partial x \partial z} \right) \quad (3.1.33)$$

Neglecting the influence of torsional rigidity, Eq (3.1.33) becomes:

$$\frac{\partial^4 w}{\partial x^4} + 2\nu \frac{\partial^4 w}{\partial x^2 \partial z^2} + \frac{\partial^4 w}{\partial z^4} = \frac{1}{D} \left(q + N_x \frac{\partial^2 w}{\partial x^2} + N_z \frac{\partial^2 w}{\partial z^2} + 2N_{xz} \frac{\partial^2 w}{\partial x \partial z} \right) \quad (3.1.34)$$

By a refinement of the bar net, the behaviour of the model can also be described by a differential equation. Eqs. (3.1.30) and (3.1.31) will be somewhat changed in this case on account of the assumption made with regard to Poisson's ratio ν in bending. Thus the equations will take the form

$$M_x = -D \frac{\partial^2 w}{\partial x^2} \quad (3.1.35)$$

$$M_z = -D \frac{\partial^2 w}{\partial z^2} \quad (3.1.36)$$

Substitution of Eqs. (3.1.32), (3.1.35) and (3.1.36) in Eq. (3.1.29) yields:

$$\begin{aligned} \frac{\partial^4 w}{\partial x^4} + 2(1 - \nu) \frac{\partial^4 w}{\partial x^2 \partial z^2} + \frac{\partial^4 w}{\partial z^4} = \frac{1}{D} \left(q + N_x \frac{\partial^2 w}{\partial x^2} + N_z \frac{\partial^2 w}{\partial z^2} + \right. \\ \left. + 2N_{xz} \frac{\partial^2 w}{\partial x \partial z} \right) \end{aligned} \quad (3.1.37)$$

If the influence of the torsional rigidity is neglected, Eq. (3.1.37) becomes:

$$\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial z^4} = \frac{1}{D} \left(q + N_x \frac{\partial^2 w}{\partial x^2} + N_z \frac{\partial^2 w}{\partial z^2} + 2N_{xz} \frac{\partial^2 w}{\partial x \partial z} \right) \quad (3.1.38)$$

We now wish to use the derived differential equations on a specific example, namely a simply supported flat plate, of square form, under the actions of compressive stresses in the x-direction. On the further assumption that the plate buckles into one sinusoidal half-wave in the directions of both the x-axis and the z-axis, we can derive an expression for the critical load from Eq. (3.1.33):

$$N_{cr} = 4.0 \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b} \right)^2 \quad (3.1.39)$$

If the influence of torsional rigidity is neglected (Eq. (3.1.34)), the corresponding critical load is

$$N_{cr} = 2.6 \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b} \right)^2 \quad (3.1.40)$$

The critical loads using the differential equations of the model (Eqs. (3.1.37), (3.1.38)) are obtained in the same manner:

$$N_{cr} = 3.4 \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (3.1.41)$$

and

$$N_{cr} = 2.0 \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (3.1.42)$$

From Eqs. (3.1.39) - (3.1.42), we conclude that the model with an infinite number of bars in this example gives a buckling load which is 15 per cent too low compared with the exact theory. The cause can be related to an underestimation of the bending stiffness, due to the assumptions made concerning Poisson's ratio ν .

3.1.8 Model for the analysis of a simply supported plate in compression.

A box beam subjected to pure bending will mainly display a regular deflection pattern in the post-critical range (Fig. 3.1.10 a). If the initial deflections are small, the wavelength of the buckles will turn out to be almost constant along the beam.

To study the behaviour of the compressed flange separately, the plate field ACOM (Fig. 3.1.10 b) has been cut out. It is assumed that the plate is simply supported with regard to movements normal to the surface. The axial load is applied by means of rigid spacers. No membrane shearing forces can be transmitted between the plate and the spacers.

For reasons of symmetry, only one fourth of the plate needs to be studied. In Fig. 3.1.10 c, the homogeneous plate field HION has been replaced by a bar system. The boundary conditions along the edges HI and NH are easily obtained from considerations of symmetry.

The choice of axial displacement increment δ of the plate field HION is limited by the additional deflection Δw accepted. In Section 3.1.10, this is discussed in more detail.

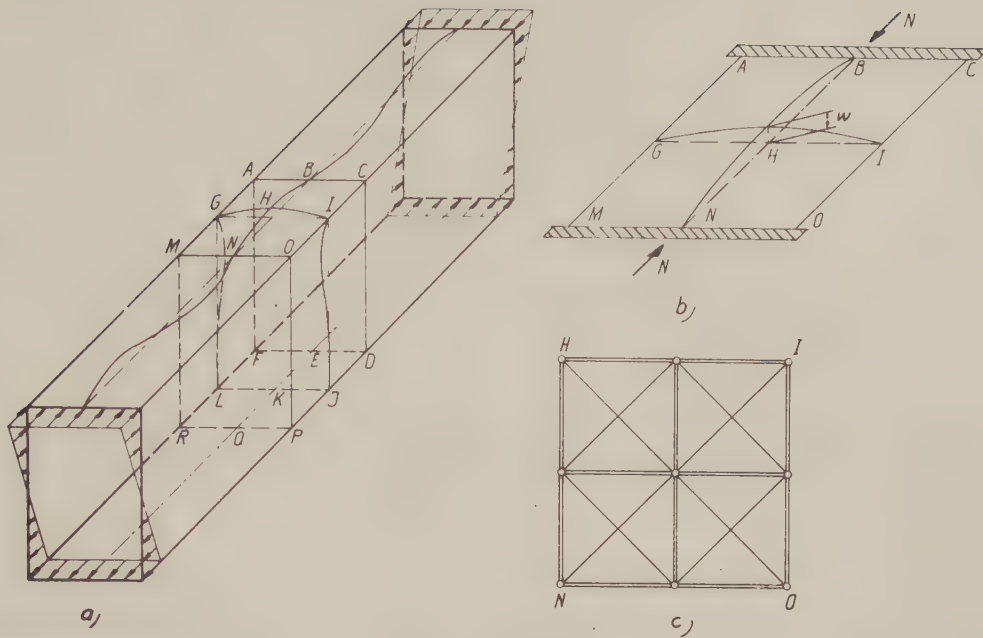


Fig. 3.1.10 a) Deflection pattern in the post-critical range for a box beam subjected to pure bending.
 b) A section cut out along the edges of the compression flange subjected to axial forces N .
 c) Symmetrical part of the compression flange where the plate has been replaced by a bar system.

3.1.9 Model for the analysis of a box beam under pure bending.

The calculation model for the analysis of a simply supported plate in compression, given in Section 3.1.8, has been developed to include a box beam under pure bending.

Let us consider a beam element ABCDEF-MNOPQR (Fig. 3.1.11 a) where the length of the element is half the wavelength of the regular deflection pattern. The bending moment is assumed to be applied by rigid plates as shown in Fig. 3.1.11 b. The boundaries ACDF and MOPR are regarded as being simply supported with respect to movements normal to the surface. Further, the boundaries are assumed not to transmit any membrane shearing forces. In the calculations, some possibilities of displacement must be prevented in order to avoid unstable equilibrium.

As for the simply supported plate (cf. Section 3.1.8) only one fourth of the beam element needs to be studied (Fig. 3.1.11 c). The boundary conditions of the edges NH, HI, IJ, JK and KQ are obtained

from considerations of symmetry.

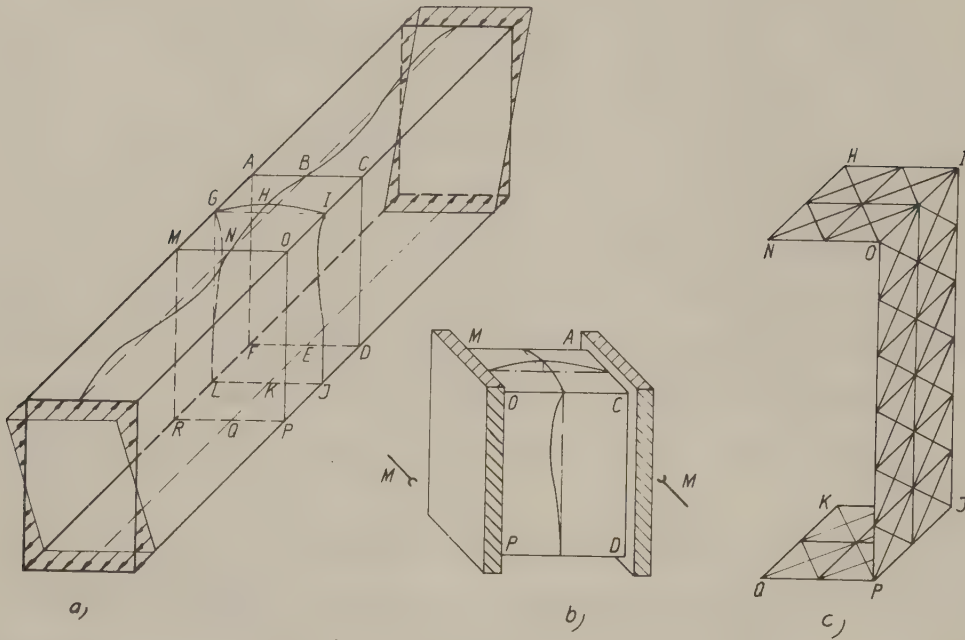


Fig. 3.1.11 a) Deflection pattern in the post-critical range for a box beam subjected to pure bending.
 b) Equivalent loading case.
 c) Symmetrical part of the box beam where the plate fields have been replaced by a bar system.

Since the position of the neutral axis is unknown and also varies with changes in the load level, actions and deformations cannot be determined directly. In solving this problem, we have chosen to study the effect of two elementary displacement cases, namely one with an axial reduction in length $\delta_1 = 1$ of the total beam element and one with a rotation of the beam element defined by $\delta_2 = 1$ according to Fig. 3.1.12. These two elementary displacement cases give rise to additional bending moments, normal forces and deflections amounting to ΔN_1 , ΔM_1 and Δw_1 and ΔN_2 , ΔM_2 and Δw_2 respectively. From the conditions that there is no resulting normal force acting along the beam and that the additional deflection is limited to Δw , we obtain:

$$\Delta N = c_1 \Delta N_1 + c_2 \Delta N_2 = 0 \quad (3.1.43)$$

$$\Delta w = c_1 \Delta w_1 + c_2 \Delta w_2 \quad (3.1.44)$$

where c_1 and c_2 are constants. This method of solution is allowed since the stiffness matrix of the structure can be regarded as constant during each displacement increment. Reduction of Eqs. (3.1.43) and (3.1.44) gives

$$c_2 = \frac{\Delta W}{\Delta W_2 - \frac{\Delta N_2}{\Delta N_1} \Delta W_1} \quad (3.1.45)$$

$$c_1 = -c_2 \frac{\Delta N_2}{\Delta N_1} \quad (3.1.46)$$

The constants c_1 and c_2 now can be used to determine the additional bending moment and all the actions and displacements:

$$\left. \begin{aligned} \Delta M &= c_1 \Delta M_1 + c_2 \Delta M_2 \\ \{A_s\} &= c_1 \{A_{s1}\} + c_2 \{A_{s2}\} \\ \{D_s\} &= c_1 \{D_{s1}\} + c_2 \{D_{s2}\} \end{aligned} \right\} \quad (3.1.47)$$

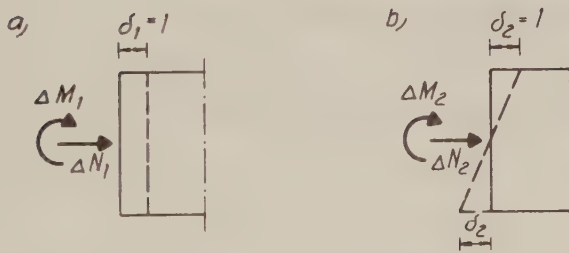


Fig. 3.1.12 a) Elementary case 1.
b) Elementary case 2.

3.1.10 Choice of displacement increment.

In the derivation of the member stiffness matrices it was found that there are two factors having a non-linear influence on the joint stiffness matrix (S_s) namely:

- 1) The change of normal forces in the continuous beam member and shear member.

- 2) The changed geometry of the bar system in the form of normal deflections.

Considering the normal forces in the continuous beam member given in Section 3.1.4, we obtained Eq. (3.1.11) where the last term was neglected. If the normal force N is equal to zero, i.e. in the first displacement increment (and no initial stresses), Eq. (3.1.11) can be written as

$$\Delta V = \Delta N \frac{w}{L} \left(1 + \frac{\Delta w}{w}\right) \quad (3.1.48)$$

Assuming that both N and w are increasing functions, the relative error in later displacement increments will be less than $\Delta w/w$ in Eq. (3.1.11).

The elongation of the continuous beam member caused by a displacement normal to the deflection surface was given in Eq. (3.1.10).

In the computer programme, the displacement increment is controlled by the additional relative deflection $\Delta w/w$ as well as by the restriction of a limited additional deflection $\Delta w_{\max}$.

3.1.11 Solution of the equation system.

In Section 3.1.2, the fundamental equation of the problem was derived:

$$\{S_s\} \{D_s\} = \{A_s\}$$

In the solution of large equation systems by a computer the joint stiffness matrix $\{S_s\}$ requires an extremely large core memory if all elements are to be saved. Since most elements are zeros and the matrix is symmetrical, only a lower band of the matrix has to be saved (Fig. 3.1.13).

In solving the equation system, Cholesky's method with triangulation has been used.

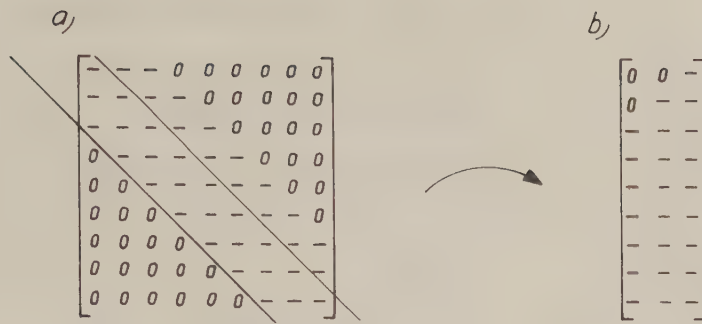


Fig. 3.1.13 a) Complete stiffness matrix.
b) Reduced stiffness matrix.

3.1.12 Computer programmes.

In order to analyse the bar models, two computer programmes have been developed. One of them is designed for the analysis of a simply supported plate in compression (cf. Section 3.1.8). The other is designed for the analysis of a box beam under pure bending (cf. Section 3.1.9). In Fig. 3.1.14, a flow chart of the computer programmes is shown. The programmes are built up in an identical manner and the main difference is clear from the flow chart.

The computer programmes can be directly applied to beams with other cross-sectional shapes and other boundary conditions. Some restrictions must, however, be observed. Further, the programmes can, with a negligible amount of work, be extended to include beam sections subjected to simultaneous compression and bending.

The programmes are written in Fortran. A copy of the programmes can be obtained from the author.

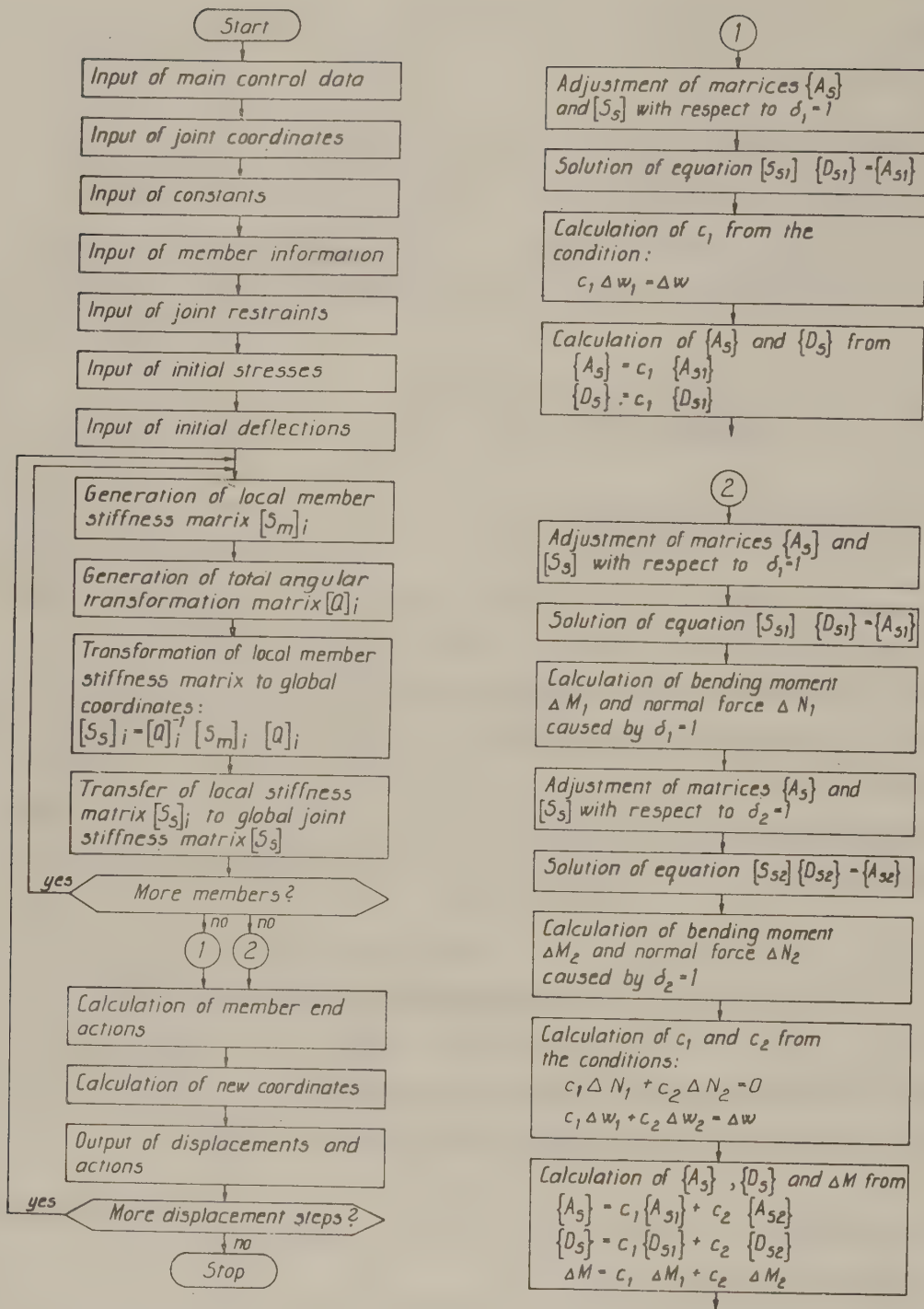


Fig. 3.1.14. Flow chart of the computer programmes.

- ① = Simply supported plate in compression.
 ② = Pure bending of box beam.

3.1.13 Numerical examples.

General.

Geometrical quantities used in this section are defined in Fig. 3.1.15. Unless otherwise stated, the following values are assumed

for the geometry and material properties:

$$a = 200 \text{ mm}$$

$$b = 200 \text{ mm}$$

$$\lambda = a/b = 1$$

$$h = 300 \text{ mm}$$

$$t = 1.5 \text{ mm}$$

$$E = 206\,000 \text{ N/mm}^2$$

$$\nu = 0.3$$

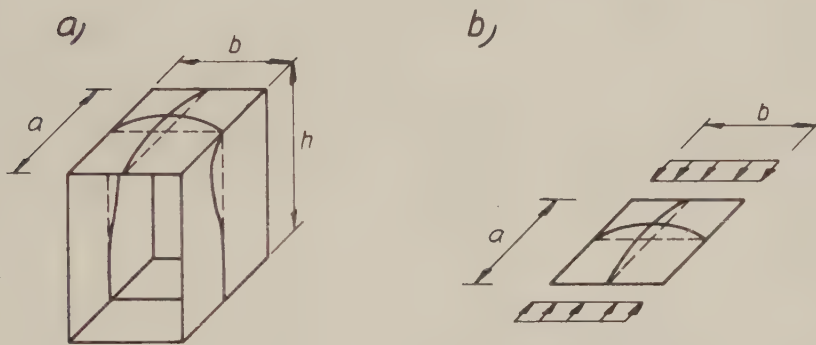


Fig. 3.1.15. Geometry of bar models used for analysis of
 a) box beam subjected to pure bending.
 b) simply supported plate subjected to uniaxial compression.

The increase in normal deflections has been limited according to the guiding principles presented in Section 3.1.10. Unless otherwise stated, the following values are applicable:

$$\Delta w/w = 0.10$$

$$\Delta w_{\max} = 0.1 \text{ mm}$$

To describe the initial deflections of the simply supported plates, a plate which is initially sinusoidal in both longitudinal and transverse directions has been assumed.

Regarding the compressed flange of the box beams, the same initial deflection surface as for the simply supported plate has been assumed. In all other cases, the initial deflections of the box beams have been assumed to be zero.

Description of different bar systems.

To analyse the behaviour of a simply supported plate under compression, four different bar systems have been studied. In Fig. 3.1.16 the bar systems are shown together with the assumed bar areas in the longitudinal and transverse direction. For the sake of simplicity, only one fourth of the bars are displayed i.e. only the region of the plates which is used in the computer calculations. The areas A_x and A_z refer to the total cross-sectional areas of the plates in the longitudinal and transverse directions respectively.

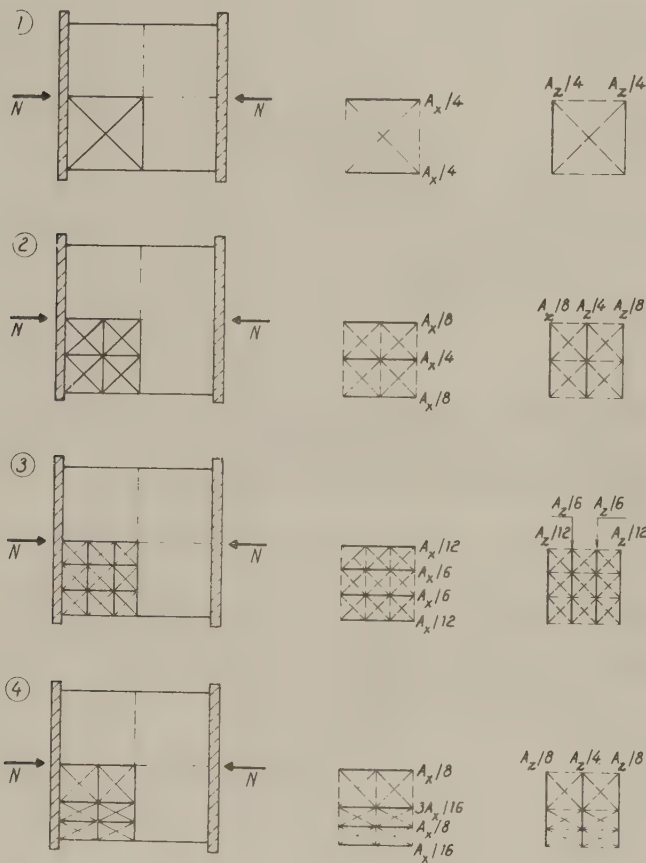


Fig. 3.1.16. Different bar systems to describe the behaviour of a simply supported plate in compression. A_x and A_z indicate the total cross-sectional areas in the longitudinal and transverse directions.

Bar system 1 represents the simplest imaginable model that can be used. Bar systems 2 and 3 have been considered in order to study the influence of bar proximity on the behaviour of the compressed plate. To reduce the number of degrees of freedom and yet obtain reasonable results, a bar system with an asymmetric structure has

been developed, see bar system 4.

To analyse the behaviour of a box-beam section under pure bending, two bar systems have been studied. The structures of the bar systems are clear from Fig. 3.1.17. The simplest of the two systems corresponds to the bar system 1 previously described and is called 1 B. In the same way, bar system 4 B is a further development of bar system 4.

In the construction of the bar systems 1 B and 4 B, the stiffness of the webs will be too high because of the procedure of dividing the area of a plate strip into two equivalent point areas located at the edges of the plate strip. For a height-to-width ratio of $h/b = 1.5$, bar system 1 B will give a moment of inertia that is 7.4 % too high. The corresponding value for bar system 4 B is 1.6 %.

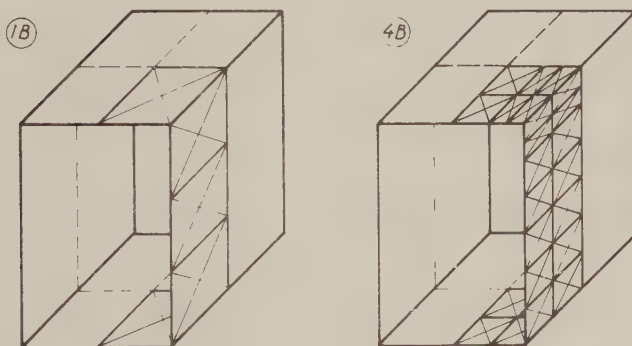


Fig 3.1.17. Different bar systems to describe the behaviour of a box beam subjected to pure bending.

Comparison of different bar systems.

In Fig. 3.1.18 the normal deflection of the central part of the plate is shown for the bar systems 1-4.

Regarding the bar systems 1-3 in Fig. 3.1.18, it is obvious that a further refinement of the bar net will not result in any great improvement in the determination of the buckling load. This can also be seen by comparing the buckling coefficients of the bar systems given in Table 3.1.1. The buckling coefficients of the first two systems in the table have been obtained by using simple beam theory for the determination of a deflected equilibrium

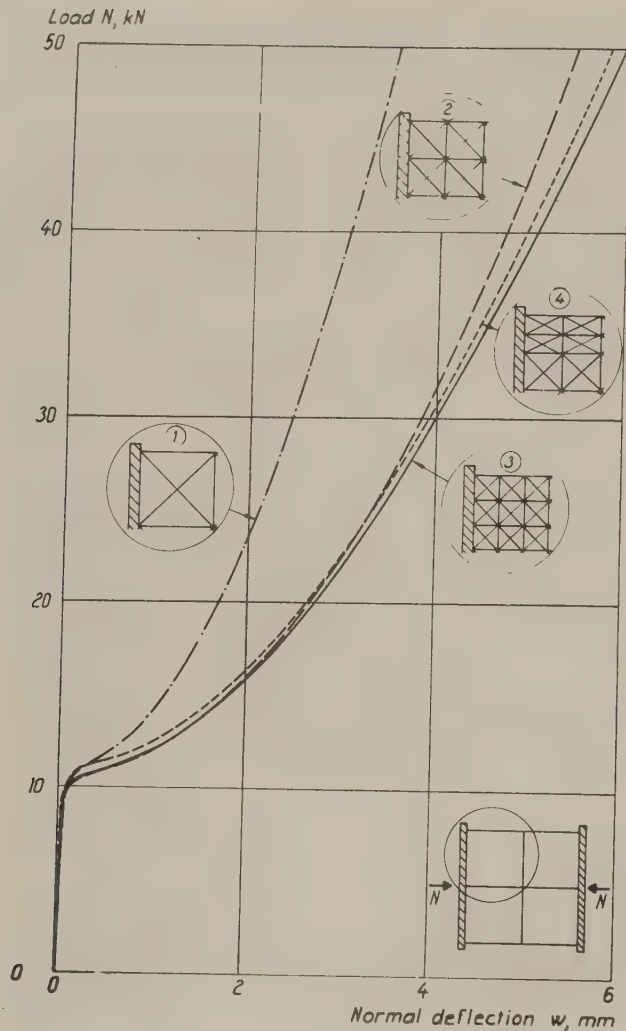


Fig. 3.1.18. Load versus normal deflection for bar systems 1-4.

position. For the analysis of the last two "bar systems" in the table, the differential equation for the deflection surface of a plate has been used. In all the four bar systems, the buckling coefficient has been divided into a part due to bending and a part due to torsion. The sum of these two parts is characterized as the total buckling coefficient. For further information, see Section 3.1.7.

Another interesting observation connected with Fig. 3.1.18 is that the buckling load is somewhat increased when the bar net is refined at the edges (cf. the load-deflection curves of bar system 2 and 4). The behaviour of bar system 4 is probably an effect of stabilizing torsional moments at the corners of the simply supported plate.

In the post-critical range, the normal deflections are rather insensitive to the choice of bar system when the bar net has been refined to a level corresponding to bar system 2 (cf. Fig. 3.1.18).

The influence of different bar systems on the stress distribution is shown in Fig. 3.1.19. The average of the longitudinal stress has been chosen to be $2\sigma_{cr}$, where σ_{cr} is the critical stress according to the exact theory.

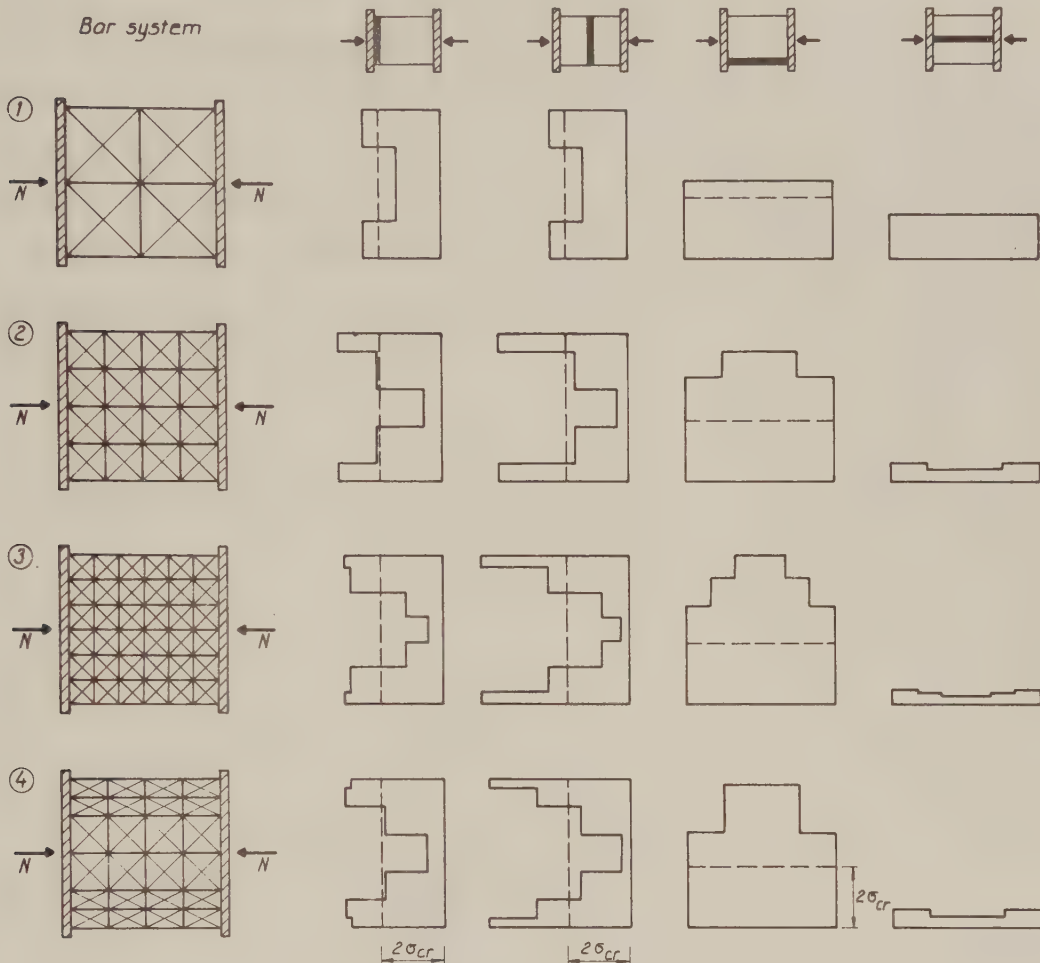


Fig. 3.1.19. Distributions of stresses in the longitudinal direction for bar systems 1-4. The average longitudinal stress is $2\sigma_{cr}$. The fat lines above in the figure indicate sections where the stresses are shown.

Fig. 3.1.20 shows the relation between curvature and bending moment for the bar systems 1 B and 4 B. As was previously mentioned in this section, the moment of inertia will be somewhat too high when the box-beam section is analysed by means of the bar models. This can be seen by enlarging Fig. 3.1.20 where the initial parts of the curves fall above the curve obtained from simple linear theory. If the theoretical moment of inertia I_0 is replaced by a corrected moment of inertia, the agreement with linear theory is very good

Table 3.1.1. Influence of bar system on the buckling coefficient of a square simply supported plate.

Bar system	buckling coefficient k		
	bending	torsion	total
 Bar system 1	2.432	1.134	3.566
 Model with an infinite number of bars	2.188	1.277	3.465
Eqs. (3.1.41) and (3.1.42)	2.0	1.4	3.4
Exact theory Eqs. (3.1.39) and (3.1.40)	2.6	1.4	4.0

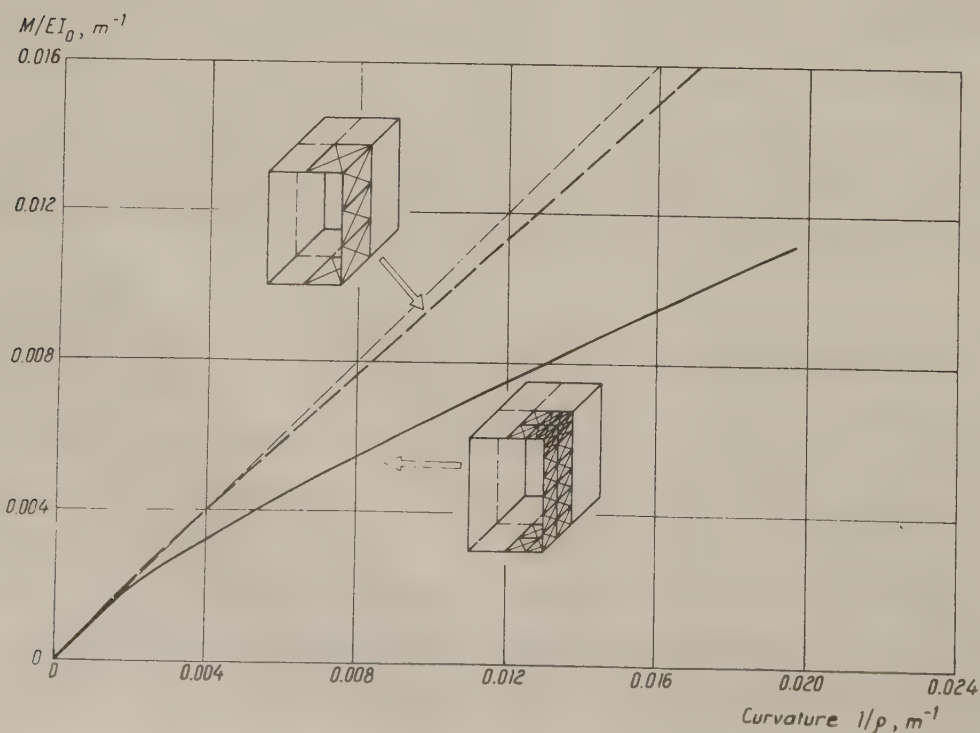


Fig. 3.1.20. Relationship between bending moment M and curvature $1/\rho$ for bar systems 1'B and 4 B.

at low bending moments. As in the case of an initially flat plate, the curves consist of two parts, before and after the buckling load has been reached. The first almost straight part corresponds to simple bending and the second part represents the increase of curvature due to deflections.

The relation between bending moment M and normal deflection w is shown in Fig. 3.1.21 for the two bar systems. The deflection w refers to the crest of the sinusoidal deflection surface of the compressed flange. As was the case for the simply supported plate, the wide-meshed bar system predicts the buckling load rather well. For the analysis of the post-critical range, however, the small-meshed model is necessary.

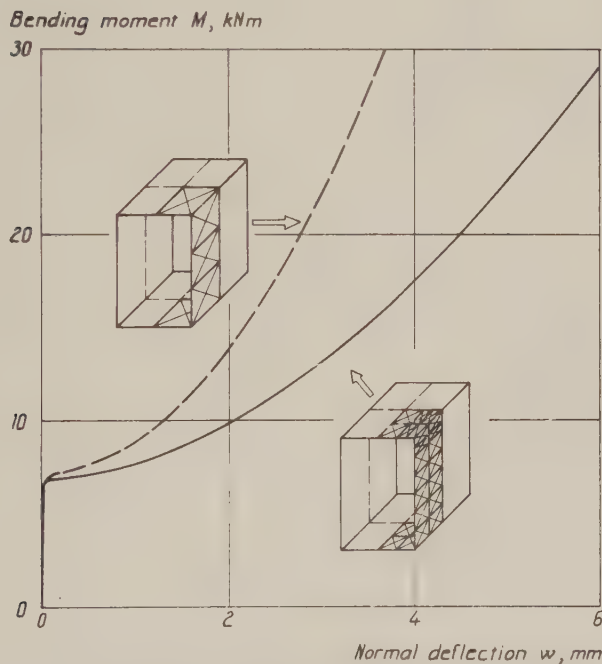


Fig. 3.1.21. Bending moment M versus normal deflection for bar systems 1 B and 4 B.

Influence of displacement increment.

To illustrate the influence of the size of the displacement increment on the load-deflection curve, bar system 3 has been analysed with the relative increment in normal deflection $\Delta w/w$ equal to both 0.10 and 0.20. At the same time the absolute increase in displacement $\Delta w_{\max}$ was limited to 0.1 and 0.2 mm respectively.

As can be seen in Fig. 3.1.22, a larger displacement increment will result in a slower change of curvature for the load-deflection relation. The displacement increment used seems to be sufficiently small.

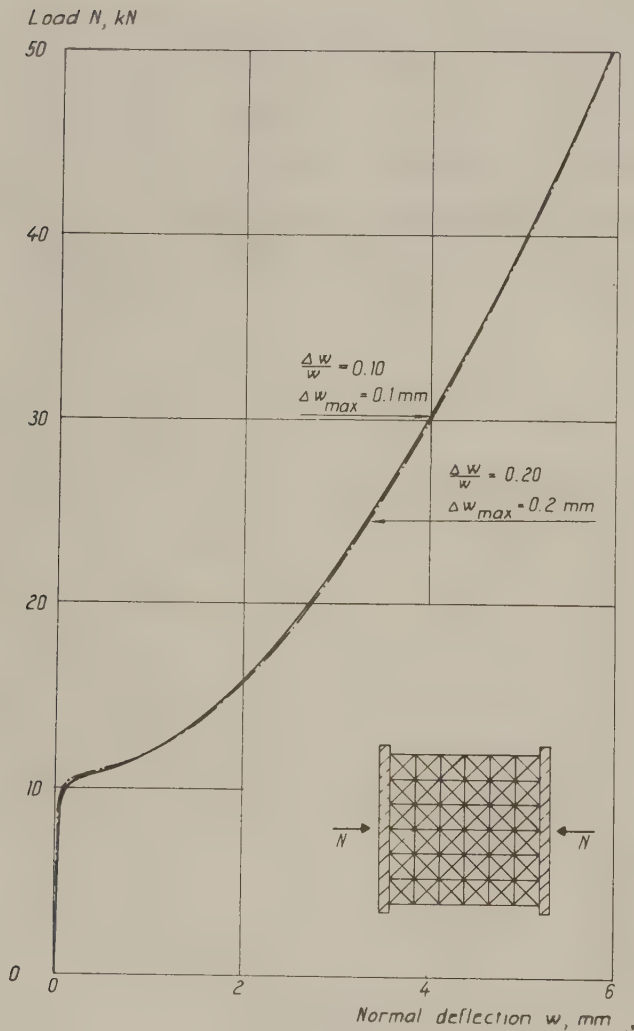


Fig. 3.1.22. Influence of displacement increment on the relationship between axial load N and normal deflection w .

Comparison between solution from bar model and "exact" theory.

The load-deflection curve of the simply supported plate obtained from bar system 3 has been compared with results deduced by Yamaki (1959). Under the assumption of sinusoidal initial deflection surface, the maximum value of which was $w_0 = 0.1$ t, and Poisson's ratio ν equal to $1/3$, Yamaki arrived at the curve given in Fig. 3.1.23. This curve is to be compared with the curve obtained from the model with ν equal to 0.3 . For comparison, Yamaki's solution

with no initial deflection is also given. In the vicinity of the buckling load, the vertical deviation between the Yamaki curve and the model curve is between 10 and 15 per cent. The deviation seems to be reasonable in view of the 15 per cent difference in buckling coefficient that was reported in Table 3.1.1. With increasing load, the curves in Fig. 3.1.23 gradually approach each other.

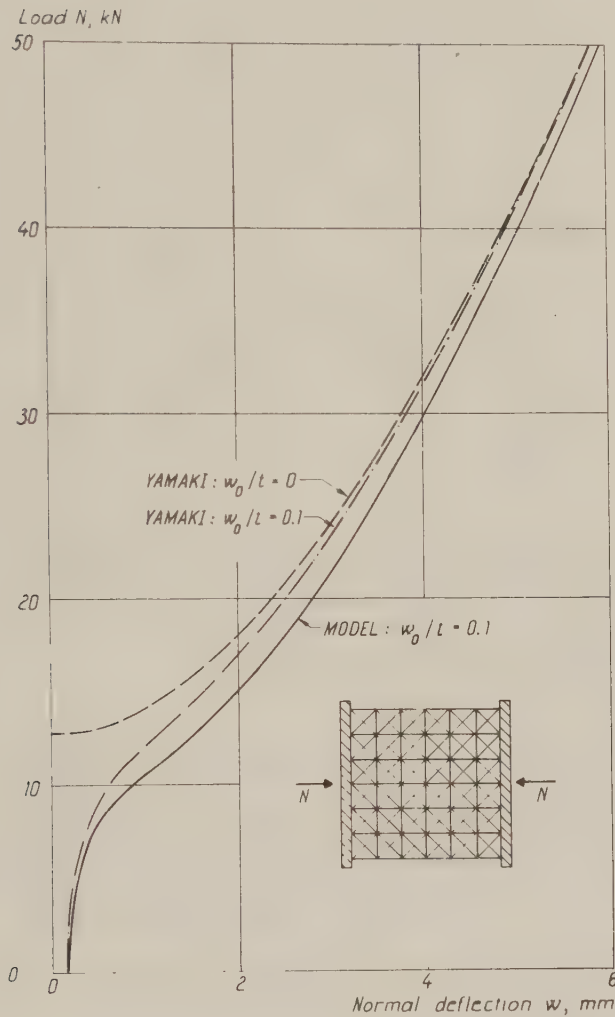


Fig. 3.1.23. Load versus normal deflection for bar system 3. Comparison with results obtained by Yamaki.

In Fig. 3.1.24 the stress distribution of the plate is shown at a load level corresponding to a longitudinal average stress of $1.737 \sigma_{cr}$. A solution presented by Coan (1951) and that for bar system 3 are compared. Coan's solution is based on $\nu = 0.316$ and $w_0 = 0.1 t$. The agreement between the stress diagrams in Fig. 3.1.24

is good. As can be expected, the stress peaks are somewhat smoothed out in the solution obtained from the model.

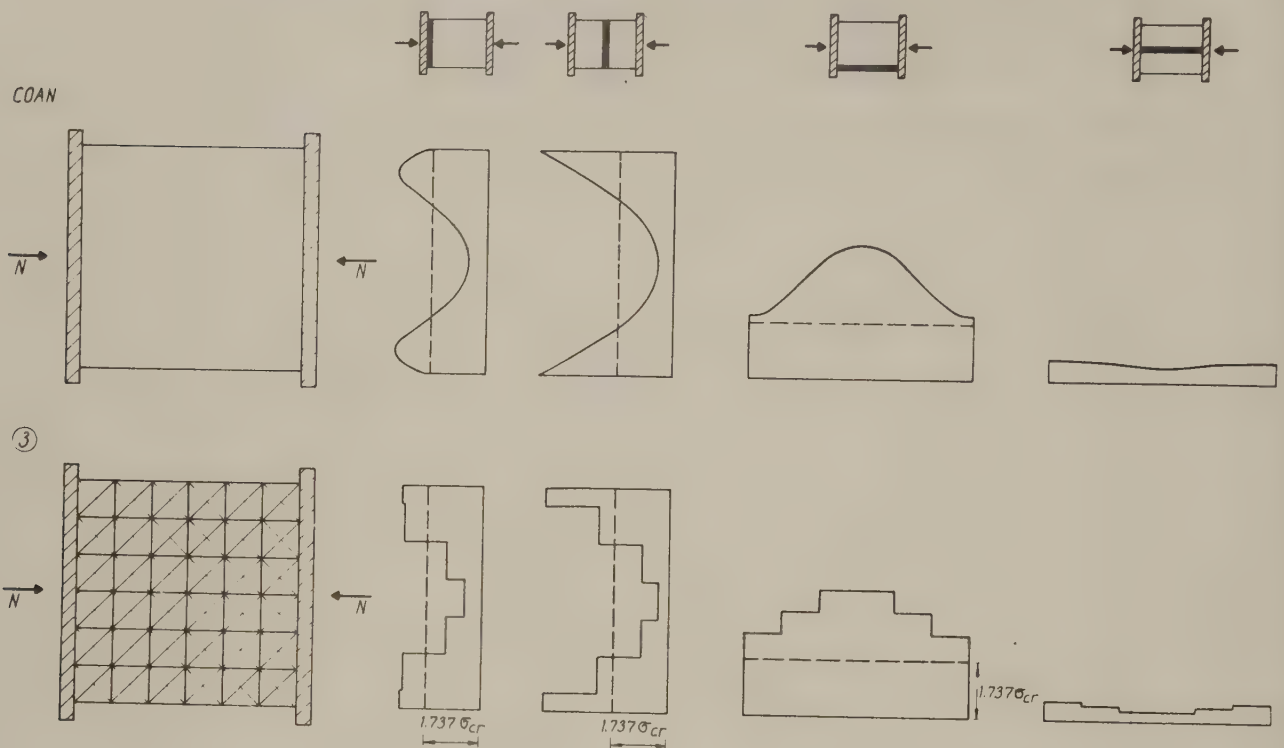


Fig. 3.1.24. Comparison of stress distributions obtained by means of bar system 3 with solution according to Coan.

Influence of the initial deflection.

The influence of the magnitude of the initial deflection on the behaviour of a simply supported plate has been analysed by means of bar system 3. Three levels of initial deflection have been studied, namely $w_0 = 0.0067 t$, $0.1 t$ and $1.0 t$ constituting a small, a moderate and a large initial deflection.

In Fig. 3.1.25, the axial load N is plotted versus the normal deflection w . It is observed that the curve relating to the initial deflection $w_0 = 0.1 t$, converges towards the curve for $w_0 = 0.0067 t$ soon after the buckling load has been exceeded. The curve $w_0 = 1.0 t$ does not approach the other curves to such an extent.

The axial load N is shown as a function of the average edge strain ϵ in Fig. 3.1.26. The comments made with regard to Fig. 3.1.25 are also relevant here.

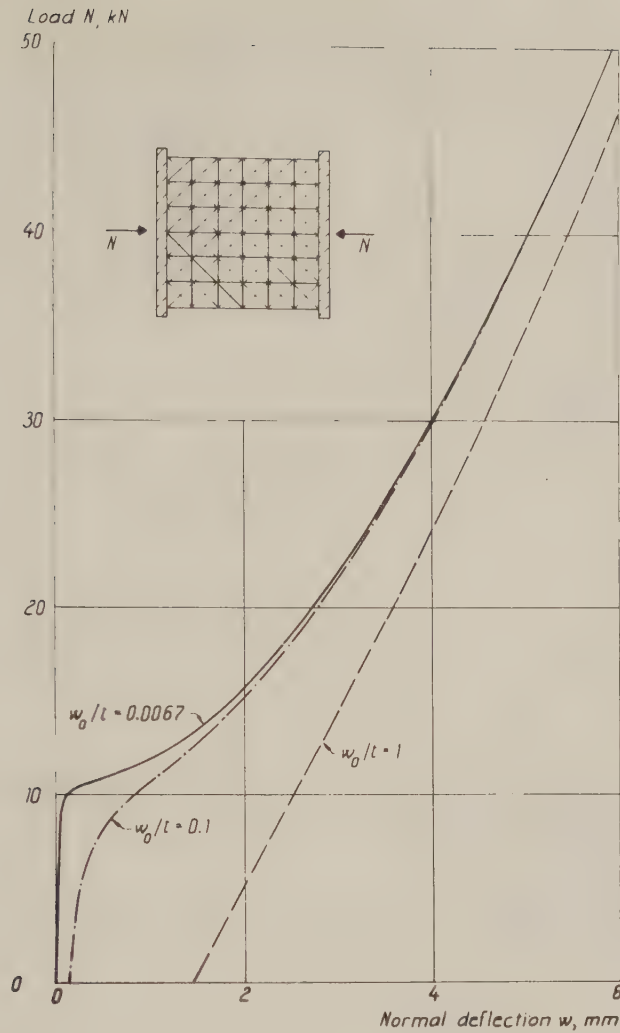


Fig. 3.1.25. Influence of initial deflection on the relationship between axial load N and normal deflection w .

The relation between the axial load N and the effective width b_e is shown in Fig. 3.1.27. It is obvious that the influence of the initial deflection on the effective width is considerable in the vicinity of the critical load.

The influence of the initial deflection on the behaviour of a box beam subjected to pure bending has also been studied. The same values were chosen for the initial deflection as in the case of the simply supported plate.

In Fig. 3.1.28 the bending moment M is plotted versus the normal deflection w . The results are very similar to those given in Fig. 3.1.25.

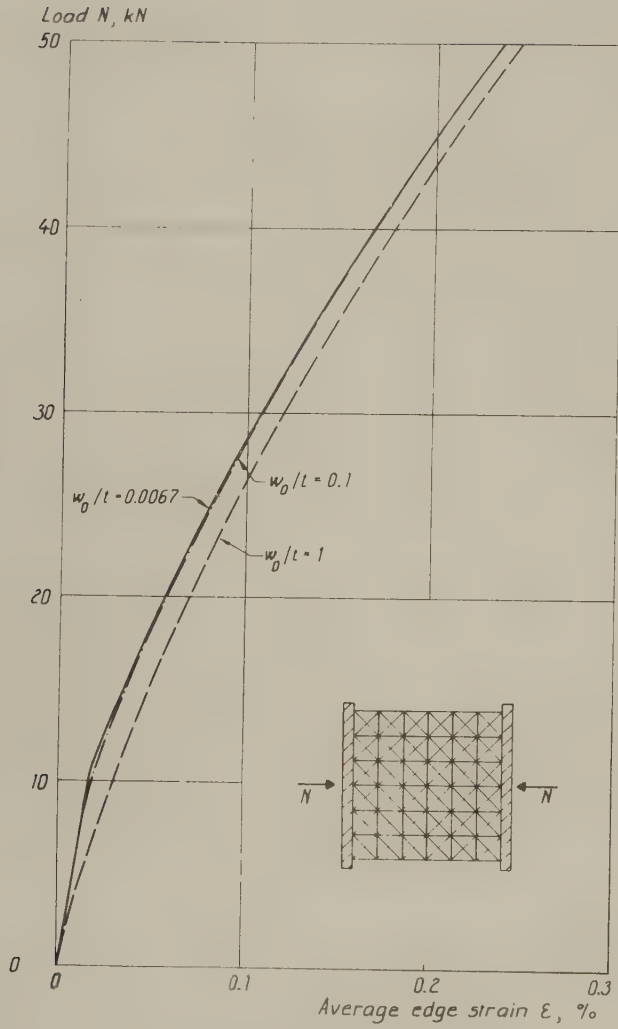


Fig. 3.1.26. Influence of initial deflection on the relationship between axial load N and average edge strain ϵ .

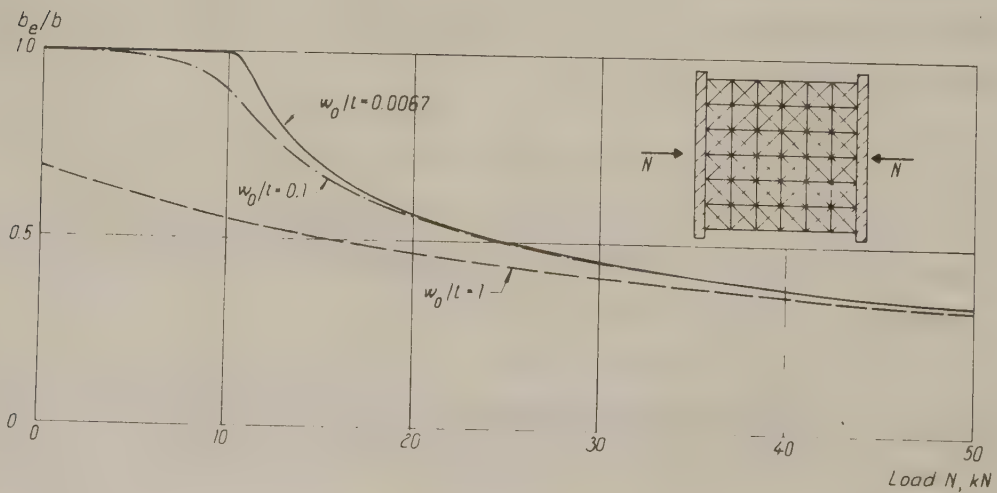


Fig. 3.1.27. Influence of initial deflection on the relationship between axial load N and effective width b_e .

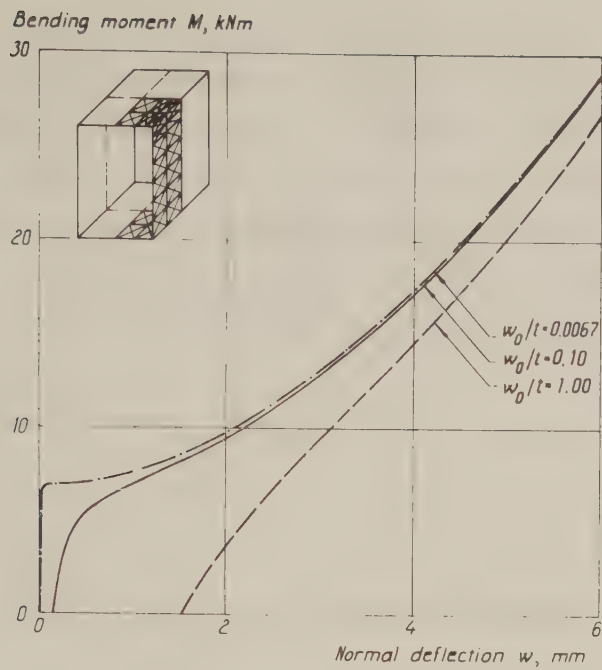


Fig. 3.1.28. Influence of initial deflection on the relationship between bending moment M and normal deflection w .

The relation between bending moment M and curvature $1/\rho$ is shown in Fig. 3.1.29. The curves $w_0/t = 0.0067$ and 0.10 differ perceptibly only in the vicinity of the buckling load. The curve $w_0/t = 1$ gradually approaches the two other curves as the bending moment increases.

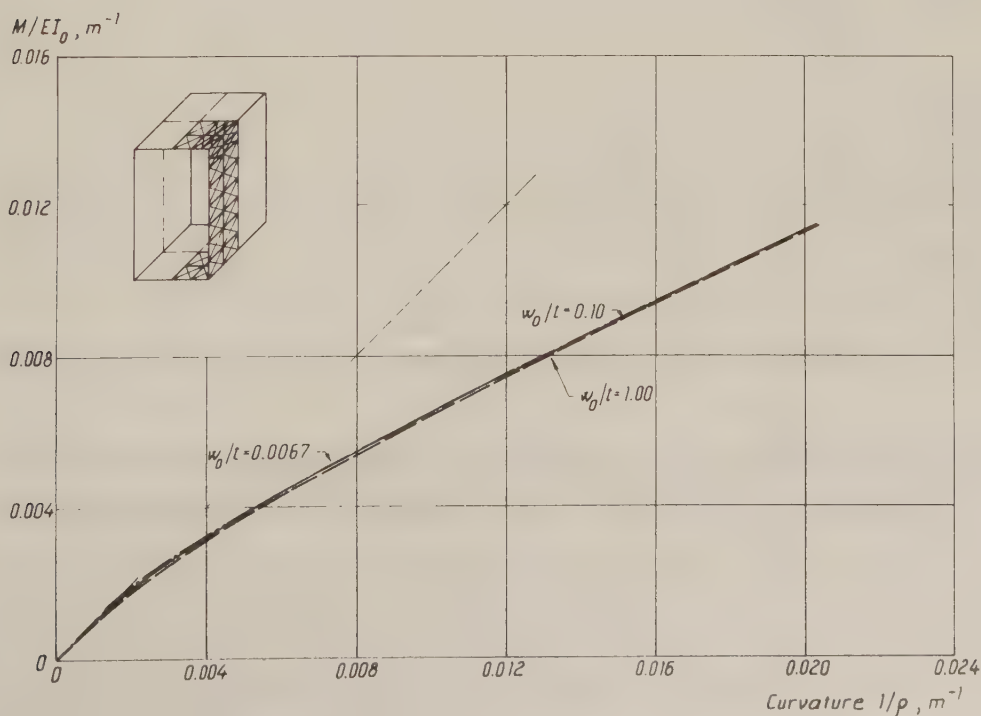


Fig. 3.1.29. Influence of initial deflection on the relationship between bending moment M and curvature $1/\rho$.

Influence of wavelength.

From the plate theory we know that a simply supported rectangular plate subjected to axial forces reaches its minimum buckling load when the buckles have a length to width ratio $\lambda = a/b = 1$. Thus, if λ is reduced to a value less than unity the buckling load will increase. This is illustrated for bar system 4 in Fig. 3.1.30, where the relation between axial load N and deflection w for three different λ -values is shown.

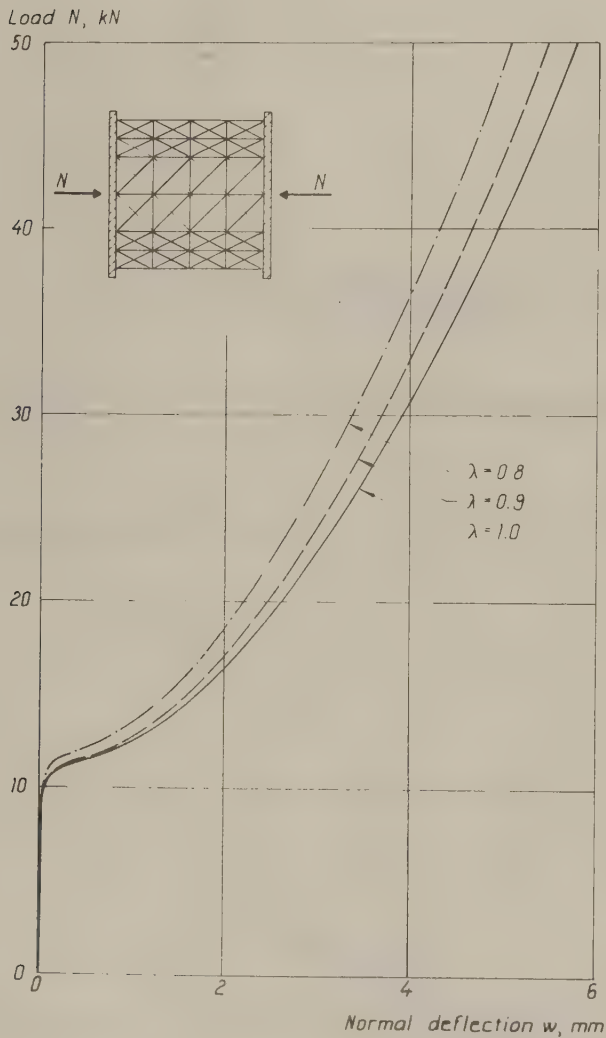


Fig. 3.1.30. Influence of wavelength on the relationship between axial load N and the normal deflection w .

For the same plate, the average edge strain is given as a function of load in Fig. 3.1.31. For any given axial load the buckles will try to adopt a wavelength making the work of the external force N a maximum. This will be achieved when the average longitudinal

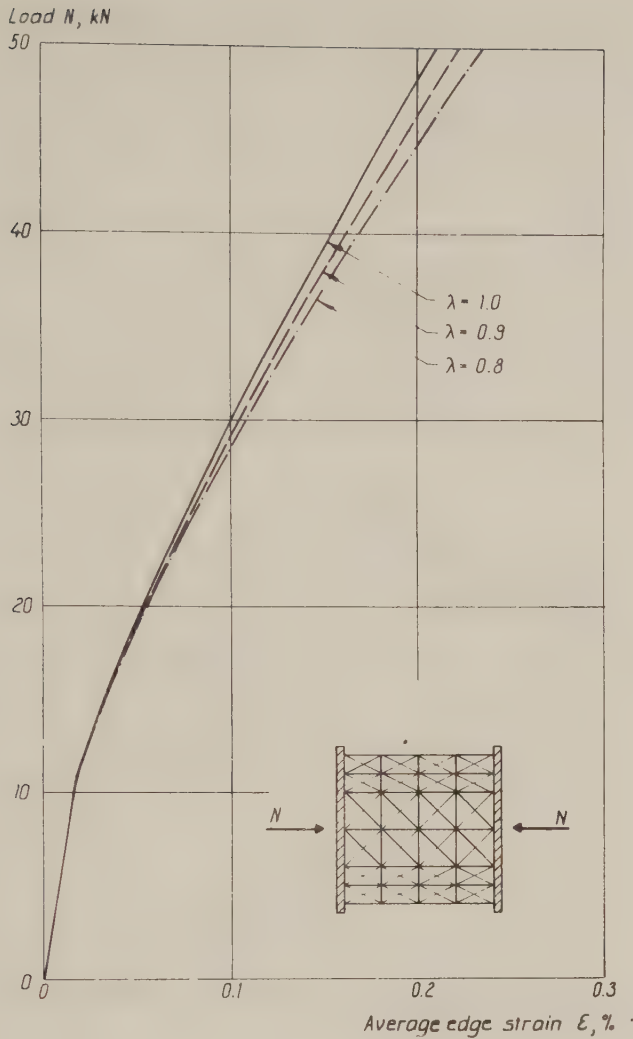


Fig 3.1.31. Influence of wavelength on the relationship between axial load N and average edge strain ϵ .

shortening of the plate is greatest i.e. by the curve which for a given axial load is farthest to the right in Fig. 3.1.31. This means that the wavelength ratio λ changes from the value of unity at small loads to a lower value at high loads. The calculated curves in Fig. 3.1.31 ought to be complemented by a few for still lower λ -values to obtain a more detailed picture of the influence.

The influence of the wavelength has also been studied for a box beam subjected to pure bending. Three different λ -values of the compressed flange have provided the basis for the calculations. The results are illustrated in Fig. 3.1.32 and Fig. 3.1.33. The box beam will adopt a buckle pattern that makes the beam section as flexible as possible. Thus, for any given bending moment, the box

beam will adopt the wavelength that is given by the curve farthest to the right in Fig. 3.1.32. Some more λ -values ought to be investigated to obtain a more accurate solution.

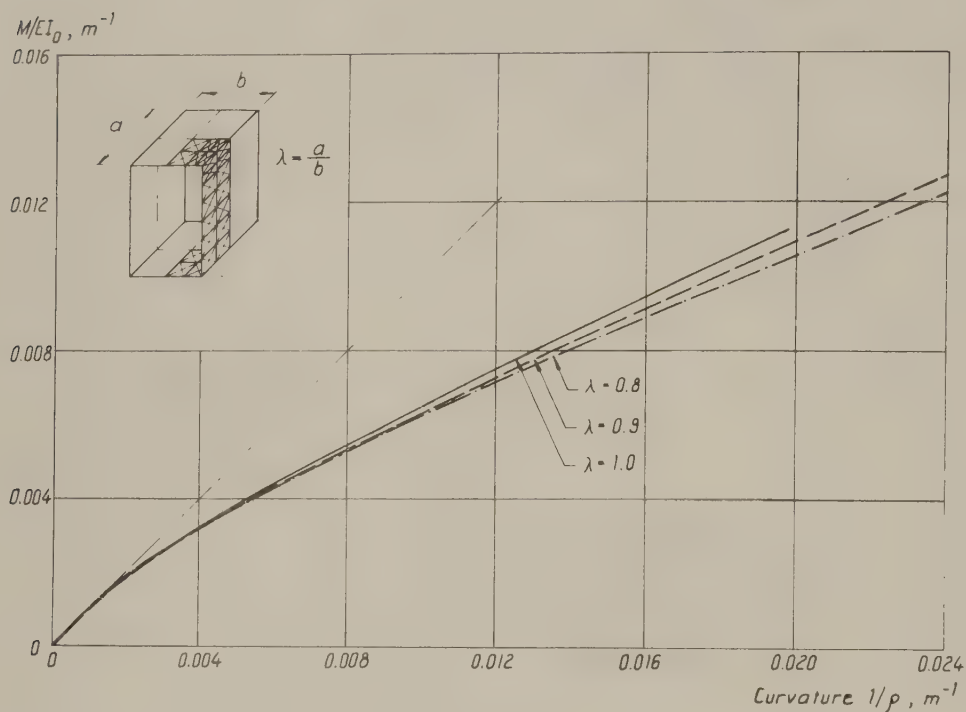


Fig. 3.1.32. Influence of wavelength on the relationship between bending moment M and curvature $1/\rho$.

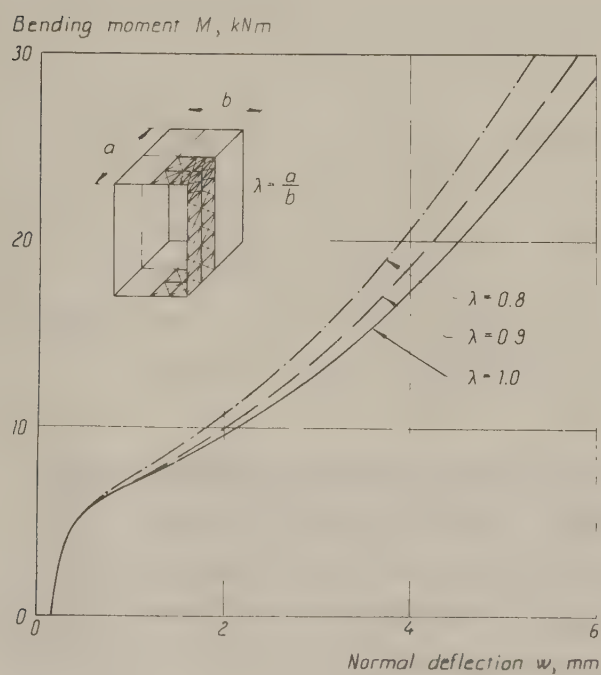


Fig. 3.1.33. Influence of wavelength on the relationship between bending moment M and normal deflection w .

Influence of initial stresses.

The influence of initial stresses due to the welding in the fabrication of the box beams has been analysed by introducing compression and tension forces which in principle have the same distribution as that obtained in the measurements described in Section 3.3.5 (cf. Fig. 3.3.8 c). The assumed distribution of the initial stresses is evident from Fig. 3.1.34 and Fig 3.1.35. The compression stress has been assigned the value 20 N/mm^2 , which may be compared with the theoretical critical stress $\sigma_{cr} = 54 \text{ N/mm}^2$.

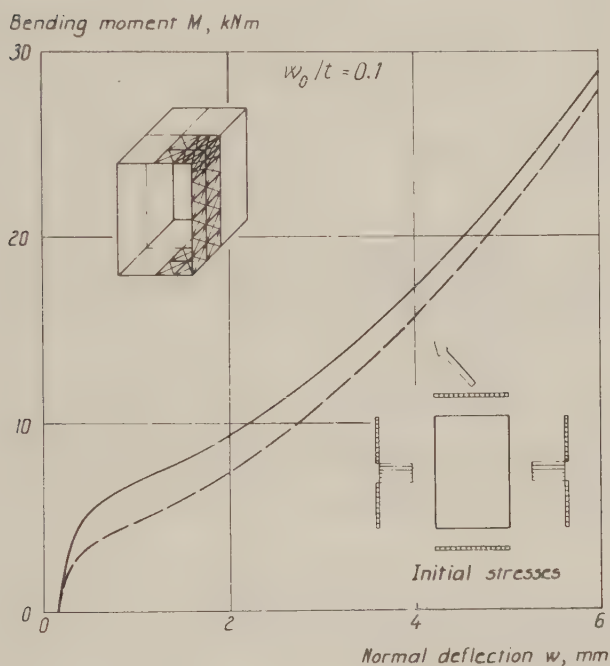


Fig. 3.1.34. Influence of initial stress on the relationship between bending moment M and normal deflection w .

The results of the calculations are illustrated in Fig. 3.1.34 by the relation between bending moment and normal deflection and in Fig. 3.1.35 by the relation between bending moment and curvature. In Fig. 3.1.34 it is clearly seen that the buckling load is decreased by the initial stresses. Further we conclude that the curves approach each other with increasing bending moment. Also in Fig. 3.1.35 the difference between the curves is greatest in the vicinity of the buckling load and gradually decreases with increasing bending moment.

The result of the calculations shows that the initial stresses are

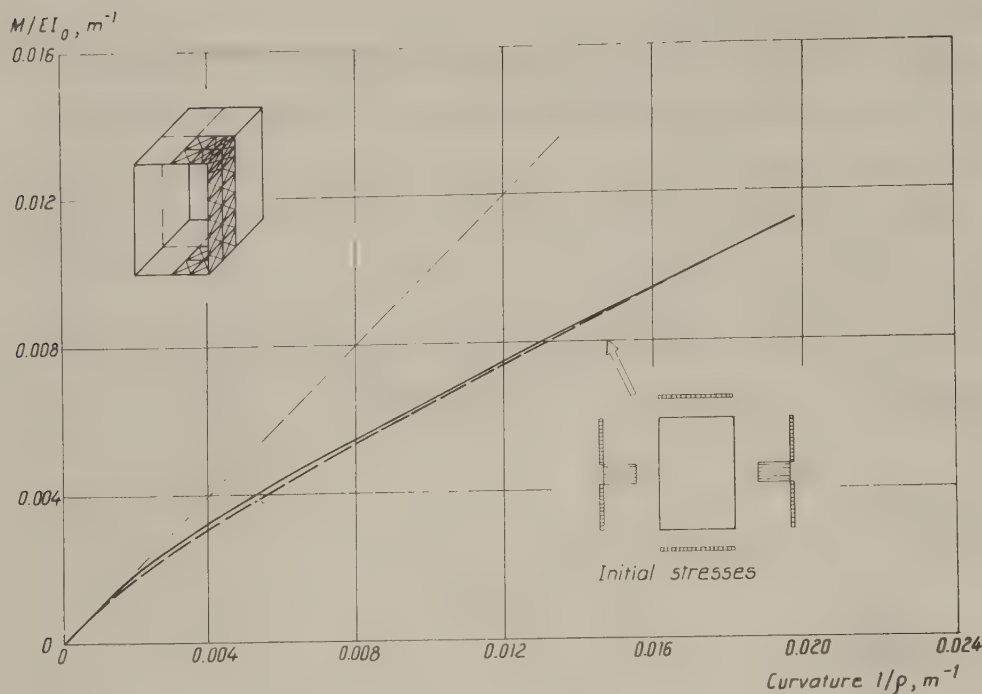


Fig. 3.1.35. Influence of initial stress on the relationship between bending moment M and curvature $1/\rho$.

of great importance for the behaviour of the box beam in the vicinity of the buckling load. The initial stresses do not, however, seem to play any decisive role for the ultimate strength of the investigated beam.

Influence of the webs on behaviour of the compressed flange in the bending of box beams.

In the bending of box beams, the behaviour of the compressed flange will be affected by the connection of the webs. To determine the magnitude of this effect, a comparison has been made between the flange regarded as a simply supported plate and the flange included in a box beam section.

Fig. 3.1.36 shows the normal deflection w of the flange as a function of the axial load N . The buckling load of the flange is increased by about 30 % when the restraints of the webs are considered. This is in good agreement with the theoretical increase of the buckling coefficient from 4.0 to 5.2 (cf. Fig. 2.4.1). At high axial forces in the compressed flange the magnitude of the deflections are relatively unaffected by the webs.

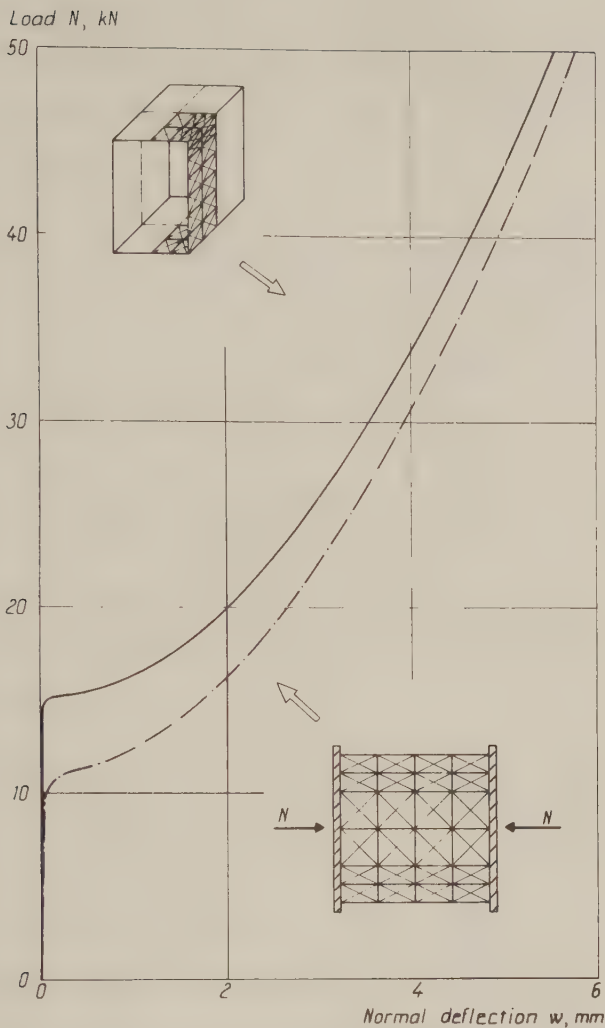


Fig. 3.1.36. Influence of the webs on the relationship between axial load N and normal deflection w .

The relation between axial load N and average edge strain ϵ is shown in Fig. 3.1.37. For the present box beam, stabilizing bending moments from the webs will reduce the deflections of the compressed flange, implying that the flange of the box beam can resist a greater axial load than a simply supported plate subjected to the same edge shortening.

Another observation associated with the calculation of the present box beam is that the axial force in the compressed flange has a longitudinal variation at large deflections. The axial force in this case turns out to be smallest at the top of the buckle and largest at the inflexion point of the buckle. This is a consequence of an unloading of the compressed flange at the top of the buckle due to shear stresses occurring along the corners of the beam. Thus, in

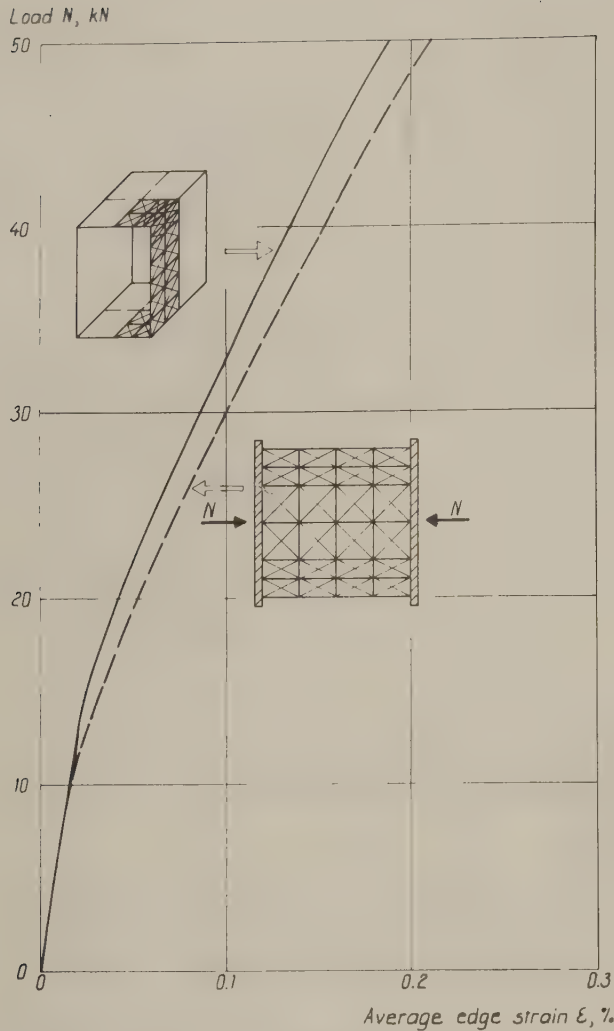


Fig. 3.1.37. Influence of the webs on the relationship between axial load N and average edge strain ϵ .

addition to stabilizing transverse bending moments transferred from the webs to the compression flange, a transfer of shear stresses also occurs. For box beams with higher h/b -ratios than for the studied beam, i.e. for box beams where the buckles are initiated in the webs, the conditions are different. In this case, the axial force at the top of the buckle in the compressed flange should increase.

Comparison between calculated and measured values.

In connection with the experimental studies (Sections 3.2-3.4), two different box beam sections have been studied theoretically by means of the bar model. In the calculations, the initial deflections and initial stresses have been chosen with regard to measured magnitudes.

To describe the initial deflections of the compressed flange, a sinusoidal deflection surface has been assumed. The initial deflections of the webs have been set equal to zero, a rather rough approximation.

The assumed initial stress distribution has been chosen with regard to the one previously described in this section. The magnitudes of initial stresses have been chosen in accordance with the estimated values of Section 3.3.5.

To illustrate the influence of the wavelength, two different values of λ have been studied, namely $\lambda = 1$ and $\lambda = 0.75$.

In Sections 3.3.8 and 3.3.9, calculated and measured values are compared.

3.1.14 A simplified model for the analysis of a box beam under pure bending.

The mode of action of a box beam under a pure bending moment in the post-critical range previously discussed in Section 3.1, was illustrated by means of the bar model described. In this section, a simplified model for the analysis of reductions in bending rigidity due to local buckling phenomena in the webs and the compression flange will be presented. The model provides an aid suitable for the determination of the deflection curve and the ultimate bending moment of the beam. In statically undetermined structures, the simplified model can be used for the determination of the bending moment distribution.

The theoretical calculation model first published by the author in connection with the Nordic research symposium on steel structures in Oslo 1973 is based on the assumption that longitudinal hinges are introduced in the corners of the beam (Fig. 3.1.38) to prevent the transmission of bending moments between flanges and webs. The behaviour of the beam under an increasing bending moment can then be described in the following stages:

- 1) Neither the critical stress of the flange nor that of the webs

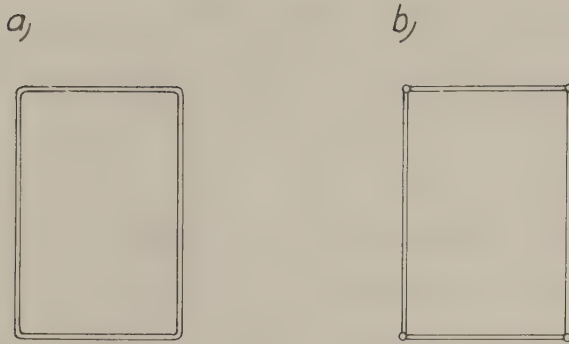


Fig. 3.1.38 a) Real beam section.
b) Simplified model

has been exceeded. All the beam section is effective. See Fig. 3.1.39 a.

- 2) The critical stress of the compression flange $\sigma_{cr,f}$ has been exceeded and we reduce the effective width of the flange b_e . The critical stress of the web $\sigma_{cr,w}$ has not yet been attained. Alternatively, the critical stress of the webs is reached first and we reduce the effective width of the compressed zone of the webs h_{ce} . See Fig. 3.1.39 b.
- 3) Both the critical stress of the compression flange and that of the webs have been exceeded. The effective widths are clear from Fig. 3.1.39 c.

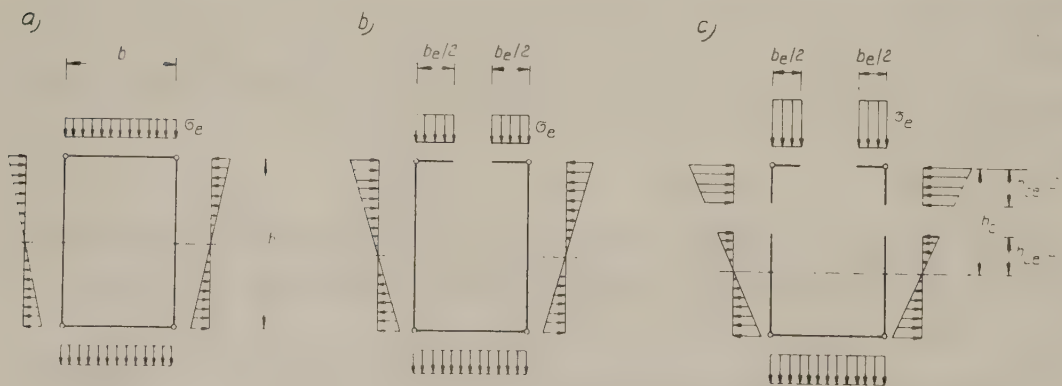


Fig. 3.1.39. Simplified behaviour for description of a box beam under increasing bending moment.
a) All the section is effective.
b) The critical stress of the compression flange has been exceeded.
c) The critical stress of the webs has been exceeded.

For the determination of critical stresses and effective widths of the webs and of the compression flange the following expressions are used

$$\sigma_{cr,f} = k_f \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2 \quad (3.1.49)$$

$$\sigma_{cr,w} = k_w \frac{\pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{h_c}\right)^2 \quad (3.1.50)$$

$$b_e = b \sqrt{\frac{\sigma_{cr,f}}{\sigma_e}} \left(1 - 0.25 \sqrt{\frac{\sigma_{cr,f}}{\sigma_e}}\right) \quad (3.1.51)$$

$$h_{ce} = h_c \sqrt{\frac{\sigma_{cr,w}}{\sigma_e}} \left(1 - 0.25 \sqrt{\frac{\sigma_{cr,w}}{\sigma_e}}\right) \quad (3.1.52)$$

where

$$k_f = k_w = 4$$

σ_e = the edge stress on the compression side

Since k_w is assigned the value 4, the critical stress of the web is attained somewhat too soon. If the ratio of the height of the zone in tension to that of the zone in compression is between 0.5 and 2.0, k_w is actually approximately equal to 6. In Eqs. (3.1.51) and (3.1.52) the expression for effective width according to Winter has been used. Alternatively any of the expressions given in Section 2.2.2 can be used.

Since no bending moments can be transmitted between the flanges and the webs, two critical loads for the simplified model will be obtained. For a real box beam section, only one critical load will be obtained. This means that the behaviour of the simplified model deviates from reality in the vicinity of the critical load. At high loads, however, the simplified model can be expected to describe reality rather well.

3.2 Singel-span beam. Introductory experiment.

3.2.1 Purpose.

The main purpose of the introductory experiments was to elucidate the post-buckling behaviour of thin-walled box beams subjected to pure bending.

The aim of the experiments was to study the influence of buckling phenomena and their interaction in webs and flanges, to compare the results with theories of post-buckling, to investigate different relations between load and deformation up to failure and, finally, to determine the nature of failure.

The experiments were carried out as undergraduate work by the now qualified civil engineers Kent Gylltoft and Lars Rydfjäll under the supervision of the author.

3.2.2 Scope.

The experiments comprised the testing of two box beams which were denoted A1 and A2. To obtain a constant bending moment, the load was applied as two equivalent, symmetrically placed point loads. Fig. 3.2.1 shows the distribution of load, shear force and bending moment for the test beams.

The dimensions of the cross sections were chosen to give interesting combinations of buckling phenomena in flanges and webs. The nominal dimensions of the test beams A1 and A2 are given in Table 3.2.1.

Table 3.2.1. Nominal dimensions of the test beams A1 and A2.

Test beam	b_o mm	h_o mm	t mm	L_1 mm	L_2 mm	L mm
A1	200	300	2.50	700	1000	2400
A2	160	240	1.25	550	800	1900

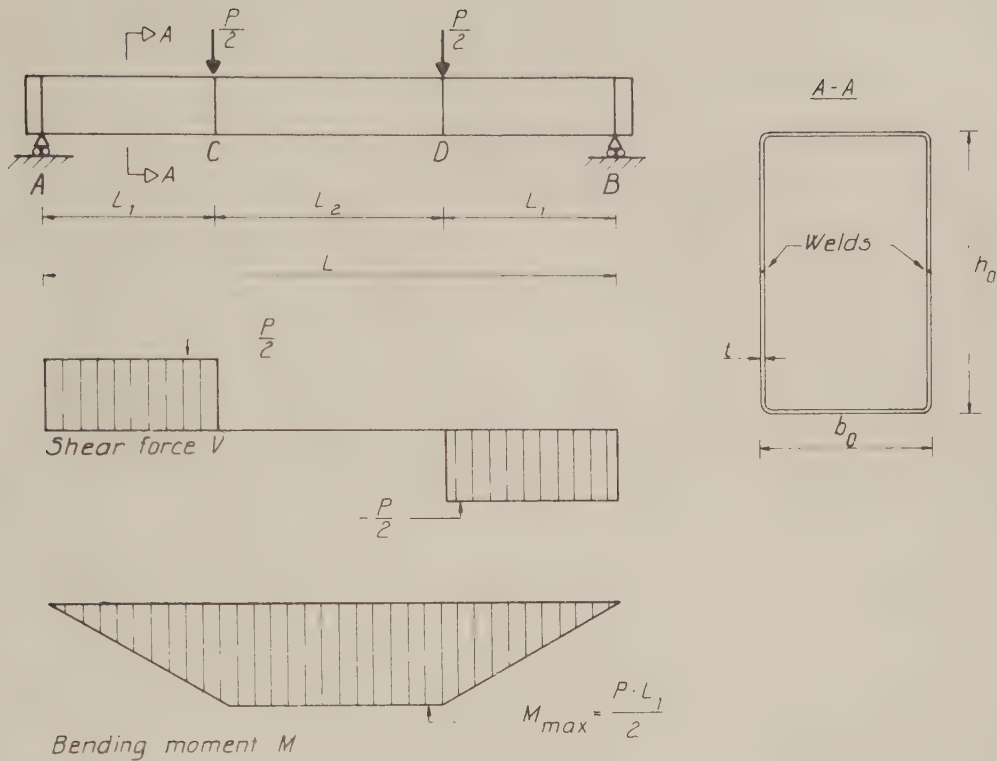


Fig. 3.2.1. Distribution of load, shear force and bending moment for the test beams A1 and A2. Notations for cross section.

3.2.3 Loading and support arrangements.

The principal arrangement of loading and support devices is illustrated in Fig. 3.2.2. The load was transmitted from the press via a knife edge bearing to two longitudinal I-beams. The load was transmitted to the webs of the test beams by screw joints. The function of the screw joints will be discussed in more detail in Section 3.3.3. The test beams were supported on roller bearings.

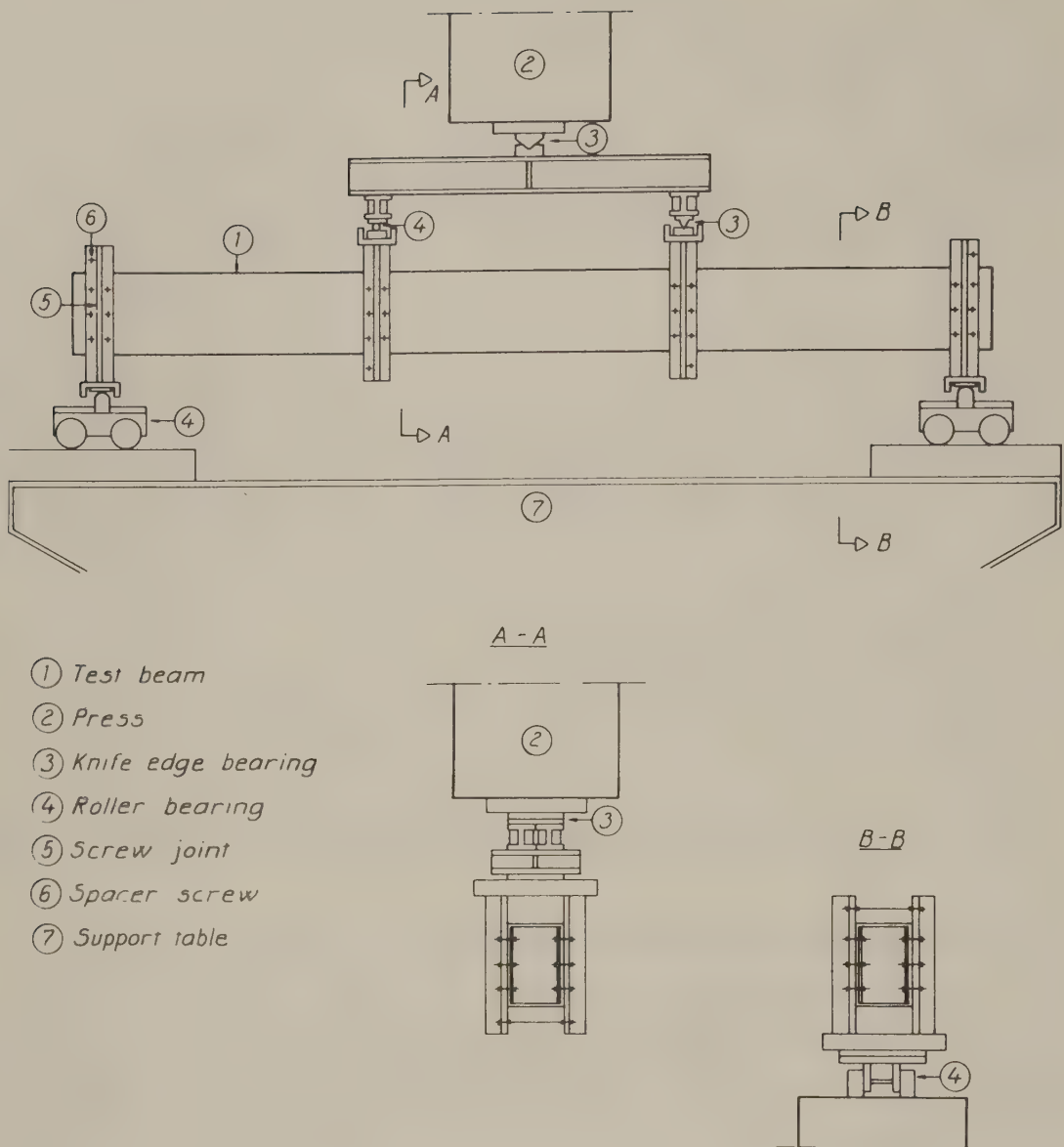


Fig. 3.2.2. Principal arrangement of loading and support devices.

3.2.4 Dimensions. Material properties.

The test beams were made by welding together U-sections. The U-sections had been produced from cold-rolled sheet by brake forming. The welds were placed in the middle of the vertical webs. The U-sections were first spot welded to each other and then continuously welded by hand. No action was taken against deformations and stresses introduced during the welding.

The material consisted of cold-rolled sheet that had been subjected

to recrystallization annealing and thereafter to a trim rolling with a reduction in thickness of 0.5-1 %.

The notations for the materials given by the manufacturer are for test beam A1, SU 220, and for test beam A2, SU 221. The qualities are related to Swedish Standards for hot-rolled structural steels SIS 1311 and SIS 1312 respectively.

Table 3.2.2 shows the results of a chemical analysis of the materials made by the manufacturer. Table 3.2.3 presents the results of tensile tests carried out by the manufacturer of the cold-rolled sheet.

Table 3.2.2. Results from chemical analysis of the cold-rolled sheet made by the manufacturer.

Test beam	C	Si	Mn	P	S	N
	%	%	%	%	%	%
A1	0.10	0.04	0.49	0.020	0.020	0.004
A2	0.12	0.05	0.50	0.027	0.028	0.005

Table 3.2.3. Results from tensile tests carried out on the cold-rolled sheet by the manufacturer.

Test piece with reference to test beam	Rolling direction	σ_y N/mm ²	σ_u N/mm ²	δ_5 %
A1	//	221	358	45
	⊥	228	366	45
A2	//	232	351	50
	⊥	224	346	47

To find out to what degree the material had been affected by welding, test pieces, with dimensions according to Fig. 3.2.3, were cut out from the test beams at different distances from the welds and subjected to a tensile test. The test pieces were taken out in a direction parallel to the welds not far from the beam ends. The result of the tensile test can be seen in Fig. 3.2.4 where the yield stress

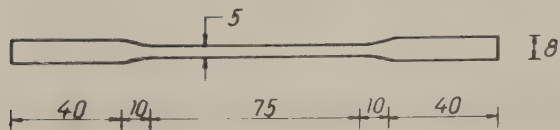


Fig. 3.2.3. Dimensions of tensile test pieces cut out at the welds.

is given as a function of distance from the weld. In test beam A1, the welds have a greater influence on the yield stress than in test beam A2, but in both cases the influence is limited to the areas in the vicinity of the welds.

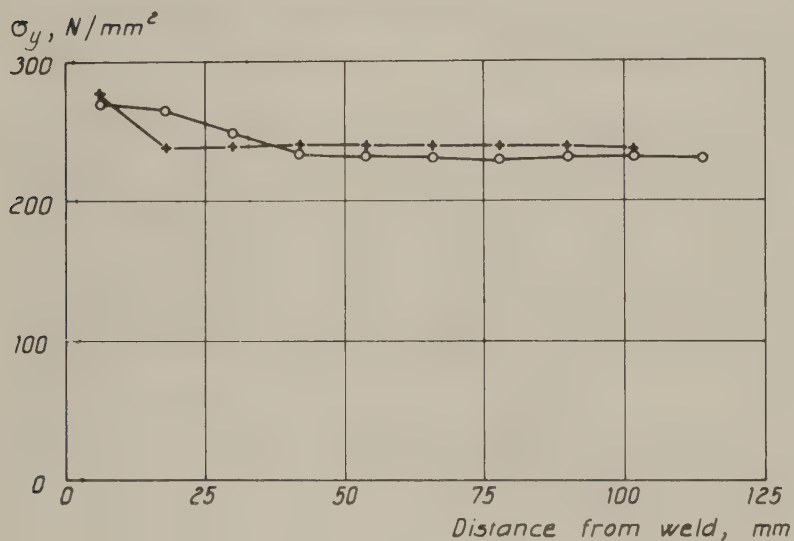


Fig. 3.2.4. Results from tensile tests. Influence of welding upon yield stress.

o = test beam A1 ($t = 2.5$ mm)

+ = test beam A2 ($t = 1.25$ mm)

Measured dimensions of the test beams A1 and A2 are given in Table 3.2.4 as mean values. Cross-sectional constants and critical stresses, defined according to Appendix A, are given in Table 3.2.5.

Table 3.2.4. Measured dimensions of test beams A1 and A2.

Test beam	b_0 mm	h_0 mm	t mm	R mm	L_1 mm	L_2 mm	L mm
A1	200.5	300.5	2.50	6	700	1000	2400
A2	164.3	243.7	1.31	3	550	800	1900

Table 3.2.5. Cross-sectional constants and critical stresses for test beams A1 and A2.

Test beam	b/t	h/t	$I_o \cdot 10^{-4}$ mm ⁴	$\sigma_{cr,f}$ N/mm ²	$\sigma_{cr,w}$ N/mm ²	σ_{cr} N/mm ²
A1	79.2	119.2	3301	119	313	154
A2	124.4	185.0	938	48	130	63

3.2.5 Measuring devices and measurement procedure.

The measurements comprised determination of applied load, measurement of deflections, measurement of strains and determination of the shape and size of the normal deflections in the compression flange.

Load measurement.

The load was applied to the test beams by means of a hydraulic press, MAN 3MN, provided with an automatic constant load control which kept the load constant independent of the deflection of the test beams. This meant that the test beams exhibited a certain degree of creep when subjected to high loads, especially near the ultimate load.

Deflection measurement.

The deflections were measured by means of dial gauges graduated in units of 0.01 mm. The deflections were recorded in three sections, according to Fig. 3.2.5. The dial gauges measured with their points against the centre of the tension flange. The vertical movement of the supports was assumed to be negligible.

Strain measurement.

The strains were measured by means of foil strain gauges with a measuring length of 60 mm. To determine the mean strains of the sheet, the strain gauges were fastened in couples opposite to each other on the outer and inner surfaces of the test beam. A strain gauge indicator Peekel 540 DNH was used for the measurements.

The positions and numbering of the strain gauges used for the test beam A2 are shown in Fig. 3.2.5.

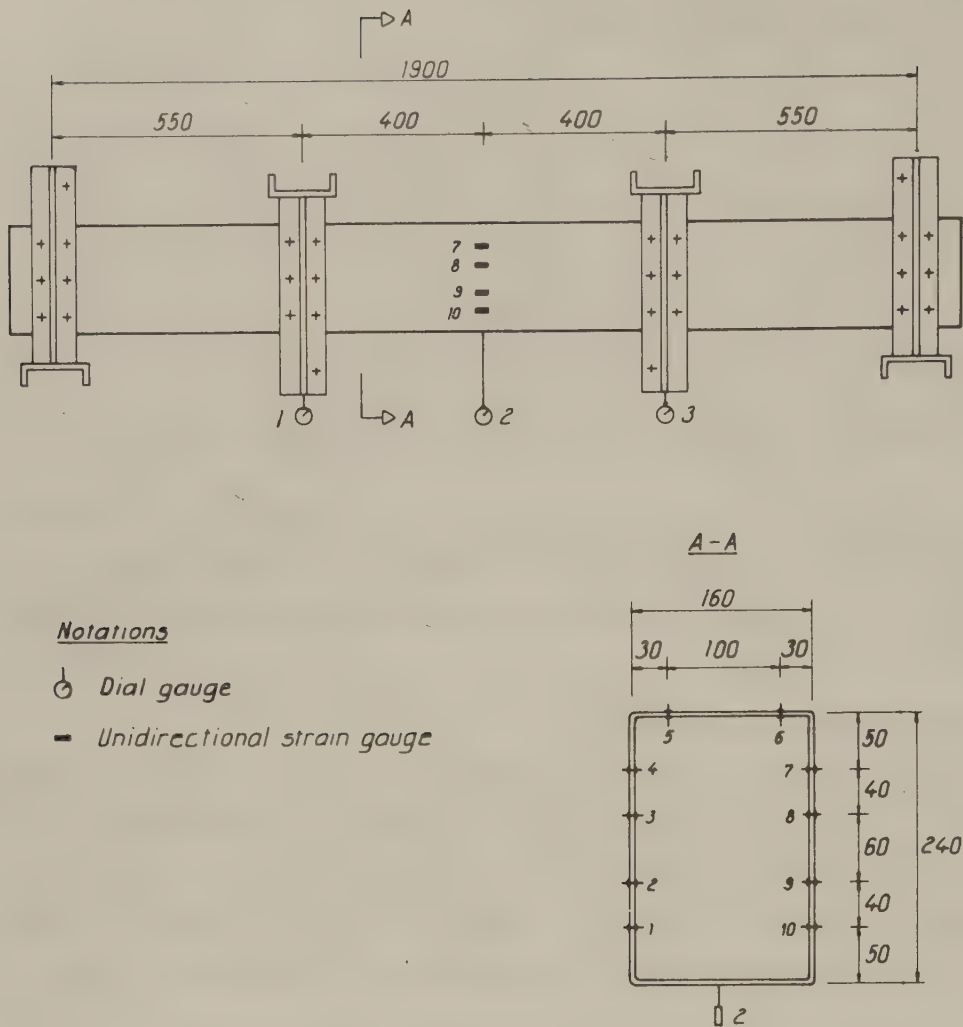


Fig. 3.2.5. Positions and numbering of dial gauges and strain gauges for test beam A2.

Measurement of normal deflections.

To measure the normal deflections, a dial gauge graduated in units of 0.01 mm was used. The dial gauge could be moved in the transverse direction of the test beam along a milled slot in a planed piece of flat bar. The flat bar was movable in the longitudinal direction along two plane milled guides which were fixed to hinges at the screw joints where the point loads were applied. A further development of this device for the manual measurement of the deflection pattern is shown in Fig. 3.3.10. With this arrangement it was possible to determine the deflection of any arbitrary point on the compression flange between the point loads.

3.2.6 Results.

Deflections.

Figs. 3.2.6 and 3.2.7 show the relationship between applied load P and measured deflection v at three positions of the test beams A1 and A2 respectively. As was described in Section 3.2.5, the dial gauges measured with their points against the centre of the tension flange. This test arrangement was not quite satisfactory owing to the possibility of cross-sectional deformations. Further, the deflections of the end supports were not recorded. Thus the results obtained from the deflection measurements have to be treated with caution.

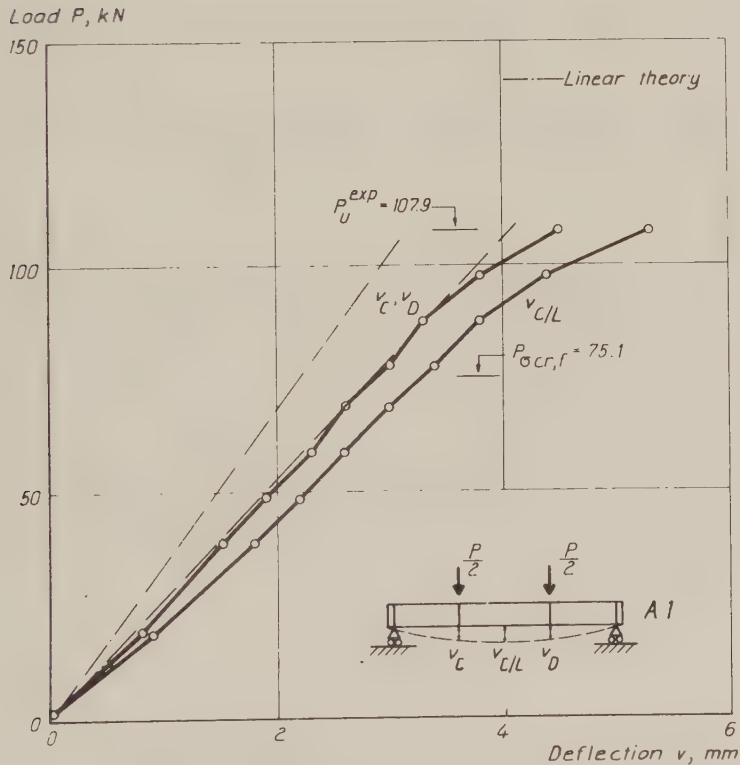


Fig. 3.2.6. Load-deflection curves for test beam A1.

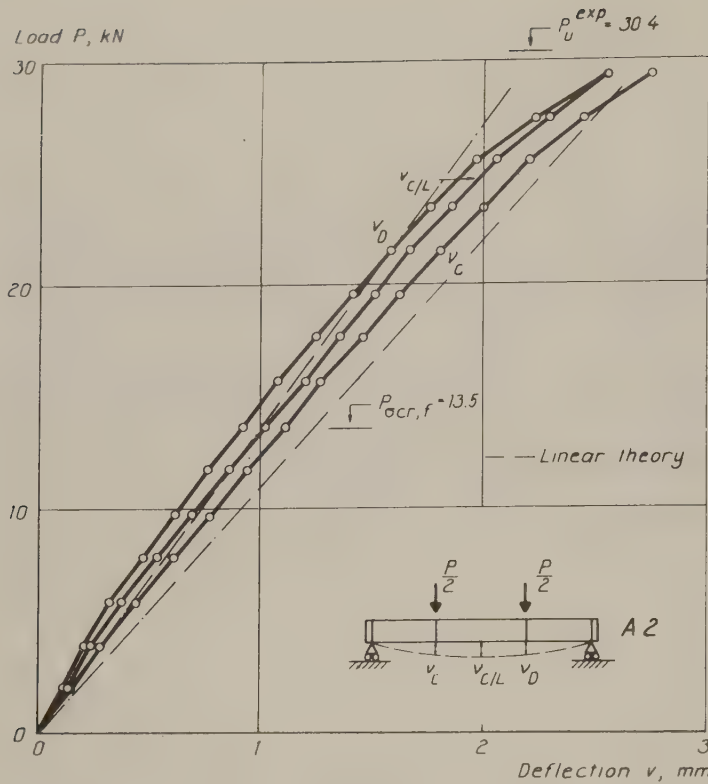


Fig. 3.2.7. Load-deflection curves for test beam A2.

Normal deflections.

In Figs. 3.2.8 and 3.2.9 the normal deflections of some points on the compression flanges of the test beams A1 and A2 respectively are given as a function of applied load. Figs. 3.2.10 and 3.2.11 show the distribution of normal deflections of the compression flanges for some load levels. In all the Figs. 3.2.8-3.2.11, the normal deflections are considered to be zero at a distance from the webs of 14 mm for A1 and 9 mm for A2.

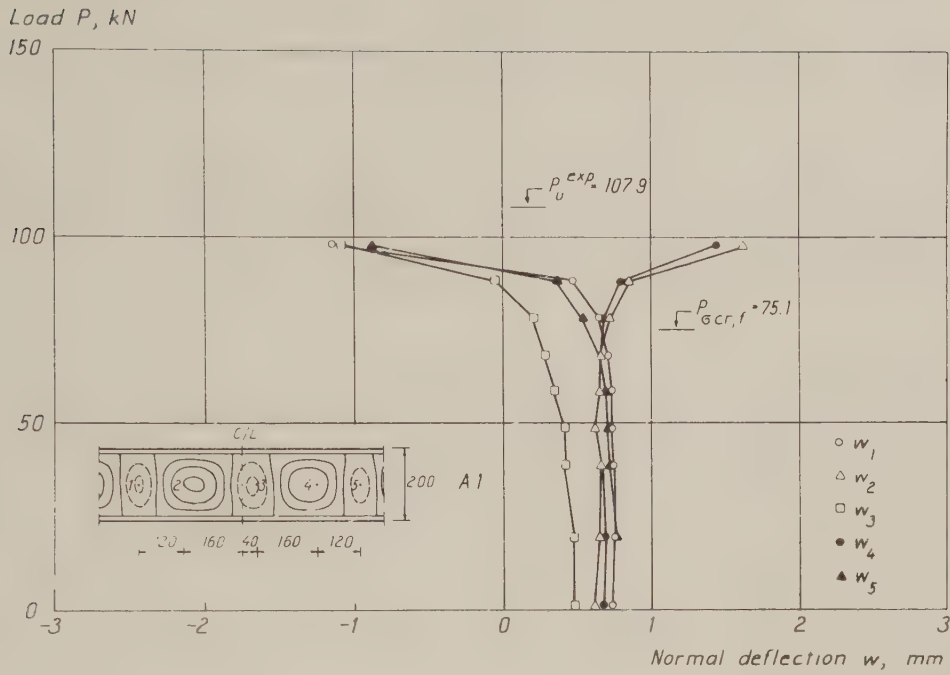


Fig. 3.2.8. Load versus normal deflection for some points on the compression flange of test beam A1.

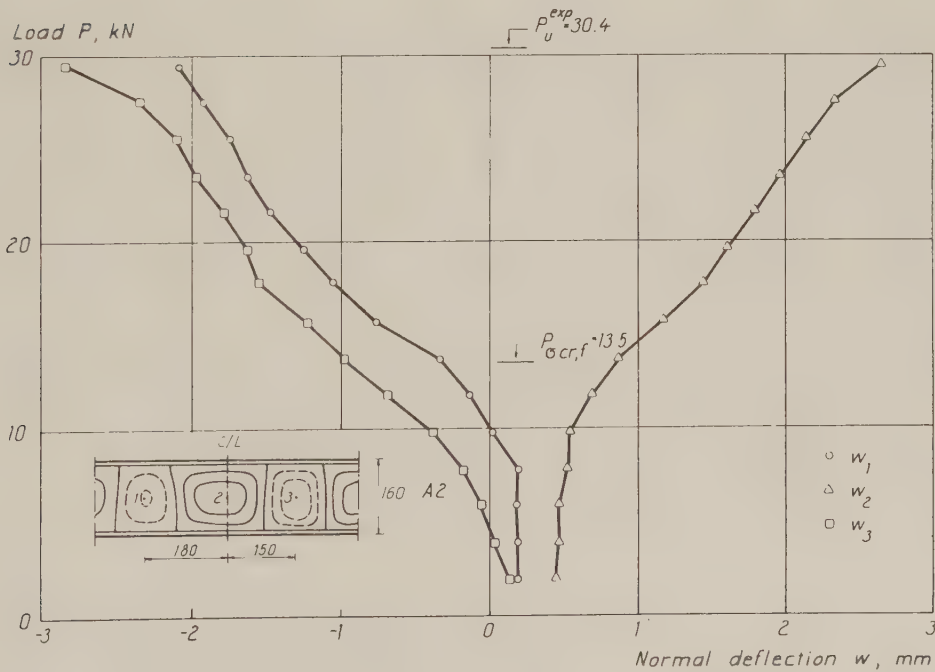


Fig. 3.2.9. Load versus normal deflection for some points on the compression flange of test beam A2.

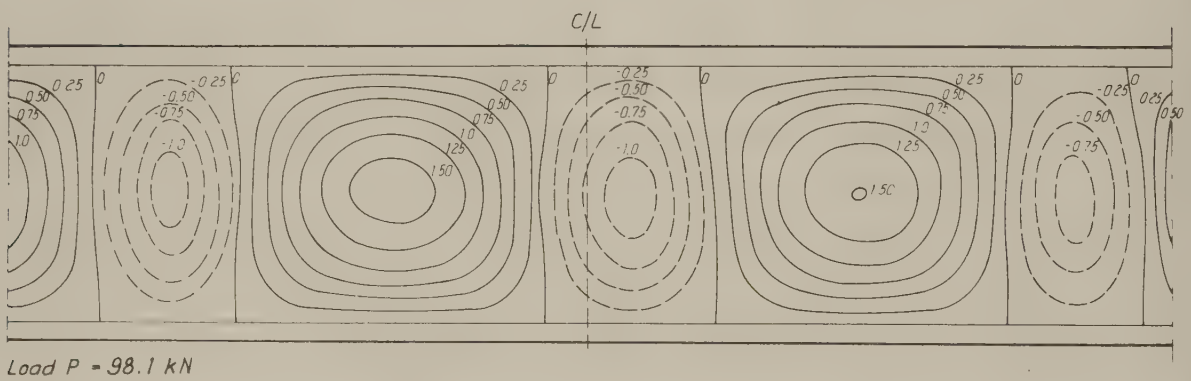
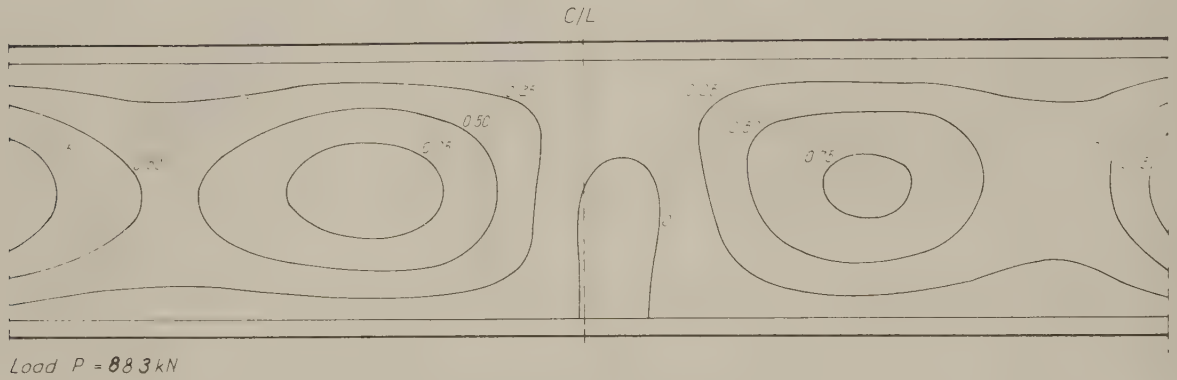
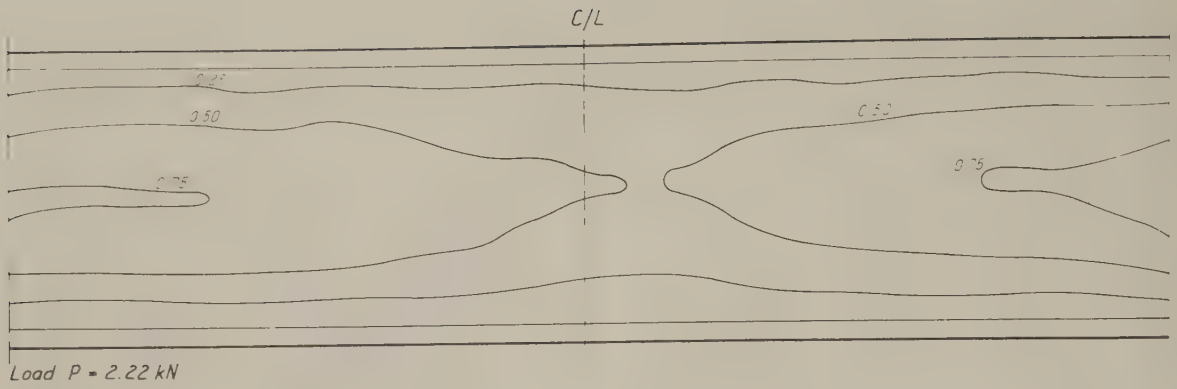


Fig. 3.2.10. Normal deflection of compression flange for test beam A1.

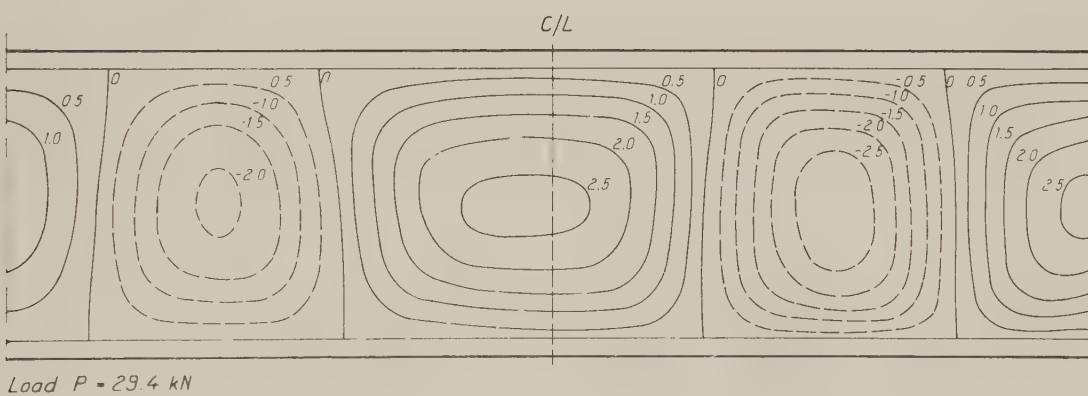
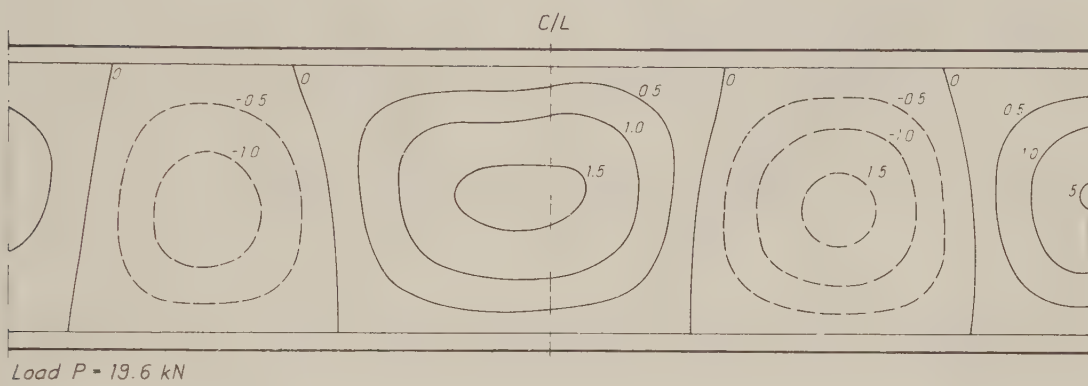
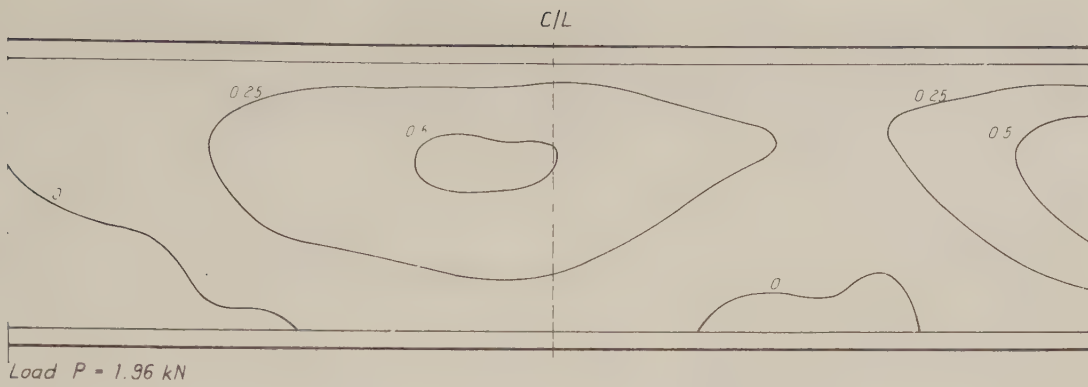


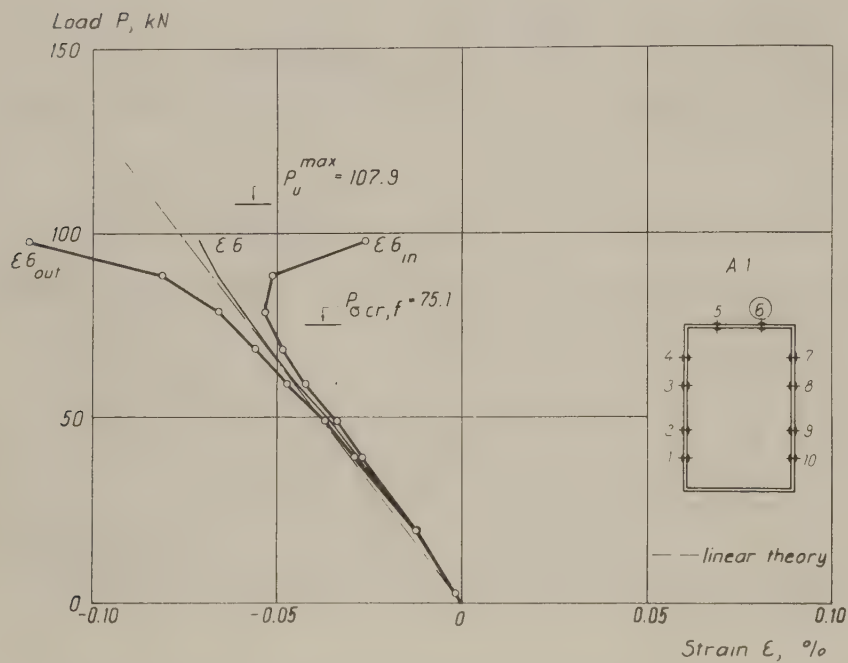
Fig. 3.2.11. Normal deflection of compression flange for test beam A2.

Strains.

Figs. 3.2.12 and 3.2.13 show load-strain relations for some points in the compressed parts of the webs and flanges of the test beams A1 and A2. In all points the strains at the outer and inner sides of the sheet are given as well as their mean value. Figs. 3.2.14 and 3.2.15 give the distribution of strains in the central sections of the test beams A1 and A2 respectively for three load levels. As the

number of points at which strain measurements have been made are very few, the strains at the corners have been obtained by extrapolation of the strains in the tension parts of the webs.

a)



b)

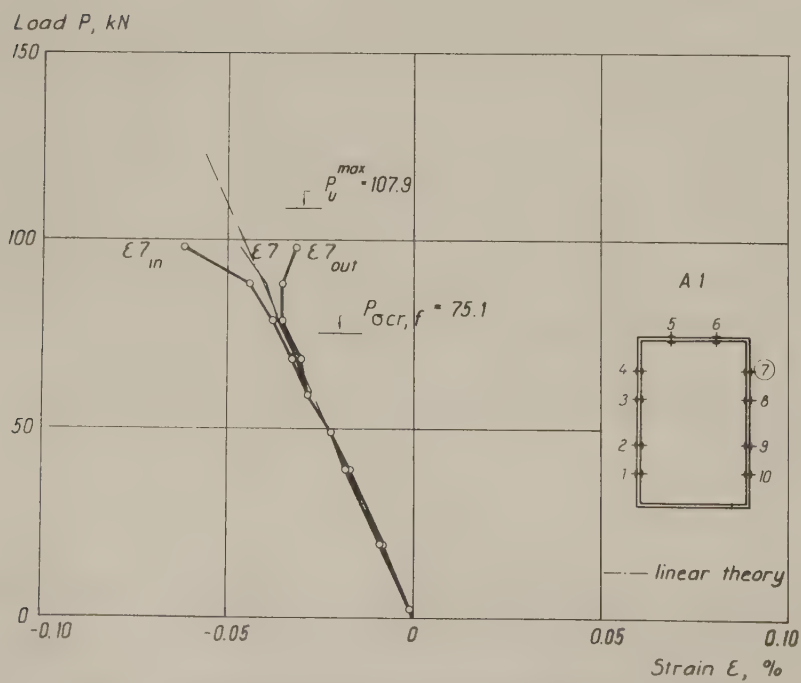
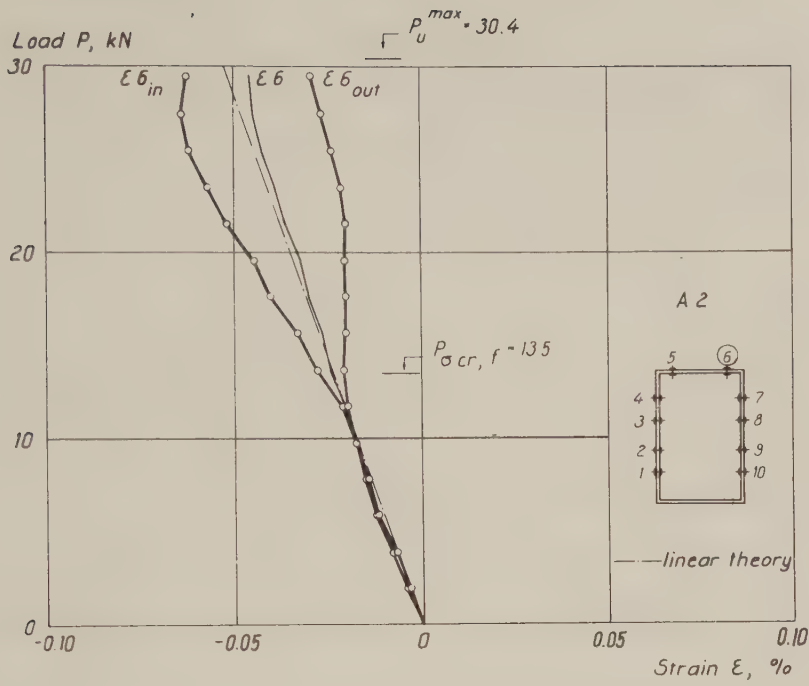


Fig. 3.2.12 a-b. Load-strain curves for test beam A1.

a)



b)

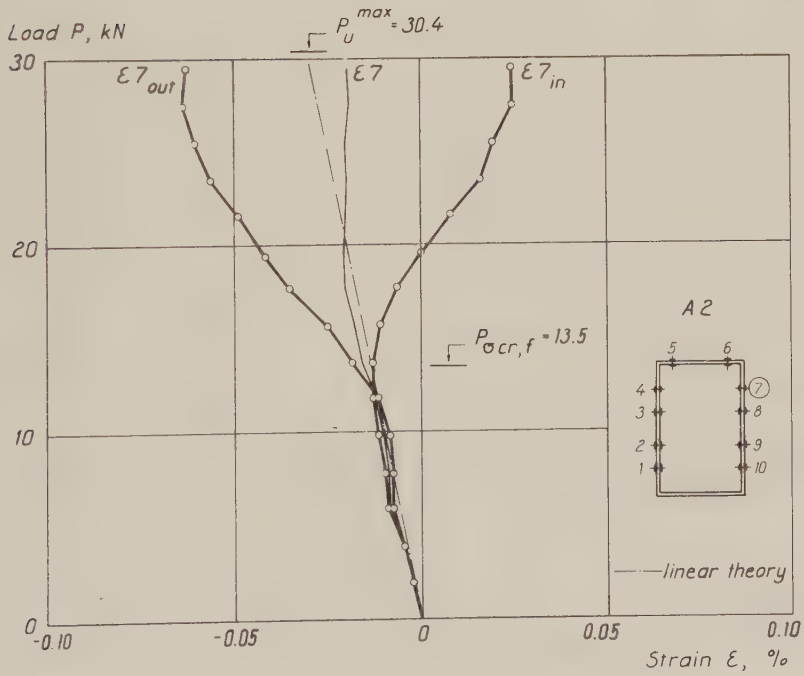


Fig. 3.2.13 a-b. Load-strain curves for test beam A2.

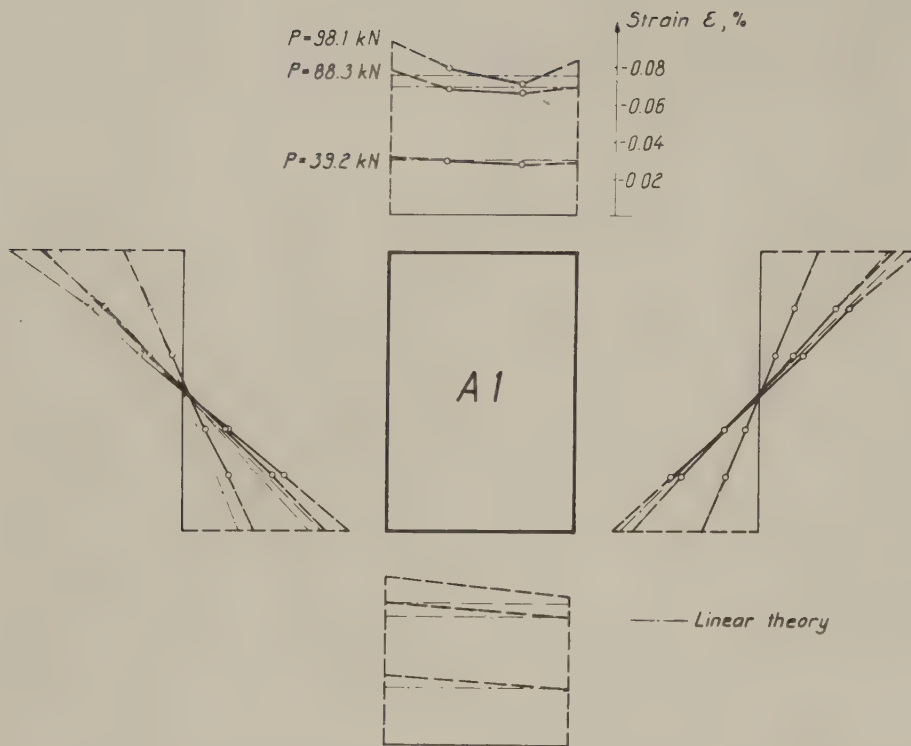


Fig. 3.2.14. Distribution of strains in the central section of test beam A1.

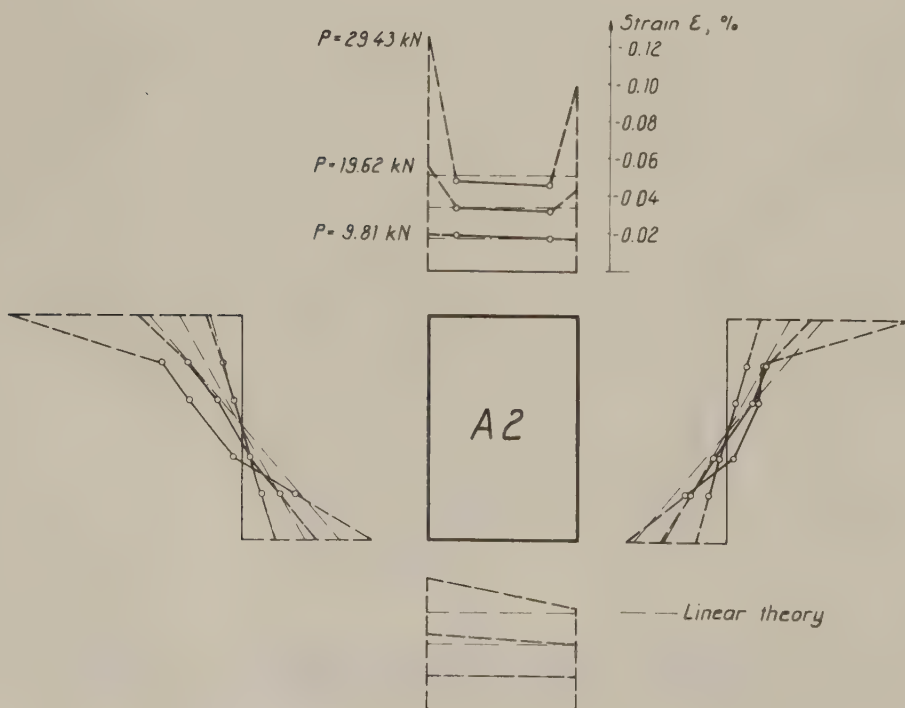


Fig. 3.2.15. Distribution of strains in the central section of test beam A2.

From the measured strain distribution of the webs, the position of the neutral axis can be calculated. This has been done by assuming a linear strain distribution between adjacent strain gauges. The results are summarized in Figs. 3.2.16 and 3.2.17, where the movement of the neutral axis of each of the two webs can be studied for test beams A1 and A2 respectively.

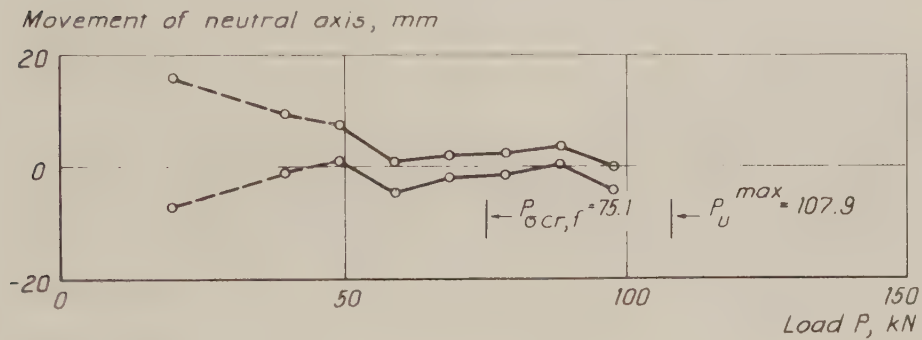


Fig. 3.2.16. Relationship between movement of neutral axis and applied load P for the two webs of test beam A1.

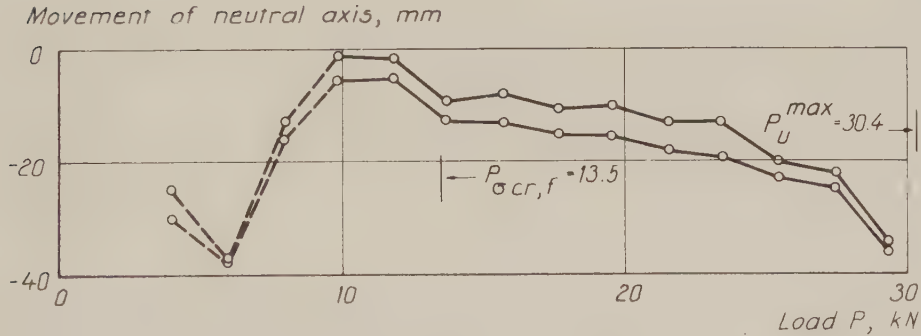


Fig. 3.2.17. Relationship between movement of neutral axis and applied load P for the two webs of test beam A2.

Character of failure.

The character of failure is described in Section 3.3.8.

Ultimate loads.

Measured ultimate loads P_U^{exp} and ultimate moments M_U^{exp} of the test beams are given in Table 3.2.6.

Table 3.2.6. Measured ultimate loads and ultimate moments for test beams A1 and A2.

Test beam	P_u^{exp} kN	M_u^{exp} kNm
A1	107.91	37.77
A2	30.41	8.36

3.2.7 Discussion of results.

In Table 3.2.5, some cross-sectional constants and characteristic critical stresses are given. As can be seen, test beam A2 is much more slender than A1. Thus it may be expected that buckling phenomena will have a greater influence on the behaviour of test beam A2.

Deflections.

According to elementary linear beam theory, the deflections are given by:

$$v_C = v_D = \frac{P}{12EI} L_1^2 (3L - 4L_1) + \frac{PL_1}{2GA_w} \quad (3.2.1)$$

$$v_{C/L} = \frac{P}{48EI} L_1 (3L^2 - 4L_1^2) + \frac{PL_1}{2GA_w} \quad (3.2.2)$$

where v_C , v_D and $v_{C/L}$ are defined in Figs. 3.2.6 and 3.2.7. In this case no consideration has been given to the stiffening effects of the screw joints. The results of the calculations above are shown in Figs. 3.2.6 and 3.2.7. The agreement between measured and calculated values is not good. Possible explanations were given in Section 3.2.6.

Normal deflections.

The normal deflections of the compression flanges of the test beams A1 and A2 may be compared by studying Figs. 3.2.8-3.2.11. Test beam A2, for which the ratio between the ultimate load P_u^{exp} and the characteristic critical load of the compression flange $P_{\sigma cr, f}$ is

rather high, shows gradually increasing normal deflections even at relatively low loads. The curves in Fig. 3.2.9 are of the same type as the theoretical ones in Fig. 3.1.33. In the vicinity of the ultimate load, the deflections tend to increase more strongly again which is an effect of beginning plasticization. For test beam A1, where the ratio between P_u^{exp} and $P_{\sigma\text{cr},f}$ is low, the deflections do not increase appreciably until the load has passed the critical load $P_{\sigma\text{cr},f}$ and plasticization has also started.

The dependence of size and shape of the initial normal deflections on the behaviour of the buckles during loading history can be studied in Figs. 3.2.10 and 3.2.11 where it is evident that the initial normal deflections are decisive for the buckle pattern which arises.

Strains.

When the applied load approaches the characteristic critical load of the compression flange $P_{\sigma\text{cr},f}$, the strains in the outer and inner sides of the buckles begin to separate from each other on account of the bending of the plate. This is confirmed by the curves in Figs. 3.2.12 and 3.2.13. Further it can be seen that, in a given test beam, the strains in the compressed flange and the strains in the compressed parts of the webs tend to separate at about the same load level.

The distributions of strains in test beams A1 and A2 are rather different (cf. Figs. 3.2.14 and 3.2.15). Thus for test beam A1 the strains are in fairly good agreement with the linear theory even at high loads. For test beam A2, however, the strains deviate from linear theory even at relatively small loads.

The position of the neutral axis for test beam A1, see Fig. 3.2.16, is almost constant during the loading history. For test beam A2, on the other hand, the neutral axis tends to fall at high loads, which is the cause of the deviation from linear theory.

Ultimate loads.

The ultimate loads and ultimate moments will be discussed in Section 3.3.9.

3.3 Single-span beam. Main experiment.

3.3.1 Purpose.

With the introductory experiments as a basis, a large series of experiments was planned comprising single-span beams subjected to pure bending. The dimensions of the test beams were chosen in such a way that buckling phenomena would be of decisive importance.

The aims of the experiments were

- to study the influence of buckling phenomena and their interaction in webs and flanges,
- to compare the results with theories of post-buckling,
- to investigate different relations between load and deformation up to failure,
- to determine the nature of failure and
- to study the influence of steel quality.

3.3.2 Scope.

The experimental investigation included fourteen simply supported beams loaded with two equal symmetrically located point loads according to Fig. 3.3.1. All the test beams were made of plate of the same thickness. Different parameters examined were

- 1) the ratio of height to width of the beam section (h/b).
- 2) the ratio of width to thickness of the compressed flange (b/t).
- 3) the ratio of the distance between a support and the nearest point load to the height of the beam section (L_1/h).

Two qualities of steel were examined, a cold-rolled sheet COR-TEN A and secondly a cold-rolled sheet of special quality. The materials

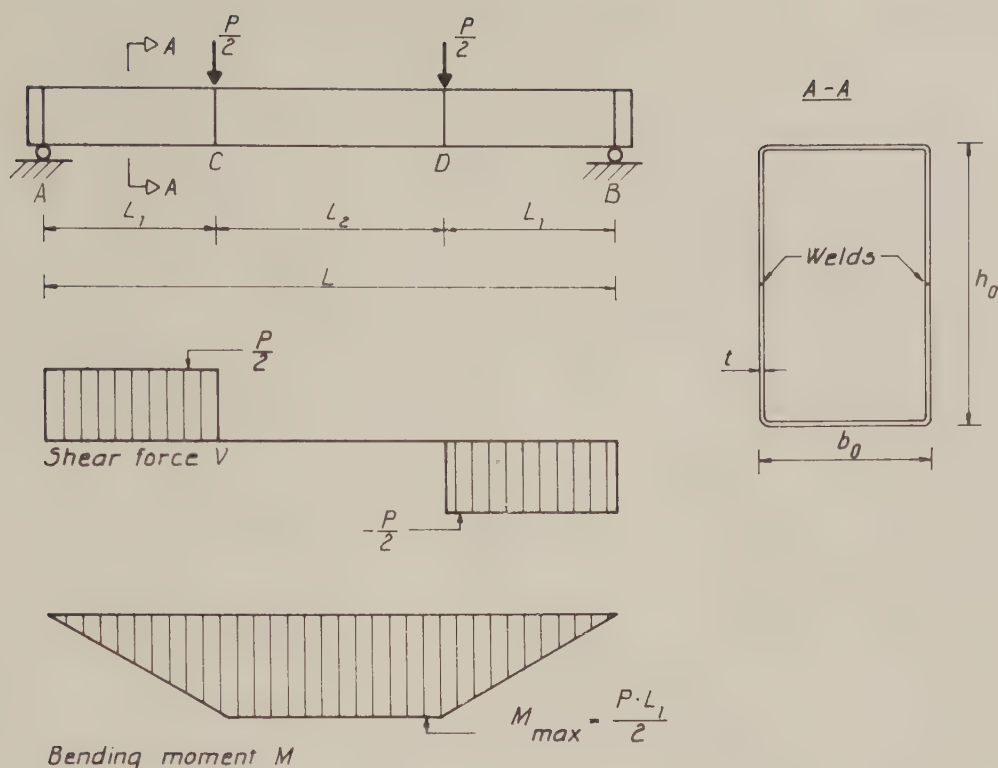


Fig. 3.3.1. Distribution of load, shear force and bending moment for the test beams B1-B7 and C1-C7.
Notations for cross section.

were chosen to permit the examination on the one hand of a material with usual strain properties and on the other a brittle and more cold worked material to see the influence of ductility. Thus the stress-strain relations for the materials were rather different. Seven experiments were performed on each of the steel qualities.

In the investigation of the influence of different parameters, one parameter was varied at a time while the others were kept constant. The choice of different parameters is illustrated in Fig. 3.3.2. The designations B1-B7 refer to beams made of the cold-rolled special quality while the designations C1-C7 refer to beams made of the cold-rolled sheet COR-TEN A. The beams B1 and C1 represent the standard level for which the following notations are introduced:

$$h_0 = h_{0s}$$

$$b_0 = b_{0s}$$

$$L_1 = L_{1s}$$

$$L_2 = L_{2s}$$

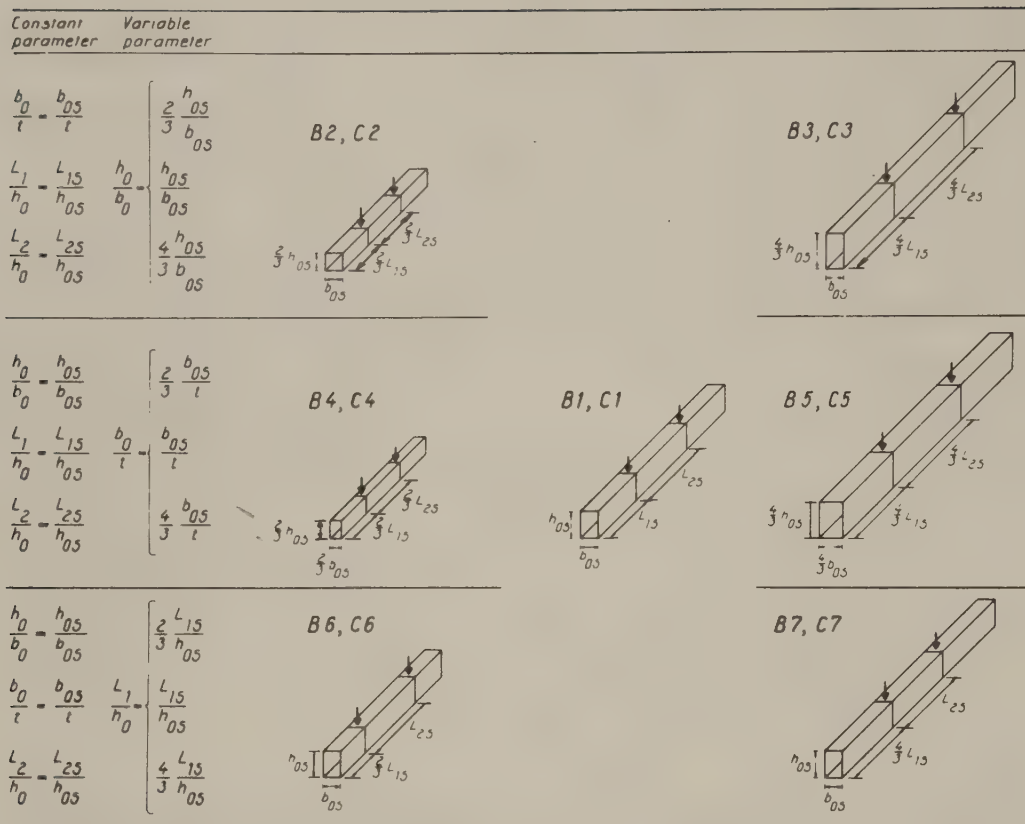


Fig. 3.3.2. A survey of investigated parameters.

The nominal dimensions of the test beams are given in Table 3.3.1.

Table 3.3.1. Nominal beam dimensions.

Test beam	b_0 mm	h_0 mm	t mm	L_1 mm	L_2 mm	L mm
B1, C1	200	300	1.5	700	1200	2600
B2, C2	200	200	1.5	467	800	1734
B3, C3	200	400	1.5	933	1600	3466
B4, C4	133	200	1.5	467	800	1734
B5, C5	267	400	1.5	933	1600	3466
B6, C6	200	300	1.5	467	1200	2134
B7, C7	200	300	1.5	933	1200	3066

3.3.3 Loading and support arrangements.

The principal loading and support arrangements are shown in Fig. 3.3.3. The load was transmitted by a fixed hydraulic jack. To permit the relation between load and deformation to be watched after the ultimate load was attained, the jack was equipped with an adjustable valve which made it possible continuously to lower the load. The load was divided into two equal concentrated loads by means of two adjacent I-beams and transferred to the webs of the test beam by screw joints.

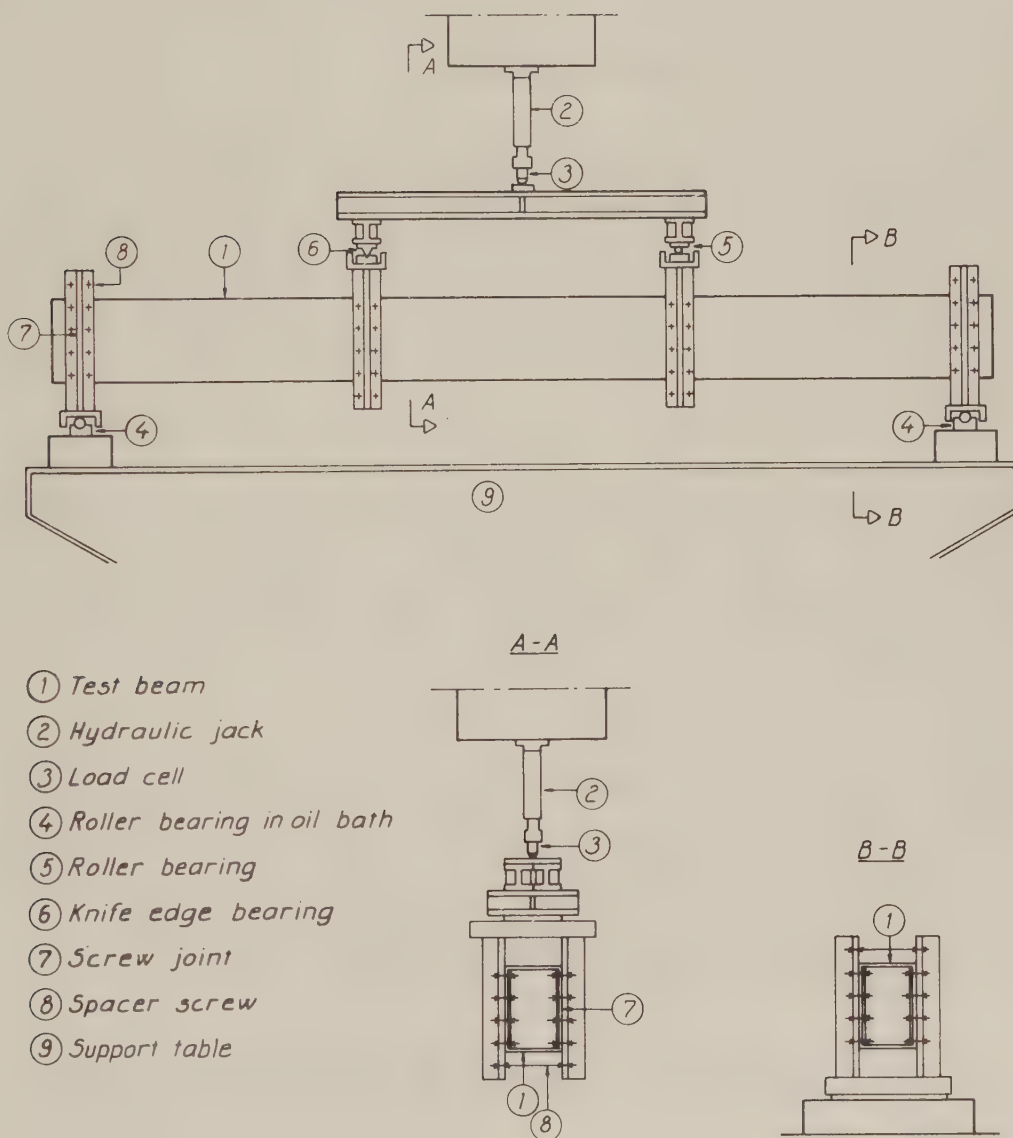


Fig. 3.3.3. Principal arrangement of loading and support devices.

The supports of the test beams consisted of roller bearings in an oil bath. The effect of the friction in the supports was not completely negligible. Thus the bending moments of the registered loads were probably a few per cent to high.

The screw joints were made to transmit the load as uniformly as possible to the webs. Thus the screw joints were provided with resisting pieces of flat bar inside the test beams. In fact the screw joints worked as high tensile bolt connections. To prevent deformation at the screw joints in the transverse direction of the test beams, they were fixed in couples by spacer screws.

3.3.4 Dimensions. Material properties.

The test beams were made of cold formed U-sections which were welded together. The cold forming process used was brake forming. For the test beams B1, B2, B6, C1, C2 and C6, the welding was carried out by spot welding followed by continuous welding by hand. For the rest of the test beams, the continuous welding was done in an automatic welding machine.

The material of the test beams in series B is usually used as rotor ring sheet in electric motors. The material consists of a killed cold-rolled sheet that has been subjected to recrystallization annealing and thereafter to reducing rolling with a reduction in thickness of 10-15 %.

The material of the test beams in series C is called COR-TEN A and has a resistance to corrosion many times better than ordinary carbon steel. The material consists of a killed cold-rolled sheet that has been subjected to recrystallization annealing and thereafter to trim rolling with a reduction in thickness of 0.5-1 %.

Results of a chemical analysis of the two materials are given in Table 3.3.2.

Table 3.3.2. Results of the manufacturer's chemical analysis of the cold-rolled sheet.

Steel quality	C %	Si %	Mn %	P %	S %	N %	Cr %	Cu %	Ni %
B	0.09	0.17	0.45	0.036	0.021	0.007			
C	0.12	0.40	0.38	0.084	0.021		0.72	0.35	0.22

Fig. 3.3.4 shows the positions where tensile and bending test pieces have been cut from unused cold-rolled sheets having the same material properties as the test beams. The results of the tensile tests, carried out according to Appendix B, are given in Table 3.3.3. As can be seen, the material properties varied considerably over the plate width for the steel quality B. Since the material properties of the compressed flanges are decisive for the load-carrying capacity of the test beams, tensile test pieces were taken according to Fig. 3.3.5 from test beams B1-B7. The tensile test pieces were taken after the loading tests had been carried out and they were therefore cut out near the ends of the beams, where they were least affected by prior compressive strains. The material properties for steel quality C were much more uniform, so that no tensile test pieces were cut from the test beams in this case. Table 3.3.4 shows the results from these tensile tests, which were carried out according to the procedure described in Appendix B. Results of tensile tests carried out by the manufacturer of the cold-rolled sheet are given in Table 3.3.5. A comparison of the results in Tables 3.3.3 and 3.3.5 shows that for steel quality B, the values in Table 3.3.5 fall between the lower and upper limits of those in Table 3.3.3. For steel quality C, the values in Table 3.3.5 for σ_y and σ_u seem to be somewhat too high. Hereafter we shall use the tensile stress according to Table 3.3.4 for the test beams B1-B7 and the tensile stress according to Table 3.3.3 for test beams C1-C7. In Fig. 3.3.6, two typical examples of stress-strain relations are shown.

The dynamic modulus of elasticity for the two materials was determined by testing bending test pieces according to the procedure described in Appendix C. Fig. 3.3.4 shows where the bending test

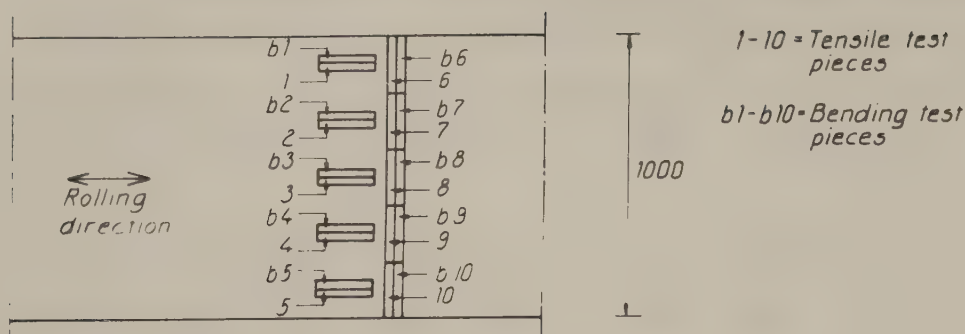


Fig. 3.3.4. Positions at which tensile and bending test pieces were taken from an unused cold-rolled sheet.

pieces have been cut out. Results from the tests are given in Table 3.3.6. It will be assumed hereafter that $E = 2.06 \cdot 10^5 \text{ N/mm}^2$.

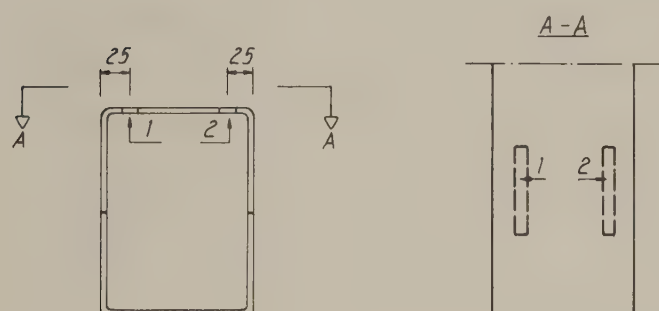


Fig. 3.3.5. Positions at which tensile test pieces were taken from test beams B1-B7.

Measured dimensions of the test beams are shown in Table 3.3.7. The dimensions are given as mean values. Cross-sectional constants and critical stresses, defined according to Appendix A, are given in Table 3.3.8.

Table 3.3.3. Results of tensile tests carried out on pieces from cold-rolled sheets of materials B and C.
MV = mean value.

Steel quality	Rolling direction	Tensile test piece	t mm	σ_y N/mm ²	σ_u N/mm ²	δ_5 %
B	//	:1	1.53	414	503	(>25)
		:2	1.55	500	551	(>20)
		:3	1.55	619	661	(>14)
		:4	1.55	561	613	(>16)
		:5	1.56	436	512	(>19)
		MV:		506	568	
	⊥	:6	1.52	459	526	27
		:7	1.55	529	583	(>14)
		:8	1.54	645	679	14
		:9	1.55	603	636	(>13)
		:10	1.55	489	547	(>17)
		MV:		545	594	
C	//	:1	1.55	298	488	38
		:2	1.56	315	496	(>28)
		:3	1.57	317	488	(>27)
		:4	1.57	309	488	(>27)
		:5	1.55	299	489	(>27)
		MV:		308	490	
	⊥	:6	1.55	310	498	39
		:7	1.57	325	498	(>28)
		:8	1.57	336	496	(>27)
		:9	1.57	325	501	34
		:10	1.56	313	499	38
		MV:		322	498	

Table 3.3.4. Results of tensile tests carried out on pieces from test beams B1-B7. MV = mean value.

Test beam	Tensile test piece	t mm	σ_y N/mm ²	σ_u N/mm ²	δ_5 %
B1	:1	1.54	475	545	26
	:2	1.53	416	505	33
	:MV		445	525	30
B2	:1	1.60	471	541	28
	:2	1.61	569	617	21
	:MV		520	579	25
B3	:1	1.57	496	564	26
	:2	1.58	569	619	22
	:MV		533	592	24
B4	:1	1.54	439	525	27
	:2	1.54	469	542	28
	:MV		454	534	28
B5	:1	1.58	420	504	31
	:2	1.59	490	554	25
	:MV		455	529	28
B6	:1	1.53	476	547	24
	:2	1.52	428	511	31
	:MV		452	529	28
B7	:1	1.57	589	636	-
	:2	1.56	623	664	20
	:MV		606	650	-

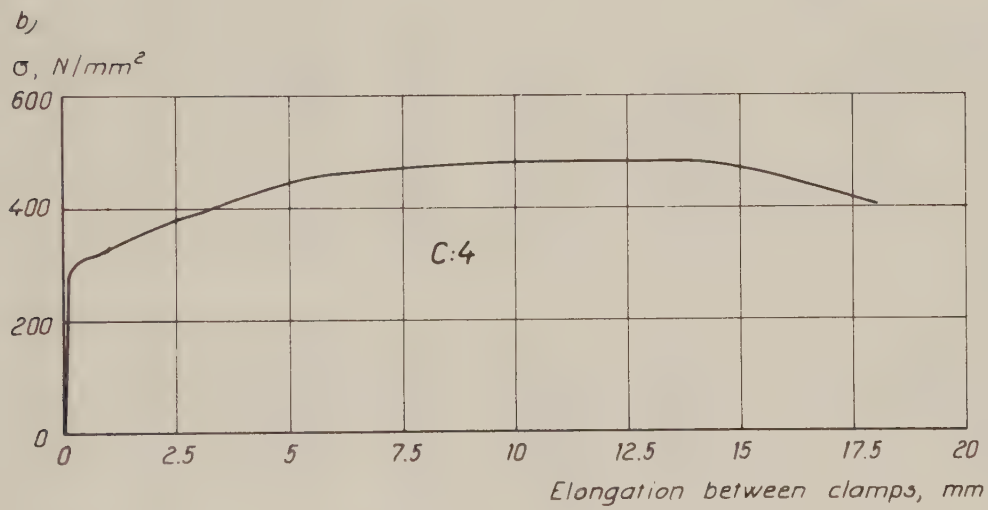
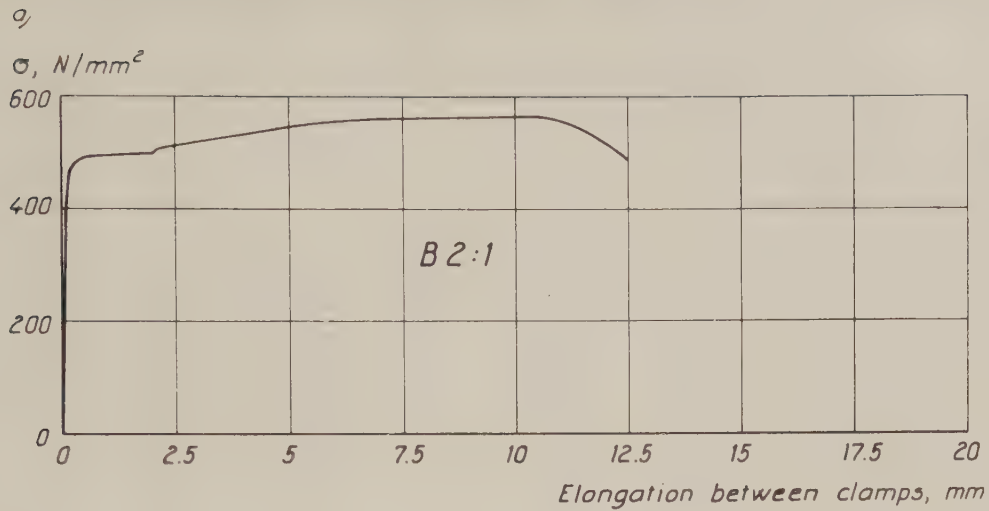


Fig. 3.3.6. Typical stress-strain curves for
a) steel quality B
b) steel quality C.

Table 3.3.5. Results of tensile tests carried out by the manufacturer of the cold-rolled sheet.

Steel quality	Rolling direction	σ_y N/mm^2	σ_u N/mm^2	δ_5 %
B	//	461	538	27
	⊥	523	571	20
C	//	330	499	37
	⊥	350	506	43

Table 3.3.6. Results of vibration tests carried out on bending pieces from steel sheet of materials B and C.
MV = mean value.

Bending test piece	Rolling direction	t mm	b mm	L mm	m g	f Hz	$E \cdot 10^{-5}$ N/mm ²
B:b1	//	1.52	20.09	205.3	48.85	191.9	2.08
:b2	//	1.54	20.08	205.3	49.42	194.7	2.10
:b3	//	1.54	20.06	205.3	49.51	195.2	2.10
:b4	//	1.54	20.05	205.3	49.53	195.6	2.11
:b5	//	1.54	20.04	205.3	49.27	194.4	2.07
MV:							2.09
B:b6	⊥	1.52	19.98	196.6	46.41	214.2	2.17
:b7	⊥	1.54	20.02	196.6	46.97	217.1	2.19
:b8	⊥	1.54	19.98	196.6	47.14	219.1	2.24
:b9	⊥	1.54	19.98	196.6	47.19	219.4	2.24
:b10	⊥	1.54	19.97	196.6	46.95	217.7	2.22
MV:							2.21
C:b1	//	1.54	20.03	205.3	49.09	193.4	2.07
:b2	//	1.56	20.04	205.3	49.82	196.3	2.08
:b3	//	1.57	20.04	205.3	50.00	196.8	2.06
:b4	//	1.56	20.01	205.3	49.74	196.1	2.06
:b5	//	1.55	20.04	205.3	49.47	195.2	2.07
MV:							2.07
C:b6	⊥	1.54	19.99	196.6	46.96	215.7	2.15
:b7	⊥	1.562	19.97	196.6	47.67	218.9	2.16
:b8	⊥	1.567	19.98	196.6	47.84	219.3	2.16
:b9	⊥	1.565	19.97	196.6	47.72	218.8	2.15
:b10	⊥	1.545	19.97	196.6	47.15	217.0	2.17
MV:							2.16

Table 3.3.7. Measured dimensions of test beams B1-B7 and C1-C7.

Test beam	b_o mm	h_o mm	t mm	R mm	L_1 mm	L_2 mm	L mm
B1	202	302	1.52	3.5	697	1201	2595
B2	202	202	1.57	3.5	467	801	1735
B3	202	402	1.57	3	928	1604	3460
B4	134.7	200.4	1.52	3.5	465	799	1729
B5	267.5	400.7	1.56	3.5	933	1604	3470
B6	203	302	1.52	3	468	1195	2131
B7	202	302	1.54	3	935	1201	3071
C1	202	302.5	1.55	4.5	697	1204	2598
C2	202	202	1.56	3	466	799	1731
C3	202	402	1.56	4.5	938	1595	3471
C4	133.2	201.5	1.56	3	463	805	1731
C5	268.1	401.6	1.57	4.5	933	1603	3469
C6	202	302	1.55	4.5	467	1203	2137
C7	202	302	1.55	4.5	931	1196	3058

Table 3.3.8. Cross-sectional constants and critical stresses for test beams B1-B7 and C1-C7.

Test beam	b/t	h/t	$I_o \cdot 10^{-4}$ mm ⁴	$\sigma_{cr,f}$ N/mm ²	$\sigma_{cr,w}$ N/mm ²	σ_{cr} N/mm ²
B1	131.9	197.7	2063	42.8	113.9	56
B2	127.7	127.7	843	45.7	273.0	61
B3	127.7	255.1	4203	45.7	68.4	57
B4	87.6	130.8	600	97.0	260.0	126
B5	170.5	255.9	4958	25.6	68.0	33
B6	132.6	197.7	2070	42.4	113.9	55
B7	130.2	195.1	2090	44.0	116.9	57
C1	129.3	194.2	2111	44.5	118.0	58
C2	128.5	128.5	838	45.1	269.5	60
C3	128.5	256.7	4176	45.1	67.5	56
C4	84.4	128.2	618	104.6	270.9	136
C5	169.8	254.8	5023	25.8	68.5	34
C6	129.3	193.8	2103	44.5	118.4	58
C7	129.3	193.8	2103	44.5	118.4	58

3.3.5 Residual stresses. Influence of welding and brake forming.

To obtain a picture of the magnitude of residual stresses caused by welding, strain gauges were fastened to test beam C7. The positions of the strain gauges are evident in Fig. 3.3.7. The strain gauges were fastened after the beam C7 had been tested. To minimize the influence of previous testing they were located near the end of the beam. As can be seen in Fig. 3.3.7, strain gauges were placed opposite to each other on the inside and outside of the test beam to obtain the mean value of the strains. The test beam was first sawn off at section S-S (see Fig. 3.3.7) and the strains were measured. These strains are shown in Fig. 3.3.8 a. Small pieces of plate were then sawn out around the strain gauges and the additional strains were determined as given in Fig. 3.3.8 b. The total residual strains are given in Fig. 3.3.8 c. At the welds there are residual tensile stresses of the same magnitude as the yield stress. These zones of tension extend only to a very limited area around the welds. The rest of the cross section is under compression, except for the corners where tension has been introduced during brake forming. The measurements at the corners are somewhat uncertain because of the difficulty of fastening a strain gauge on a curved surface. The measured tensile stresses at the corners do not seem to appear by chance. Thus a theoretical and experimental investigation by Ingvarsson (1975) indicates that there are residual stresses due to cold-forming not only in the tangential direction but also in the longitudinal direction of channel members.

The magnitude of the measured residual compression stresses in the webs and flanges is mostly between 10 and 40 N/mm². The uneven distribution of the compression strains depends on the proximity of the beam end and on the fact that the single U-sections may have become somewhat bent during welding because of not perfectly straight edges. A mean value of the residual compression stresses in the flanges of the actual beam ought to be about 20 N/mm².

The following discussion can provide a basis for the estimation of the magnitude of the residual stresses appearing in the different test beams. As the same welding procedure has been used for all test beams, the stresses introduced round the welds ought to be of the same order of magnitude. Consequently the compressive stresses,

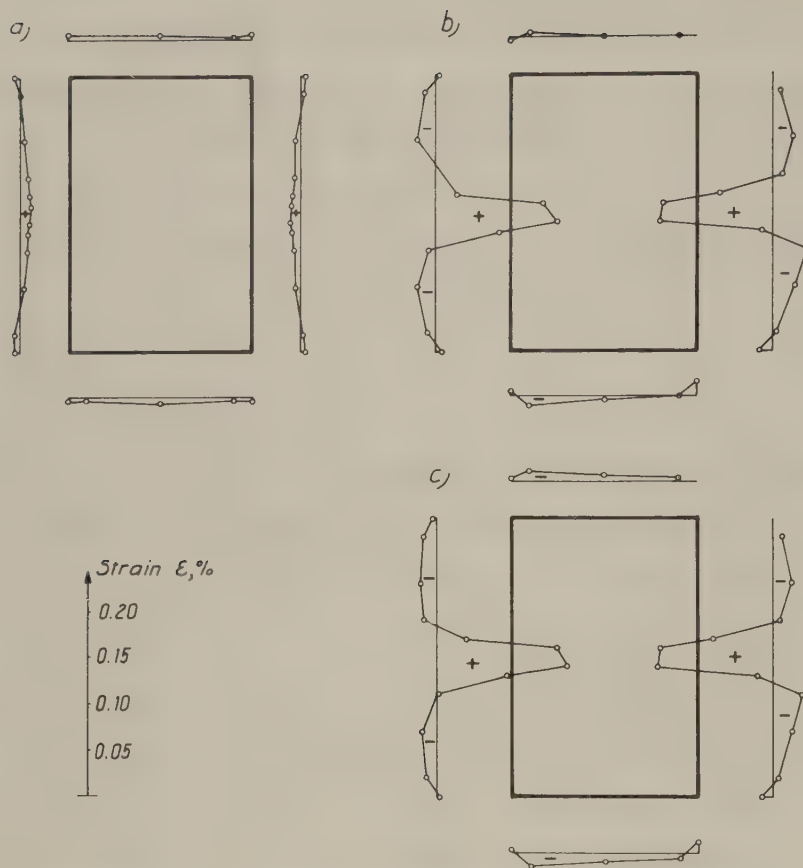


Fig. 3.3.8. Initial strains for test beam C7.

- a) Released strains when the beam was sawn off in section S-S.
- b) Additional released strains when small pieces around the strain gauges were sawn out.
- c) Total released strains.

Another interesting effect of the brake forming process is a strengthening of the material in the corners. Karren (1967) has studied this problem and recommends the following expression for estimating the ratio of corner yield stress σ_{yc} to virgin yield stress σ_y

$$\frac{\sigma_{yc}}{\sigma_y} = \frac{\beta}{\left(\frac{R}{t}\right)^\alpha} \quad (R < 7t) \quad (3.3.1)$$

$$\text{where } \beta = 3.69 \frac{\sigma_u}{\sigma_y} - 0.819 \left(\frac{\sigma_u}{\sigma_y}\right)^2 - 1.79$$

$$\alpha = 0.192 \frac{\sigma_u}{\sigma_y} - 0.068$$

Table 3.3.9. Estimated residual stresses σ_i . Influence on critical stress σ_{cr} . Comparison with critical stress of the flange $\sigma_{cr,f}$.

Test beam	σ_i N/mm ²	$\sigma_{cr} - \sigma_i$ N/mm ²	$\sigma_{cr,f}$ N/mm ²
B1	20	36	42.8
B2	25	36	45.7
B3	17	40	45.7
B4	30	96	97.0
B5	15	18	25.6
B6	20	35	42.4
B7	20	37	44.0
C1	20	38	44.5
C2	25	35	45.1
C3	17	39	45.1
C4	30	106	104.6
C5	15	19	25.8
C6	20	38	44.5
C7	20	38	44.5

According to the AISI Specification for the Design of Cold-Formed Steel Structural Members, the formula is not applicable when σ_u/σ_y is less than 1.2. Table 3.3.10 gives results obtained using Eq. (3.3.1). For the test beams B1-B7 and C1-C7, the corner areas are always less than 5 per cent of the total cross-sectional area. In the post-critical range, the effective area of the cross section decreases and the influence of the corners consequently increases. The increment in yield stress thus obtained can be regarded as a strength reserve.

To obtain some idea of the stiffening effect of the welds, some bending test pieces were cut out from the welds and tested according to the procedure described in Appendix C. The results of these tests are given in Table 3.3.11. The total moment of inertia, I_{tot} , was determined from the vibration tests. The moment of inertia of the plate, I_{plate} , was calculated from the measured dimensions. The moment of inertia caused by welding, I_{weld} , can then easily be obtained as

the difference between I_{tot} and I_{plate} .

Table 3.3.10. Increment in yield stress for the corner material due to brake forming.

Test beam	σ_y N/mm ²	σ_u N/mm ²	R mm	t mm	$\frac{R}{t}$	$\frac{\sigma_u}{\sigma_y}$	$\frac{\sigma_{yc}}{\sigma_y}$
B1	445	525	3.5	1.52	2.30	1.18	(1.24)
B2	520	579	3.5	1.57	2.23	1.11	(1.16)
B3	533	592	3	1.57	1.91	1.11	(1.18)
B4	454	534	3.5	1.52	2.30	1.18	(1.24)
B5	455	529	3.5	1.56	2.24	1.16	(1.23)
B6	452	529	3	1.52	1.97	1.17	(1.27)
B7	606	650	3	1.54	1.95	1.07	(1.11)
C1	308	490	4.5	1.55	2.90	1.59	1.56
C2	308	490	3	1.56	1.92	1.59	1.72
C3	308	490	4.5	1.56	2.88	1.59	1.56
C4	308	490	3	1.56	1.92	1.59	1.72
C5	308	490	4.5	1.57	2.87	1.59	1.57
C6	308	490	4.5	1.55	2.90	1.59	1.56
C7	308	490	4.5	1.55	2.90	1.59	1.56
A1	221	358	6	2.50	2.40	1.62	1.65
A2	232	351	3	1.31	2.29	1.51	1.59

The occurrence of I_{weld} depends on three main factors namely

- 1) the weld causes a local increase in the thickness of the web,
- 2) the two sheets lie in non-parallel planes which intersect each other at the weld,
- 3) the two sheets lie in planes which are parallel but slightly displaced from each other at the weld.

The influence of the second factor would be greater if the widths of the test pieces were increased. Thus I_{weld} in Table 3.3.11 is

only a local value.

Table 3.3.11. Results of vibration tests carried out on bending pieces from the welds of test beams B2, C2 and C6.

Test piece	t mm	b mm	L mm	m g	f Hz	I_{tot} mm ⁴	I_{plate} mm ⁴	I_{weld} mm ⁴
B2:b1	1.565	20.40	200.2	54.95	260	11.3	6.5	4.8
:b2	1.55	20.07	200.9	54.80	248	10.4	6.2	4.2
:b3	1.57	19.58	201.2	53.76	255	10.8	6.3	4.5
C2:b1	1.585	20.11	200.4	54.47	254	10.7	6.7	4.0
:b2	1.58	19.61	199.2	52.08	242	9.1	6.4	2.7
:b3	1.57	19.75	201.6	52.26	243	9.5	6.4	3.1
C6:b1	1.545	20.06	201.4	54.49	247	10.2	6.2	4.0
:b2	1.545	20.35	199.1	52.01	246	9.4	6.3	3.1
:b3	1.55	20.06	200.7	52.68	240	9.3	6.2	3.1

3.3.6 Influence of shear lag.

When a box beam is subjected to a pure bending moment, normal stresses will be introduced in the flanges through shear stresses transferred from the webs. These shear stresses will produce shear strains in the flanges. If the flanges are wide in relation to their length, these shear strains have the effect that the normal bending stresses in the flanges decrease with increasing distance from the web. This phenomenon is known as shear lag. It results in a non-uniform stress distribution across the width of the flange. For design purposes, the simplest way of accounting for this stress variation is to assume that the non-uniformly stressed flange of stated width b is replaced by one of reduced effective width b_e which is subjected to uniform stress.

In the AISI Specification for the Design of Cold-Formed Steel Structural Members, the influence of shear lag in box beam sections is accounted for by the use of an equation involving the ratio of span length (L) to half the distance between the webs ($b/2$). Using the AISI Specification on the present test beams, the ratio b/b_e of effective design width to flange width takes the

values given in Table 3.3.12. For most of the test beams there is only a slight reduction in the flange width. Further, the circumstances seem to be more favourable in our loading case than in the loading cases that the AISI Specification is to support. Consequently the influence of shear lag will be neglected hereafter.

Table 3.3.12. Assumed reduction of flange width to take into account the influence of shear lag according to the AISI-Specification.

Test beam	$\frac{L}{b/2}$	$\frac{b_e}{b}$
A1	24.2	0.95
A2	23.3	0.94
B1	25.9	0.97
B2	17.3	0.88
B3	34.5	1.00
B4	26.0	0.97
B5	26.1	0.97
B6	21.1	0.92
B7	30.6	1.00
C1	25.9	0.97
C2	17.3	0.88
C3	34.6	1.00
C4	26.3	0.97
C5	26.0	0.97
C6	21.3	0.92
C7	30.5	1.00

3.3.7 Measuring devices and measurement procedure.

In the testing of beams B1, B2, B3, B6, B7, C1, C2, C3, C6 and C7 all measured data were manually recorded, whereas a computer controlled data acquisition system was used when beams B4, B5, C4 and C5 were tested.

The measurements in every experiment comprised the determination

of the applied load, the measurement of the deflection of the test beam, the measurement of strains in webs and flanges and finally the recording of shape and size of the normal deflections in the webs and in the compressed flange.

Load measurement.

Owing to the time dependence of the load-deformation relation, the load was applied in such a way that the deformation was kept constant during each loading step. In this connection deformation means the deflection of the test beam at the central section.

Measured load is defined as the maximum load recorded during a load step. If the load level was decreased on account of "creep", the level reached just prior to the next load increment was also recorded.

For the measurements with the computer-controlled data acquisition system, a recorder was also used to register the load as a continuous function of time.

Deflection measurement.

The deflections of the test beams were measured by means of dial gauges graduated in units of 0.01 mm and capable of recording deflections up to 50 mm. The deflections were recorded in five sections with two dial gauges in each section, see Fig. 3.3.9.

In the testing of the beams B1-B4, B6, B7, C1-C4, C6, C7 the dial gauges measured directly on the lower side of the test beams. The dial gauges were put in position with their points acting on the plane flange as near the rounded corners as possible.

As the lower sides of the test beams were somewhat deformed during loading, the manner of carrying out the measurements was changed for the final test beams B5 and C5. Thus small pieces of perspex were fastened to the webs and the dial gauges measured in the elongation of the webs.

Strain measurement.

The strains were measured by means of foil strain gauges with measuring lengths from 5 to 20 mm. To determine the mean stresses of the sheet, the strain gauges were fastened in pairs opposite to each other on the inner and outer surfaces of the test beams. At the corners of the test beams unidirectional strain gauges were fastened only to the outer side. A typical example of the positions of the strain gauges is shown in Fig. 3.3.9. In the compression flange, three-directional strain gauges have been used, in the corners and in the webs unidirectional strain gauges, and in the tension flange two-directional strain gauges.

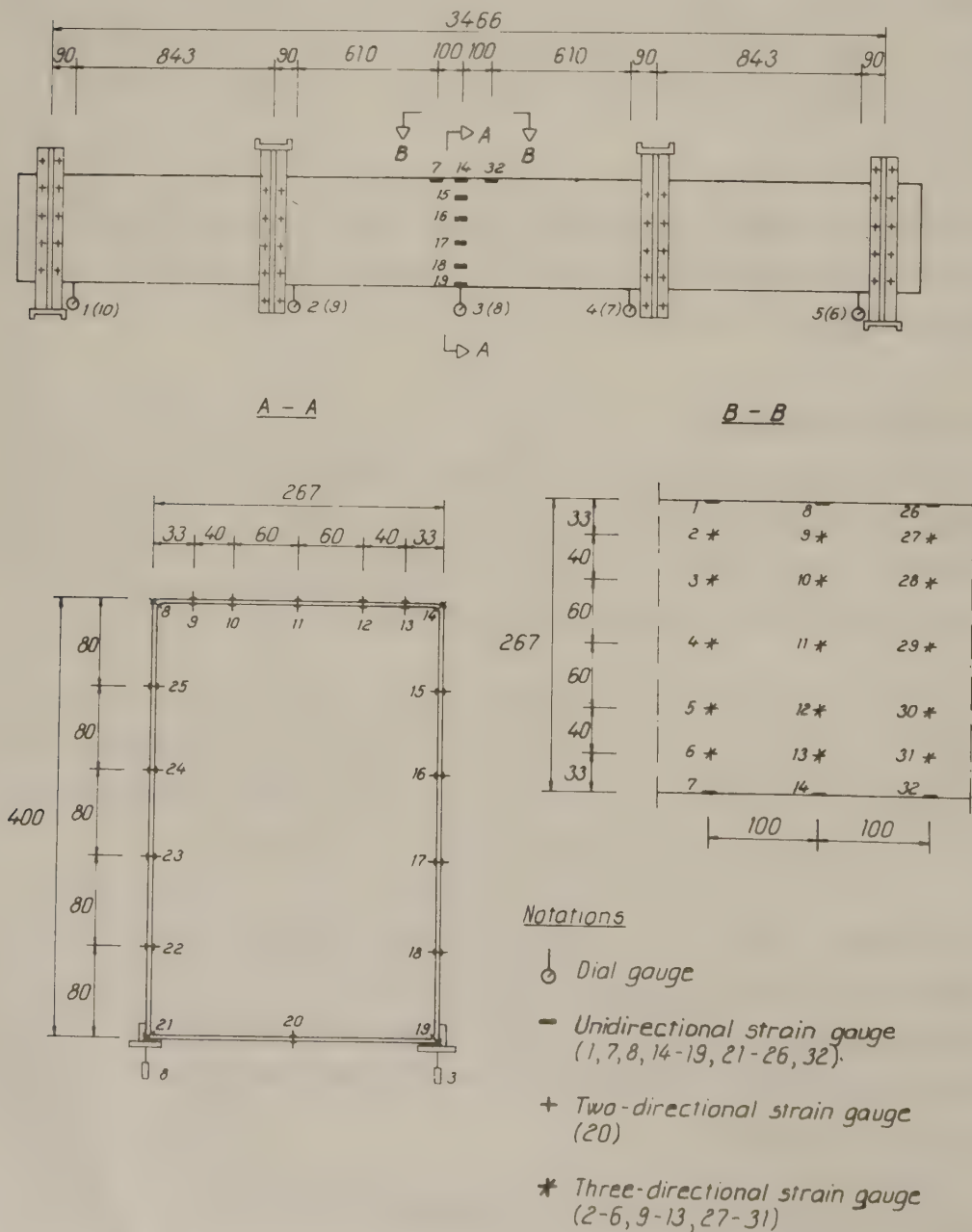


Fig. 3.3.9. Positions and numbering of dial gauges and strain gauges for the test beams B5 and C5.

In the manual measurements a strain gauge indicator Peekel B 105 was used.

Measurement of normal deflections.

For the manual measurements of the normal deflections, see Fig. 3.3.10, a dial gauge graduated in units of 0.01 mm could be moved in the transverse direction of the test beam along a milled slot in a planed aluminium profile. The aluminium profile was movable in the longitudinal direction along two plane milled guides which were fixed to hinges at the screw joints where the load was applied. Thus it was possible to determine the deflection of any arbitrary point on the compression flange between the point loads. The same method of measurement was used for the webs.

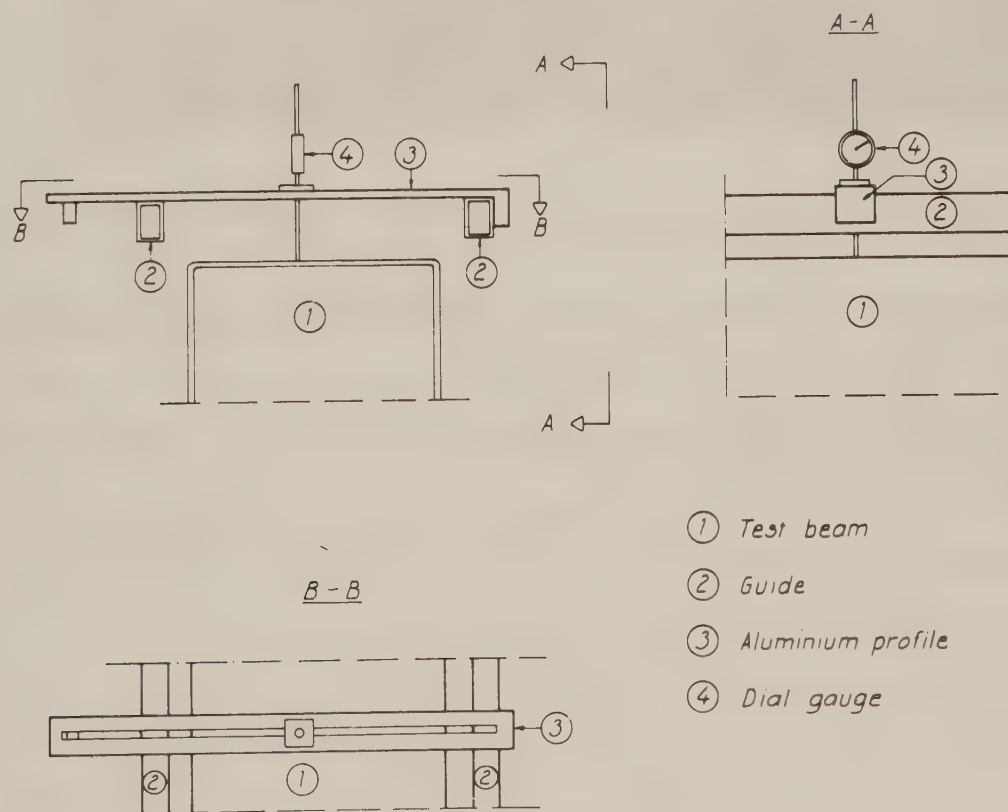


Fig. 3.3.10. Device for manual deflection measurement of buckles.

In the automatic measurements by means of the computer-controlled data acquisition system, the deflections of the buckles were measured by a number of linear motion potentiometers which were fixed to a movable frame structure, see Fig. 3.3.11. The frame structure moved on ball bearings along two guides that were entirely indepen-

dent of the motions of the test beams.

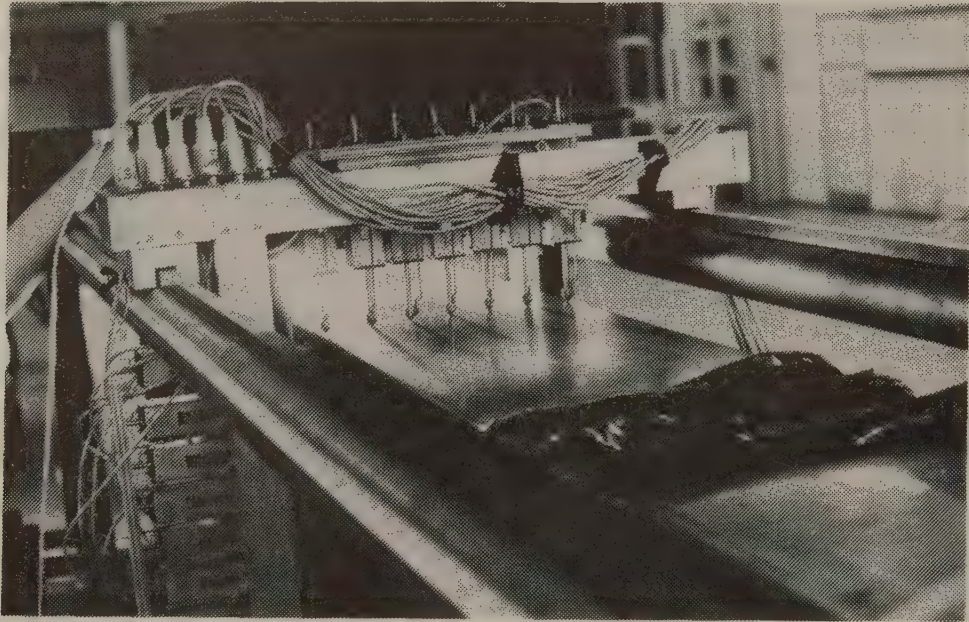


Fig. 3.3.11. Device for automatic deflection measurement of buckles.

Measurement by means of computer-controlled data acquisition system.

The computer-controlled data acquisition system used in the last four tests is described in Appendix D. A principal flow chart of the measurement programme is shown in Fig. 3.3.12. No processing of measured data except a certain degree of rearrangement was done during the tests. All measured data were punched on paper tape by means of a high speed punch.

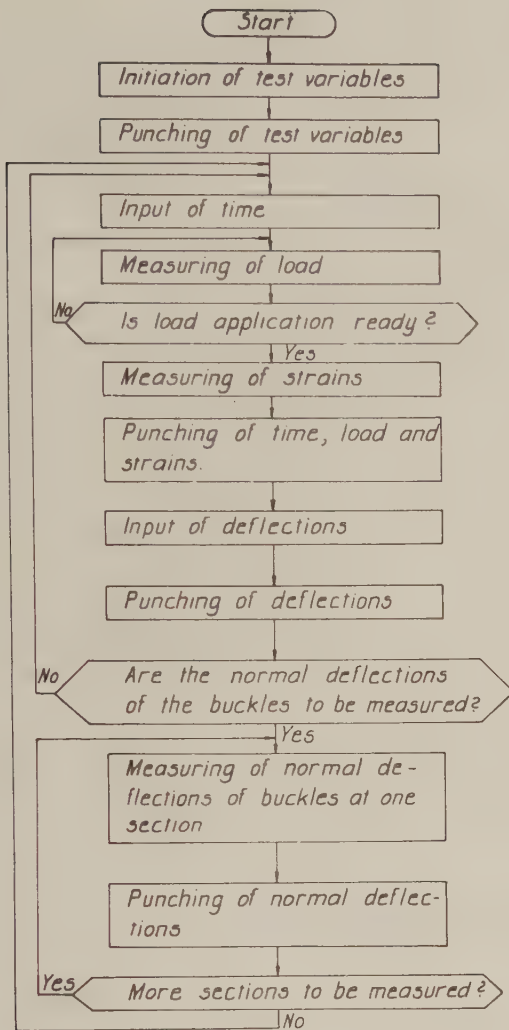


Fig. 3.3.12. Principal flow chart of the measurement programme.

3.3.8 Results.

To illustrate the behaviour of the test beams under bending, only a fraction of all the experimental data and figures collected can be shown. In this connection it has turned out to be most interesting to compare on the one hand the different materials and on the other hand the test beams with the most different slenderness ratios.

Deflections.

In Figs. 3.3.13-3.3.16, the deflection of the central section $v_{c/L}$ has been plotted versus the total applied load P for the test beams B4, C4, B5 and C5 respectively. To obtain a picture of the influence

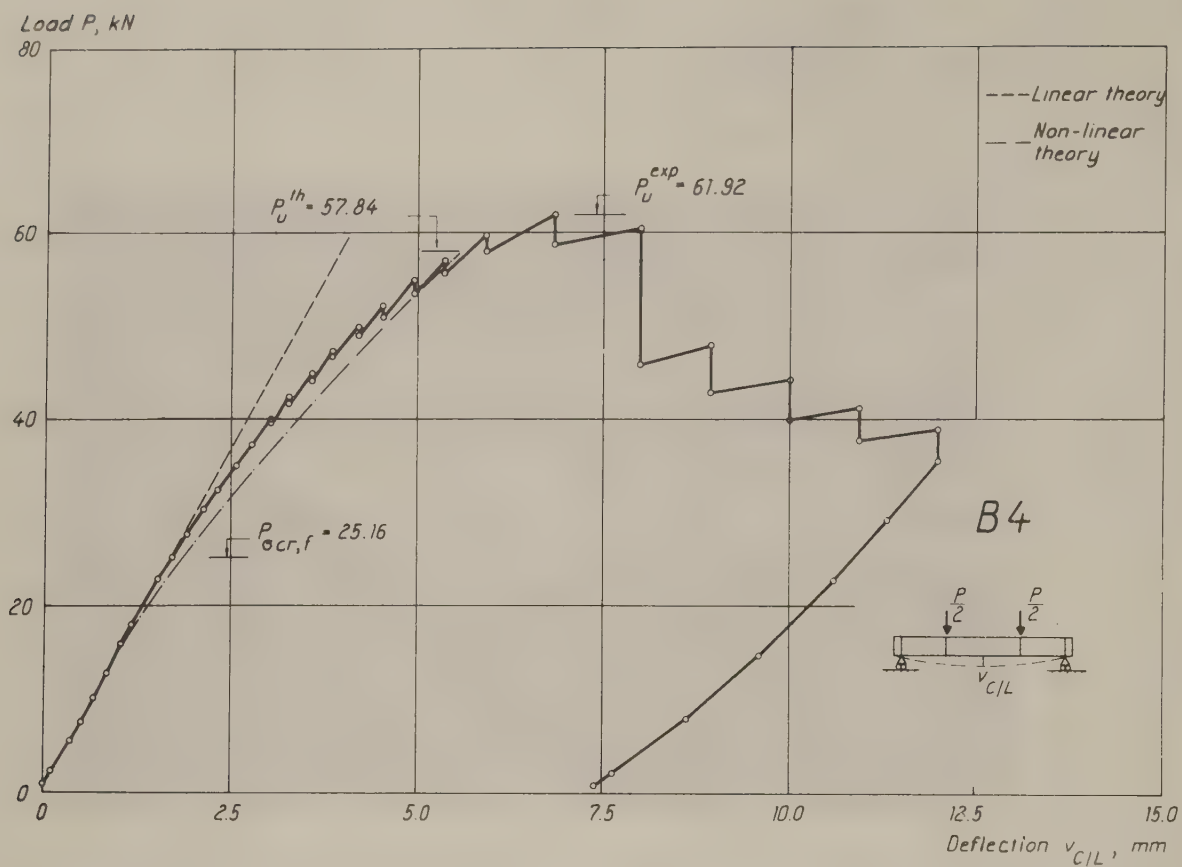


Fig. 3.3.13. Load-deflection curve for test beam B4.

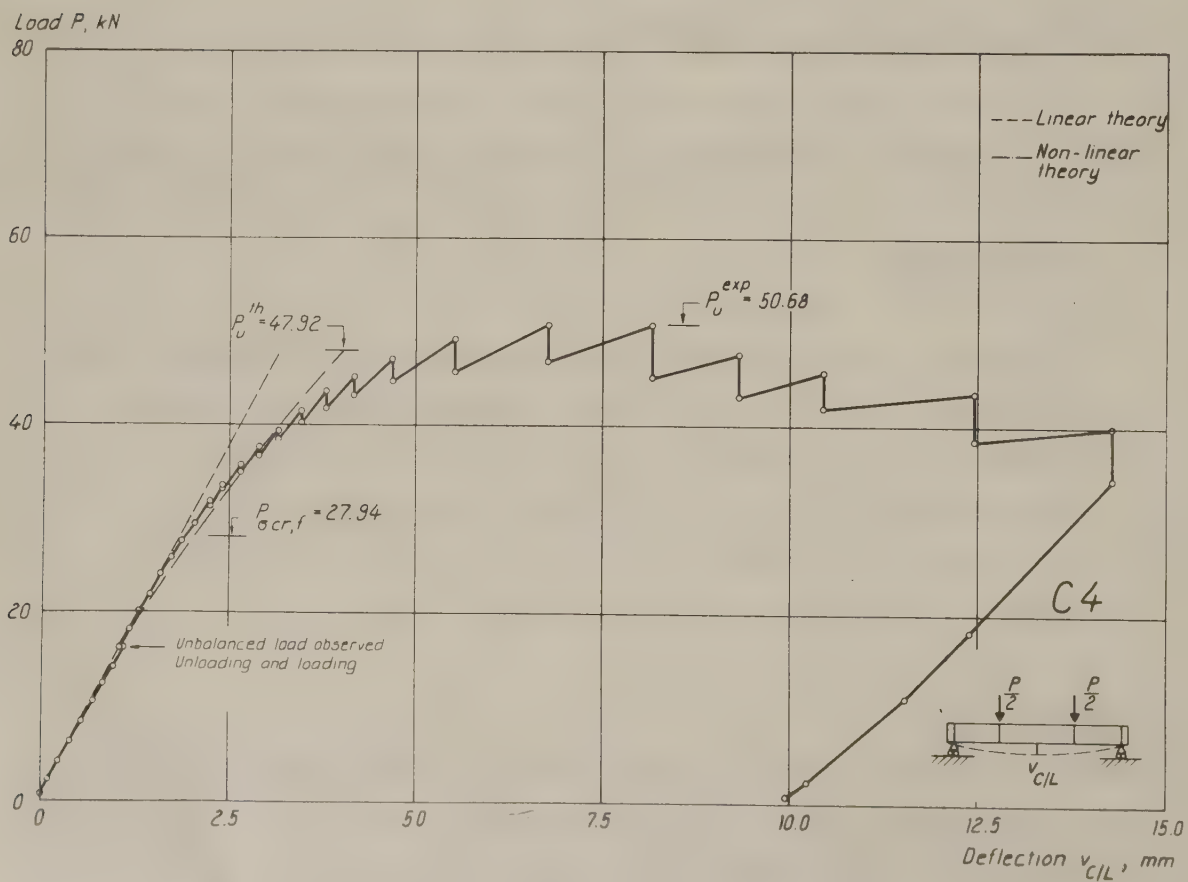


Fig. 3.3.14. Load-deflection curve for test beam C4.

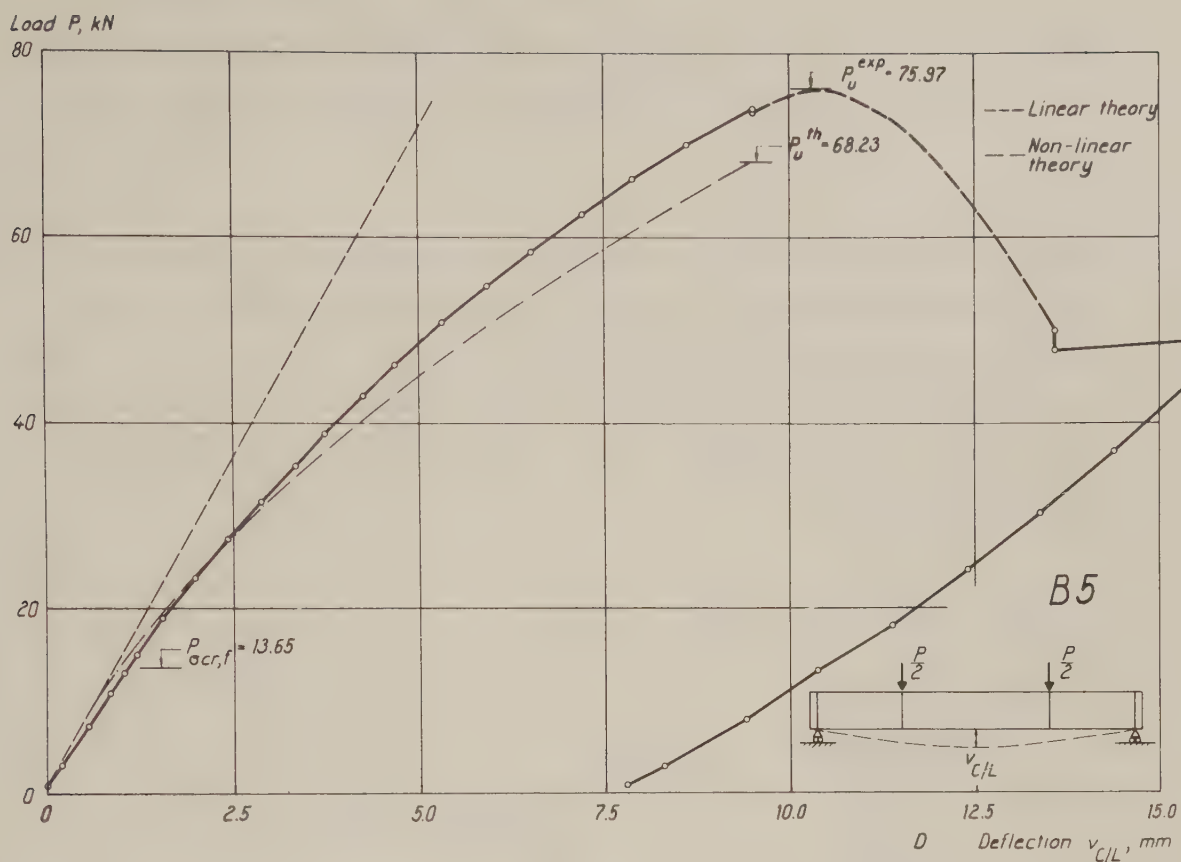


Fig. 3.3.15. Load-deflection curve for test beam B5.

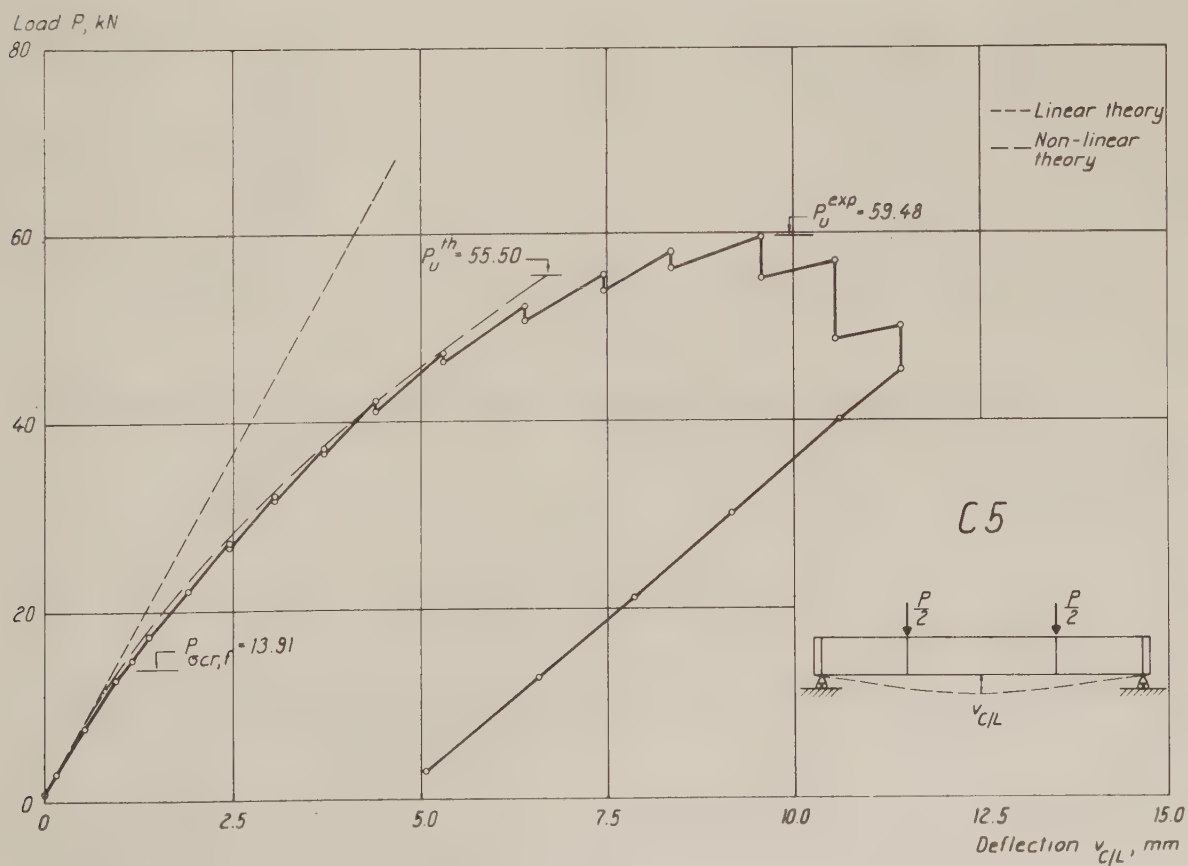


Fig. 3.3.16. Load-deflection curve for test beam C5.

of creep, the load has been plotted immediately after load application on the one hand and just before the next load step on the other. The deflections have been corrected with respect to support movements.

In Figs. 3.3.17-3.3.20, the relationship between M/EI_0 and the measured curvature $1/\rho$ of the span between the point loads is shown for the test beams B4, C4, B5 and C5. M/EI_0 has been calculated from the applied load and the cross-sectional dimensions. The measured curvature $1/\rho$ has been obtained from the deflections of three sections between the point loads.

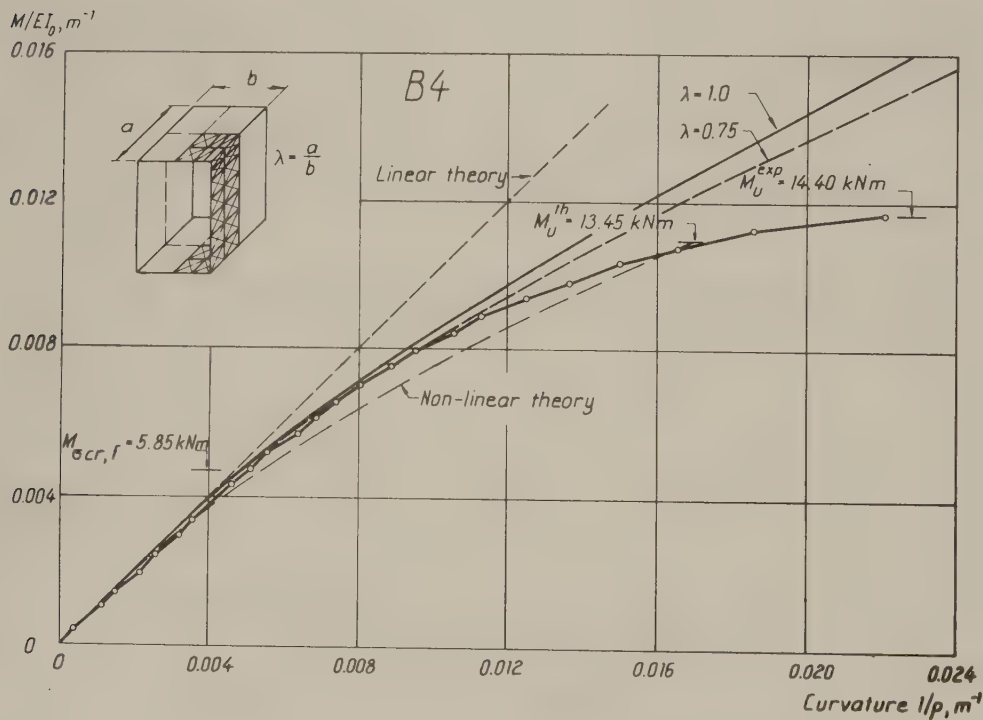


Fig. 3.3.17. Relationship between bending moment M and curvature $1/\rho$ for test beam B4.

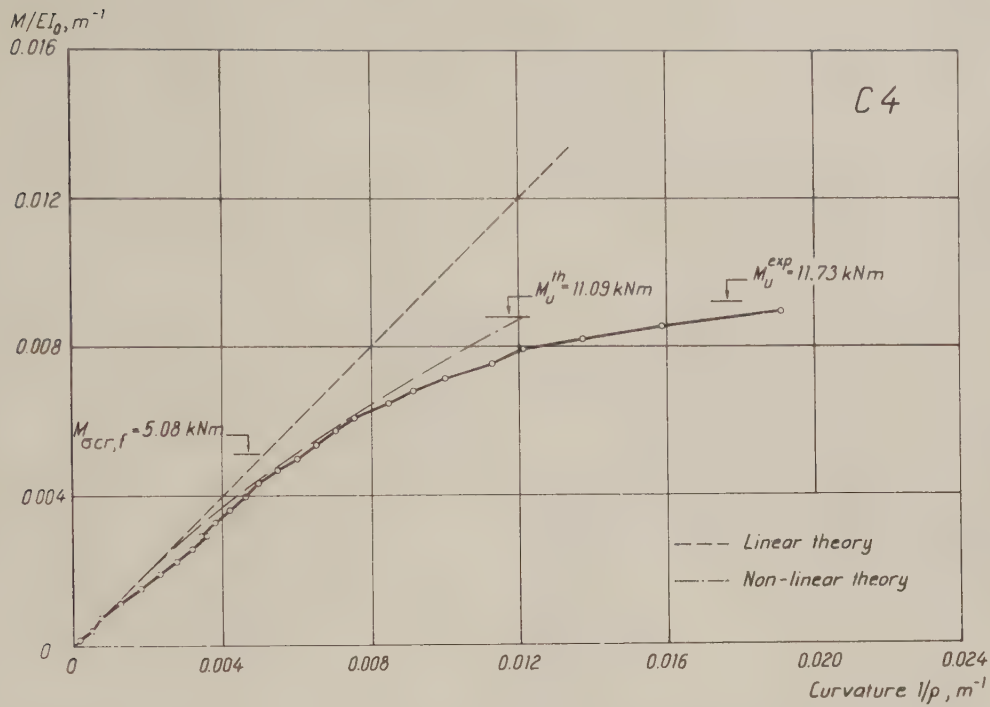


Fig. 3.3.18. Relationship between bending moment M and curvature $1/\rho$ for test beam C4.

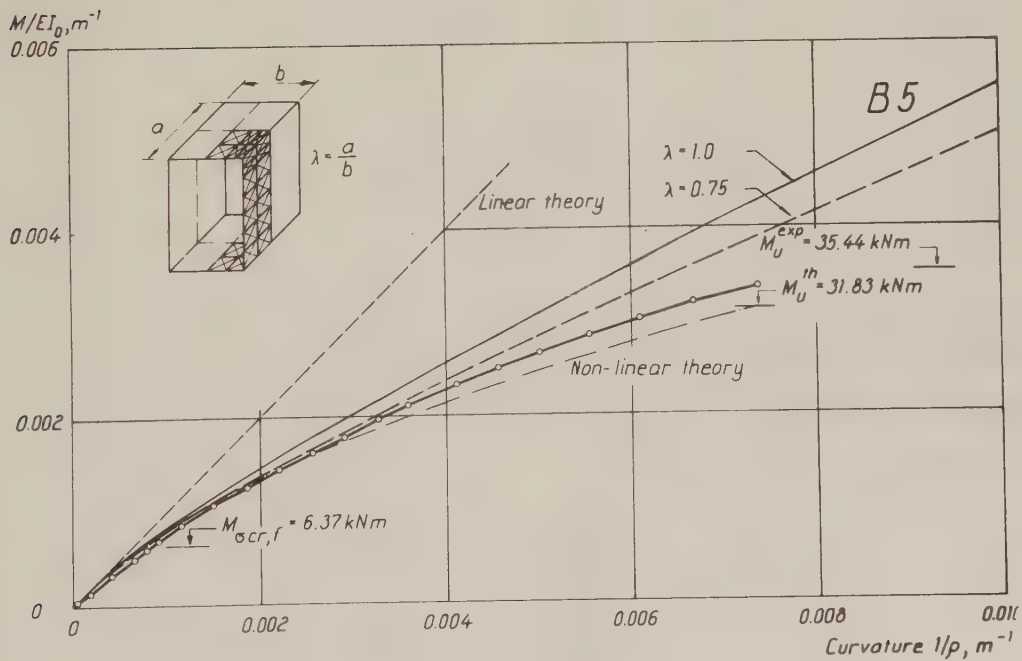


Fig. 3.3.19. Relationship between bending moment M and curvature $1/\rho$ for test beam B5.

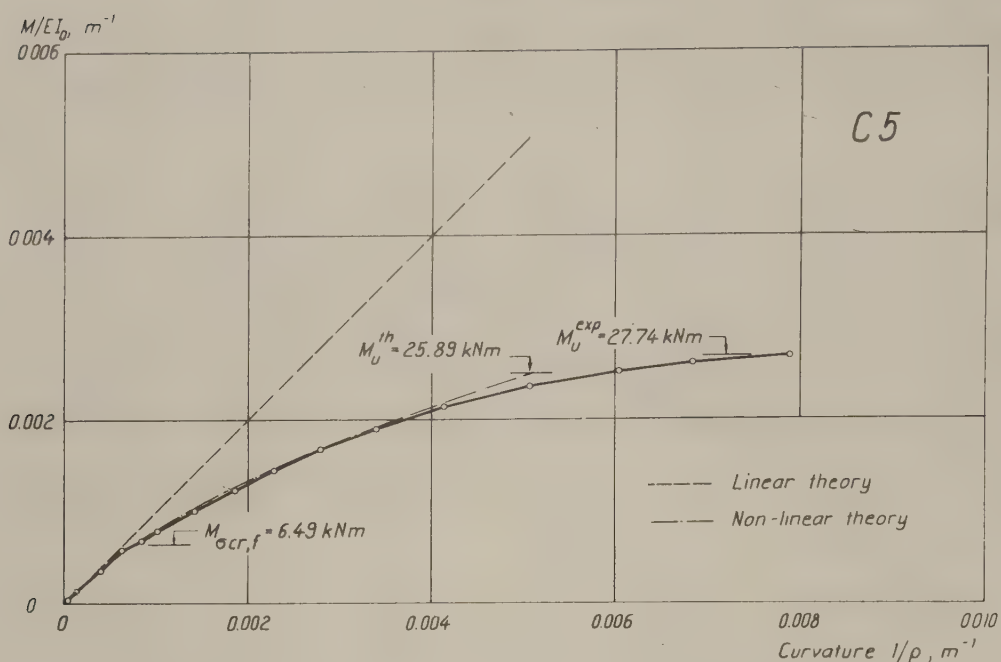
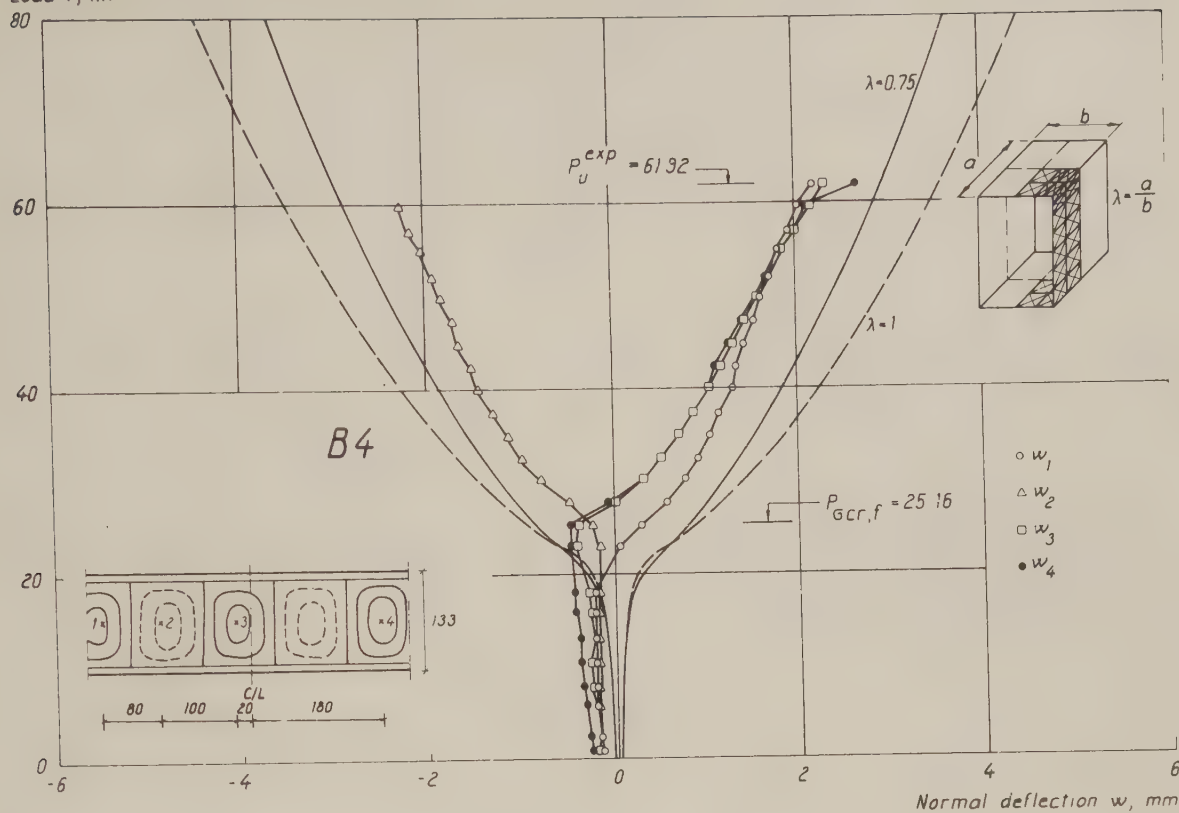


Fig. 3.3.20. Relationship between bending moment M and curvature $1/\rho$ for test beam C5.

Normal deflections.

In Figs. 3.3.21-3.3.24, the normal deflections of some points on the compressed flange and on the compressed parts of the webs are shown as a function of applied load for the test beams B4, C4, B5 and C5. For the same beams, the deflection surfaces of the upper flange and one of the webs are shown in Figs. 3.3.25-3.3.28. This is done for each of the beams at three characteristic loads namely at "zero load", in the vicinity of the buckling load and at a load immediately before the ultimate load. For all the test beams B4, C4, B5 and C5, a zero level was assumed to be situated about 9 mm from the corners of the flanges and about 10 mm from the corners of the webs.

a)

Load P , kN

b)

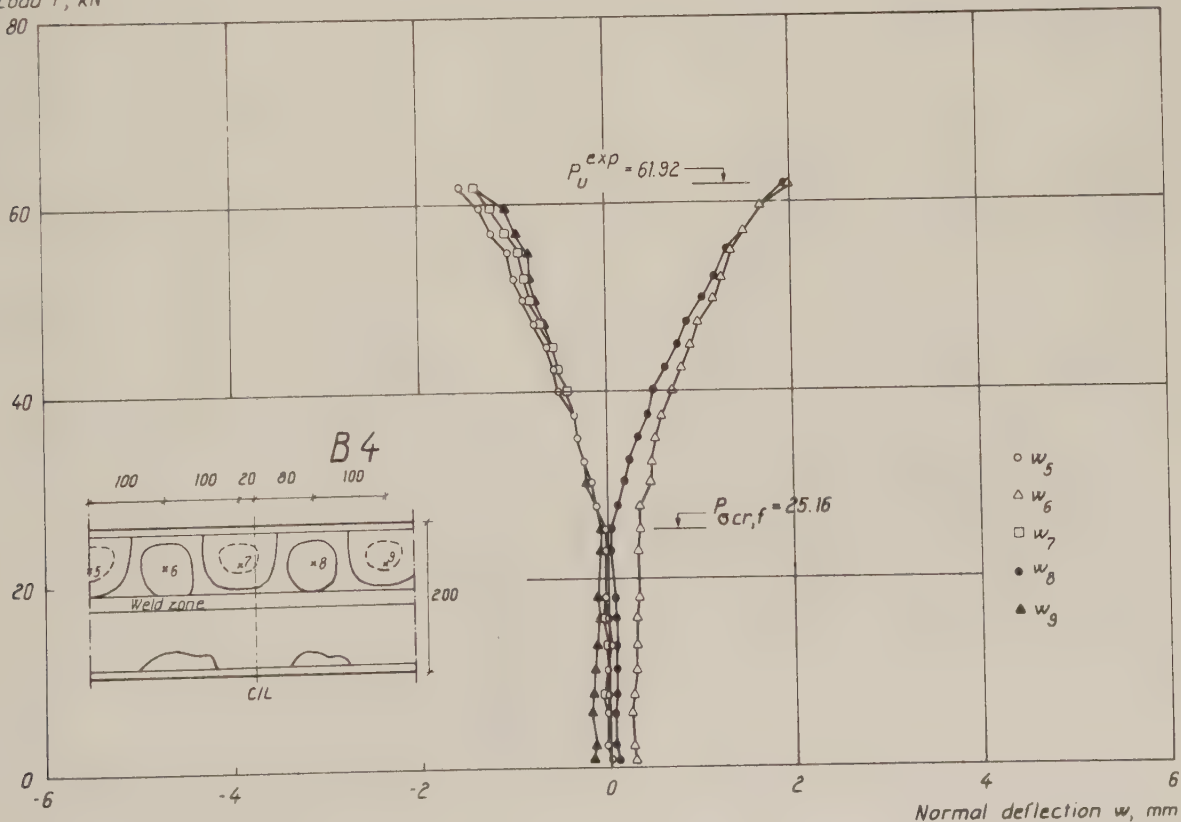
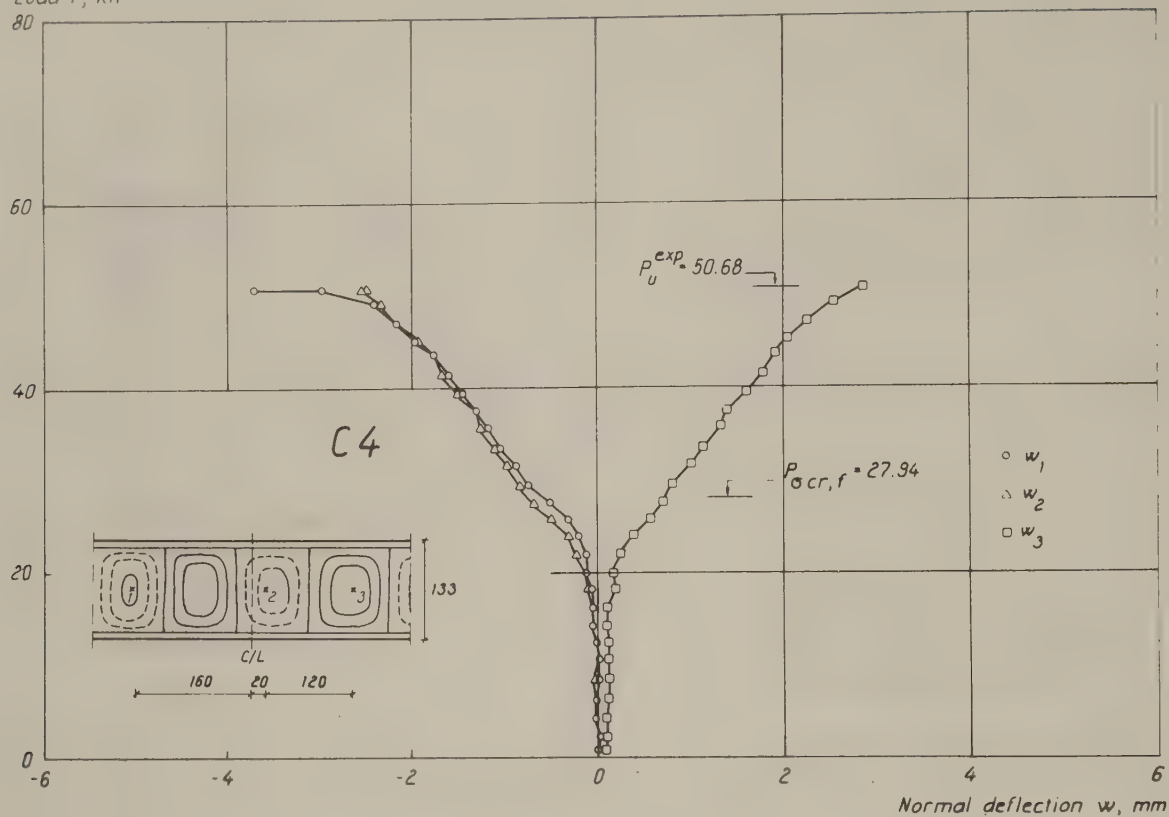
Load P , kN

Fig. 3.3.21. Load versus normal deflection for some points on
 a) the compression flange of test beam B4.
 b) the compressed part of one of the webs of test beam B4.

a)

Load P , kN

b)

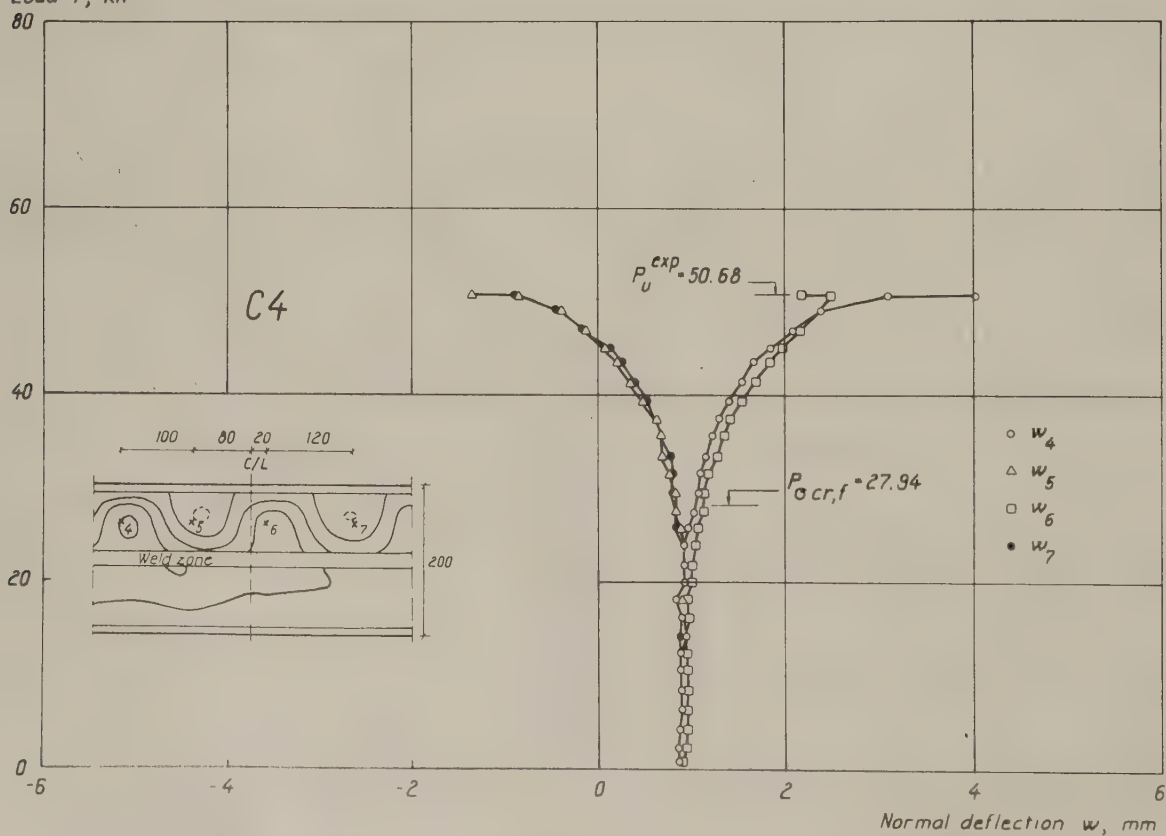
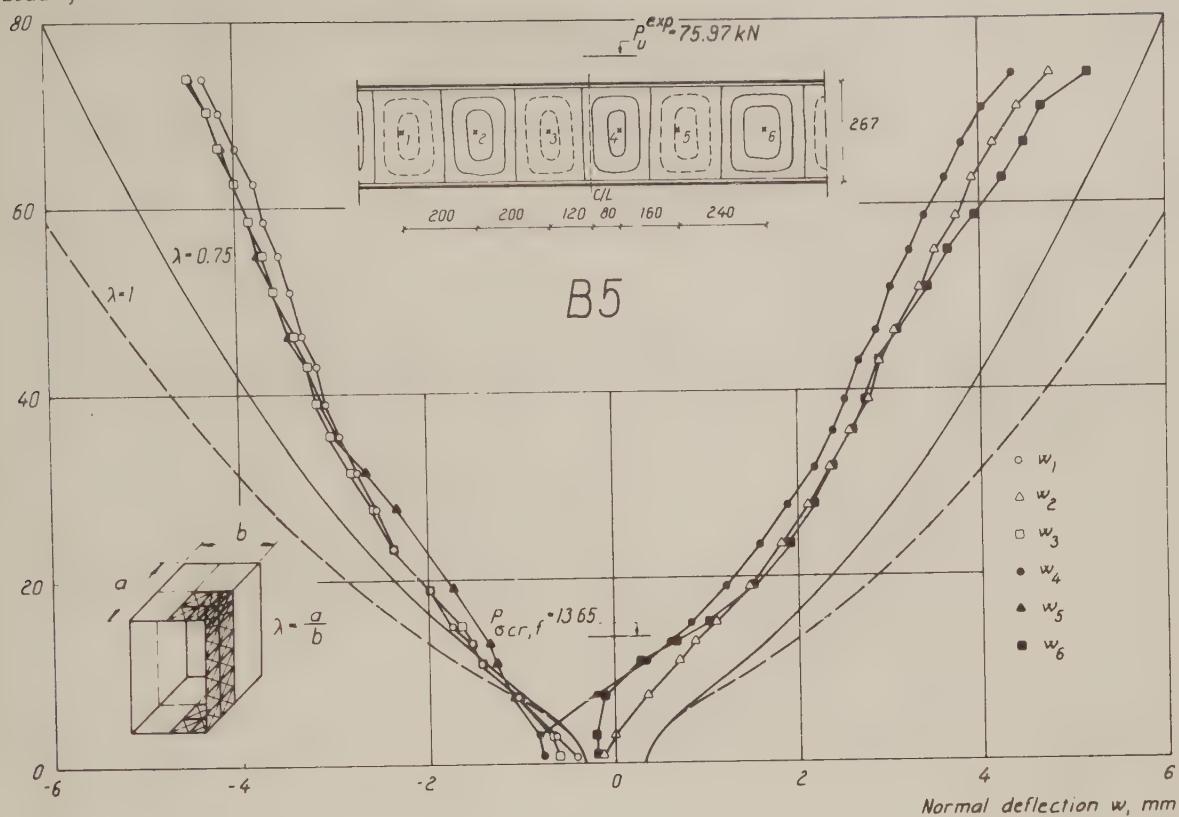
Load P , kN

Fig. 3.3.22. Load versus normal deflection for some points on
 a) the compression flange of test beam C4.
 b) the compressed part of one of the webs of test beam C4.

a,

Load P , kN

b,

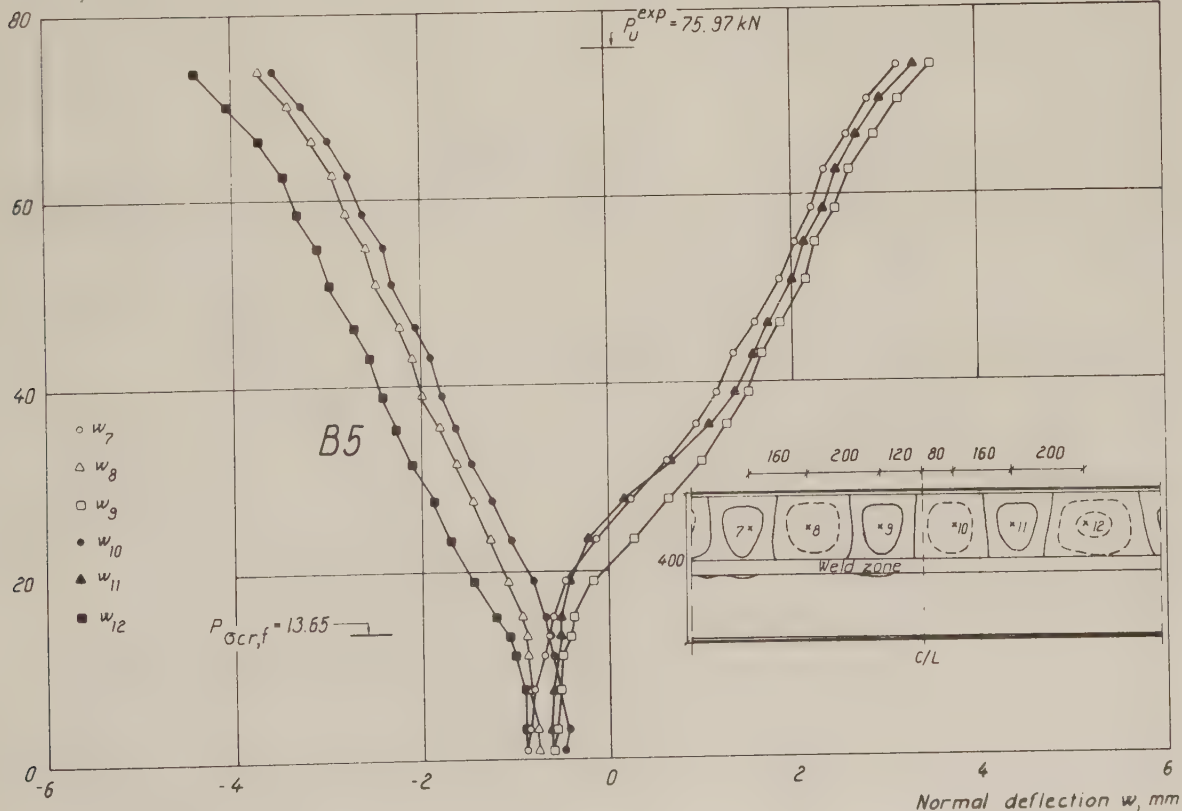
Load P , kN

Fig. 3.3.23. Load versus normal deflection for some points on
 a) the compression flange of test beam B5.
 b) the compressed part of one of the webs of test beam B5.

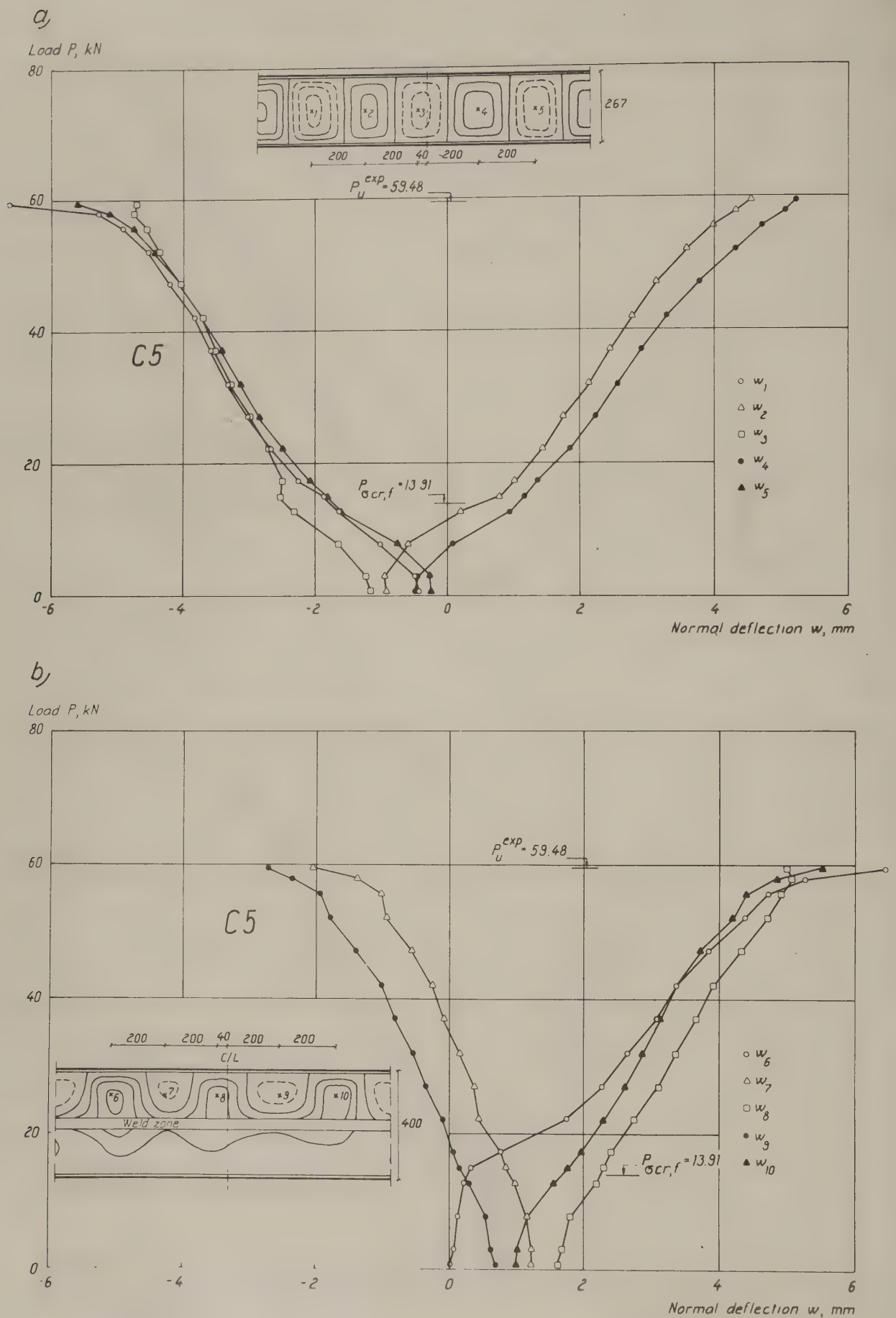
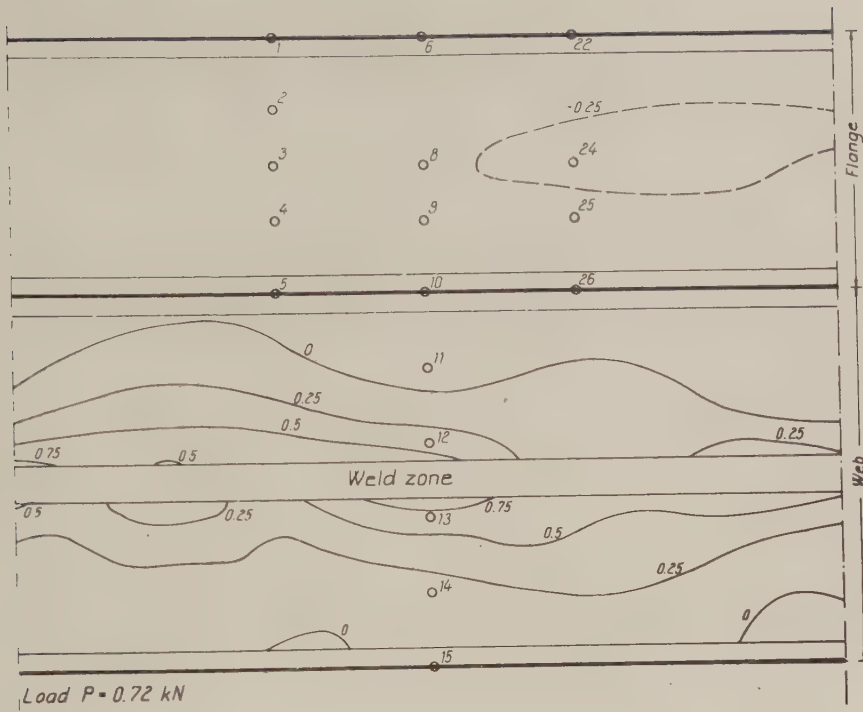
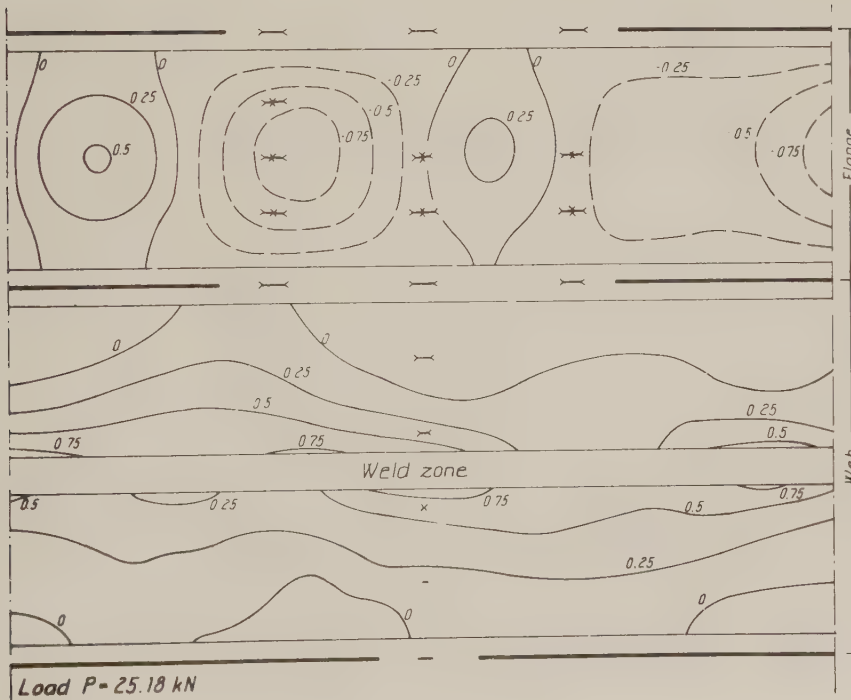


Fig. 3.3.24. Load versus normal deflection for some points on
 a) the compression flange of test beam C5
 b) the compressed part of one of the webs of test beam C5.

a)



b)



c,

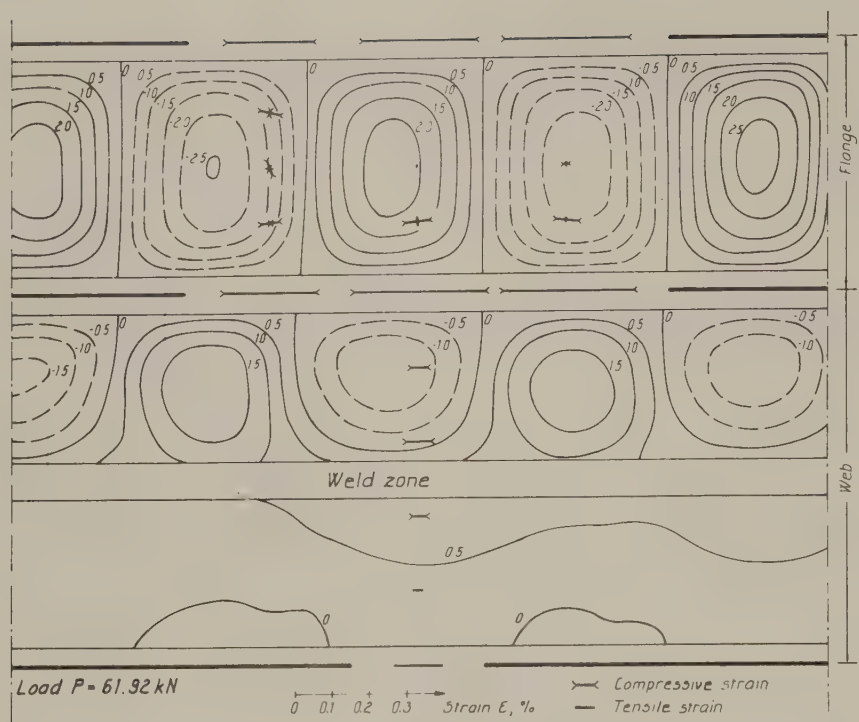
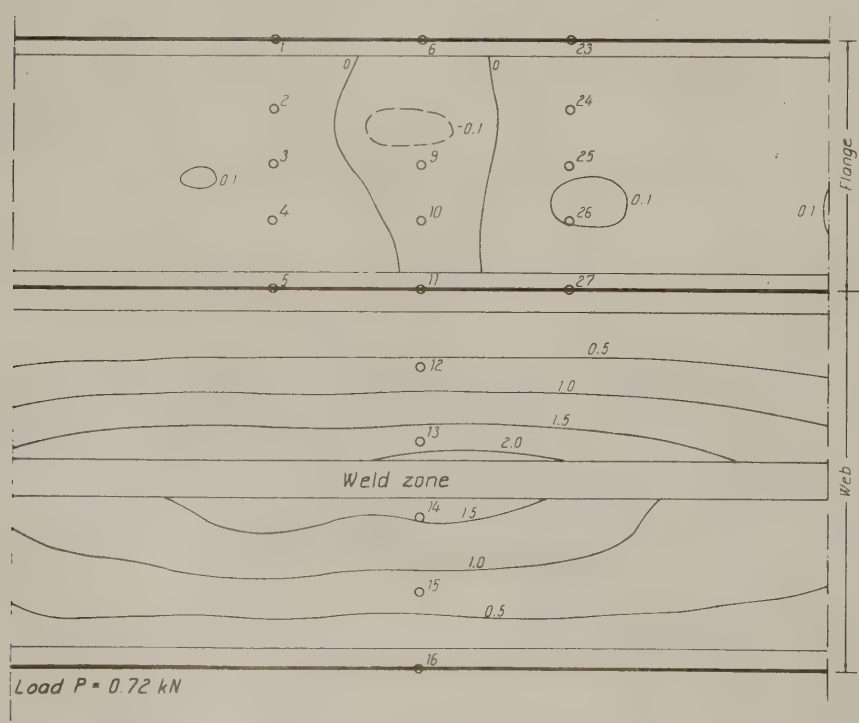
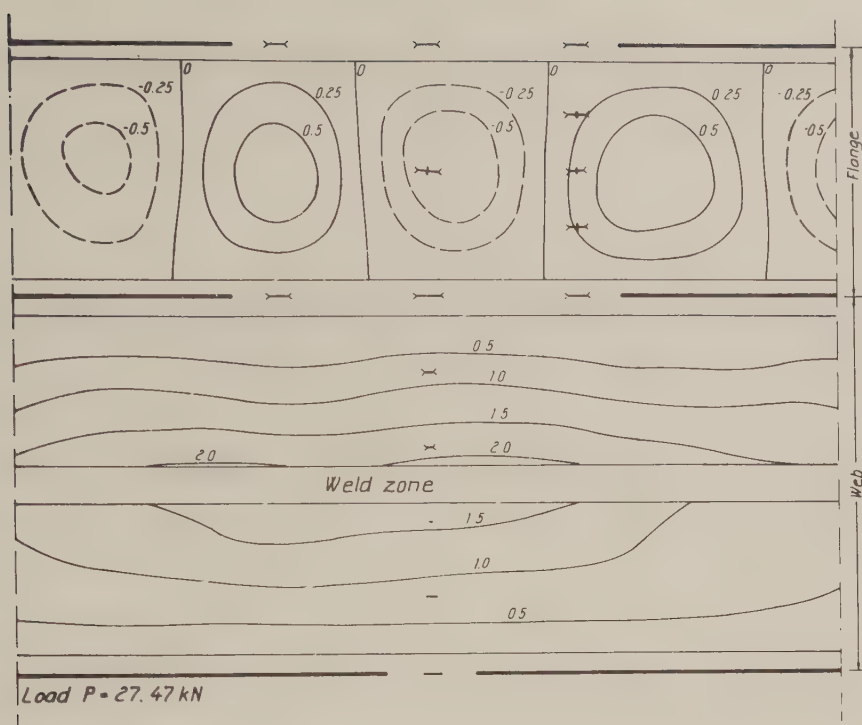


Fig. 3.3.25 a-c. Normal deflections and strains for test beam B4.

a,



b)



c)

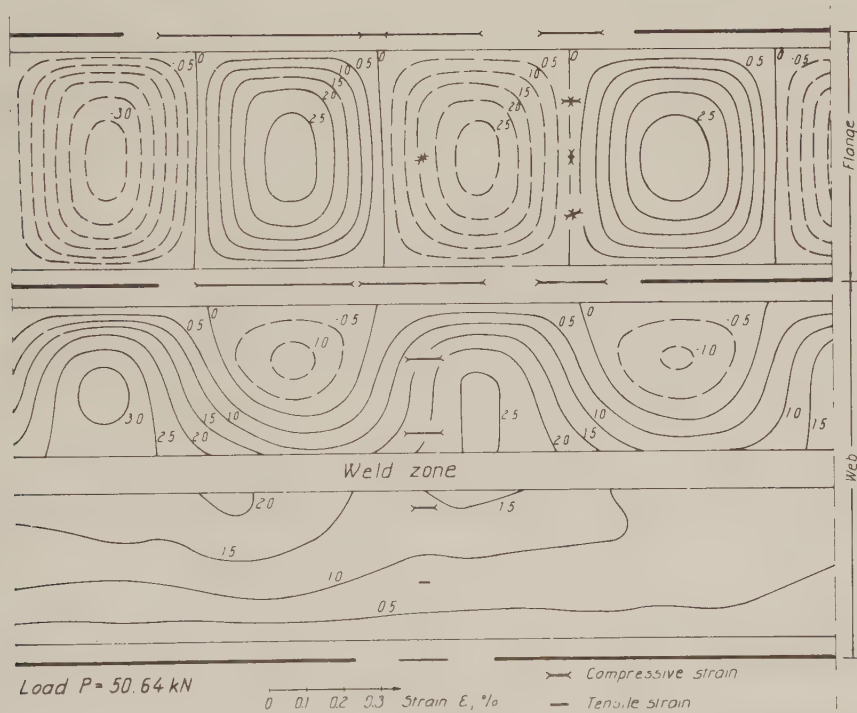
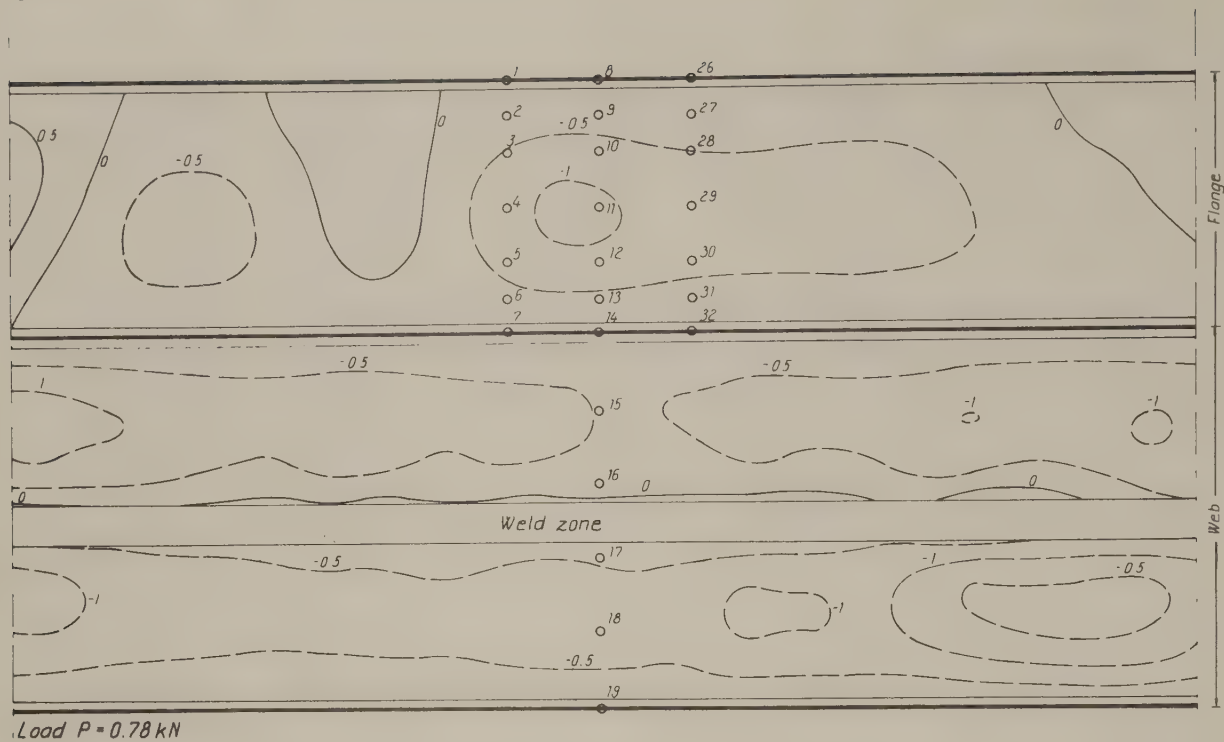
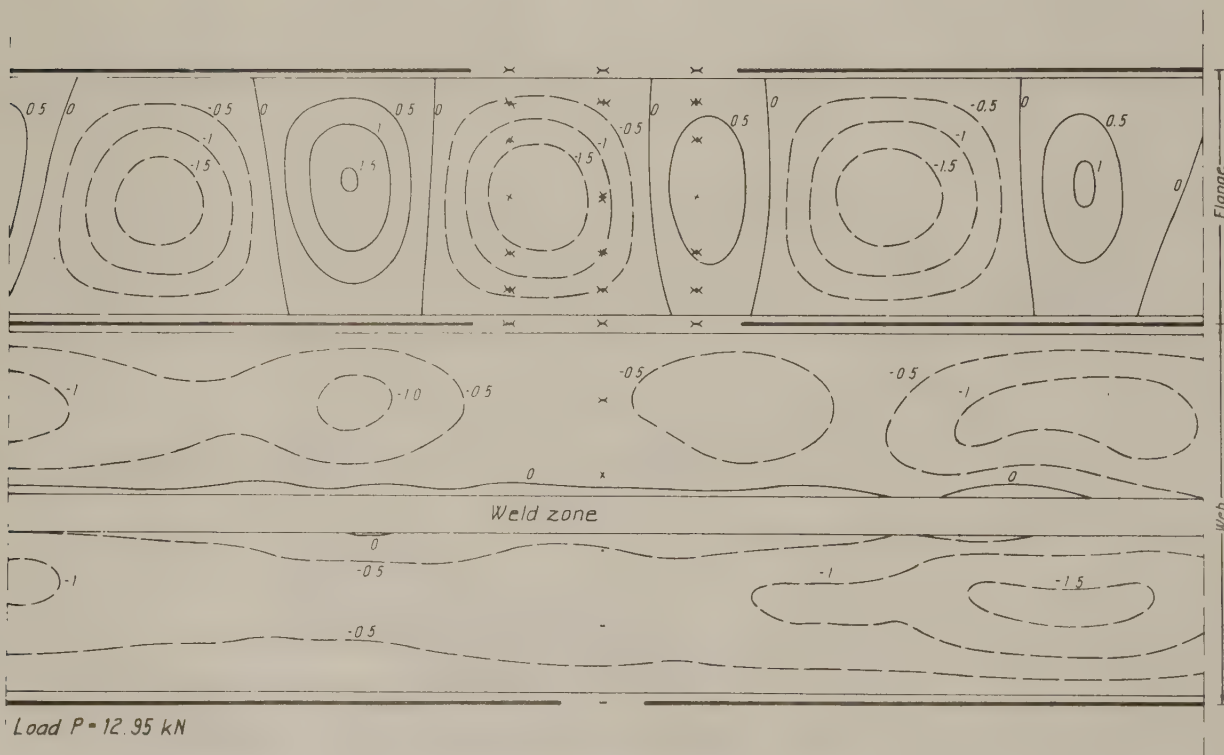


Fig. 3.3.26 a-c. Normal deflections and strains for test beam C4.

a,



b,



c,

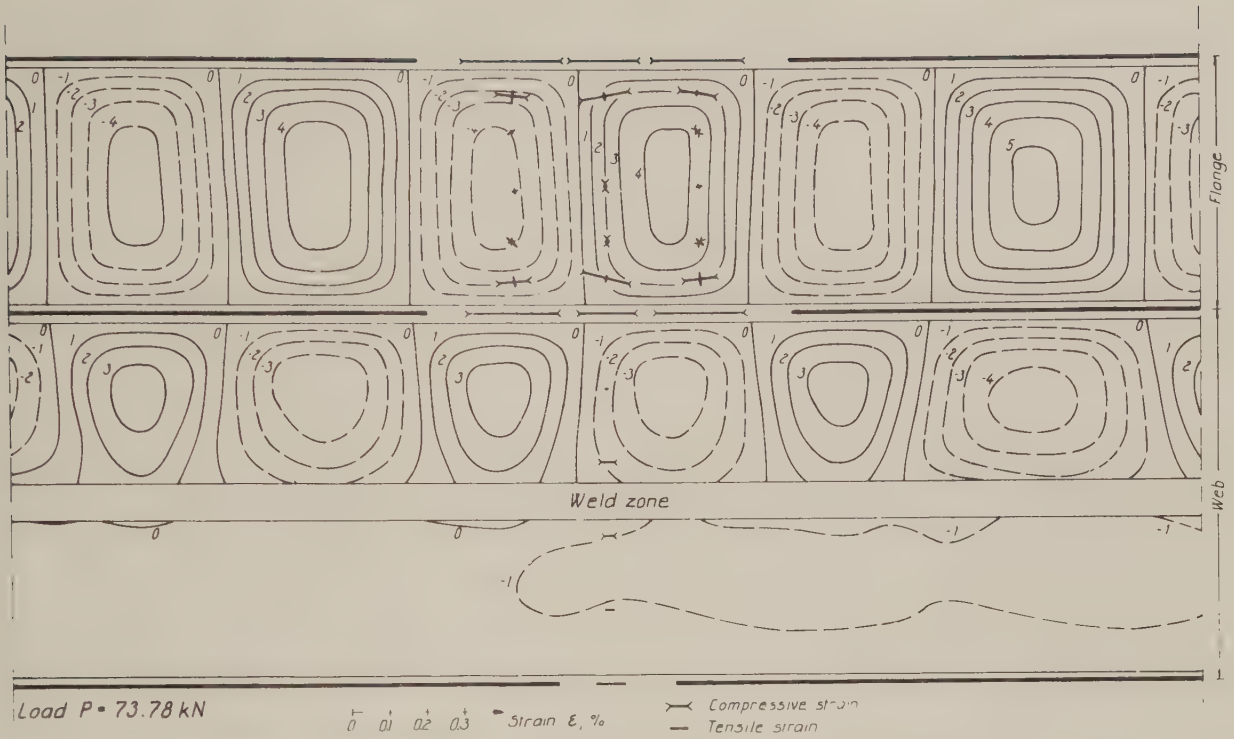
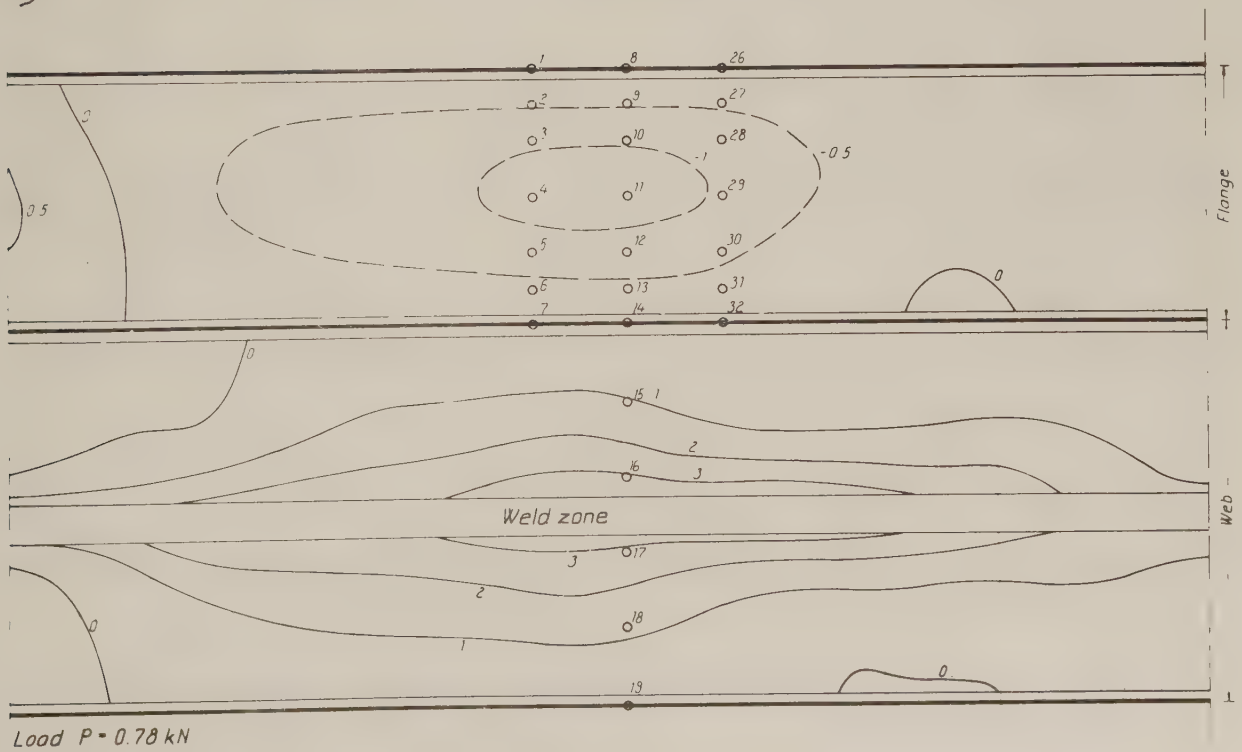
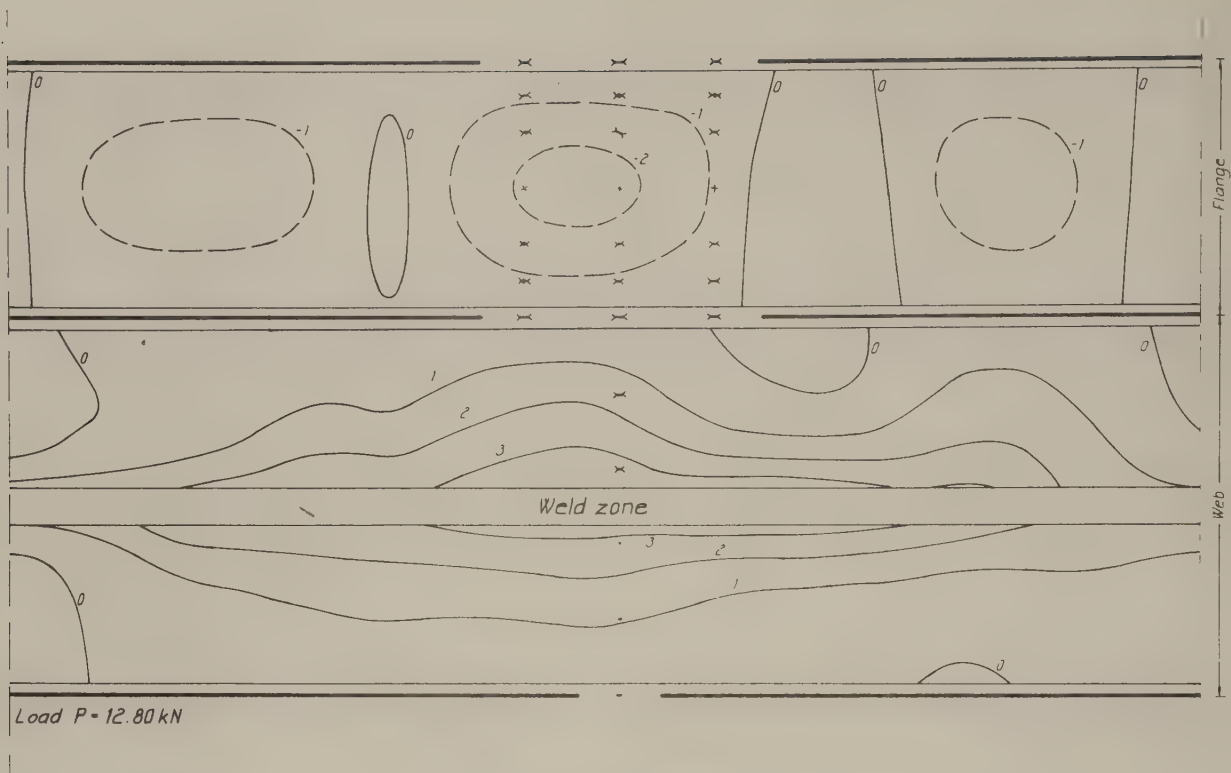


Fig. 3.3.27 a-c. Normal deflections and strains for test beam B5.

a,



b)



c)

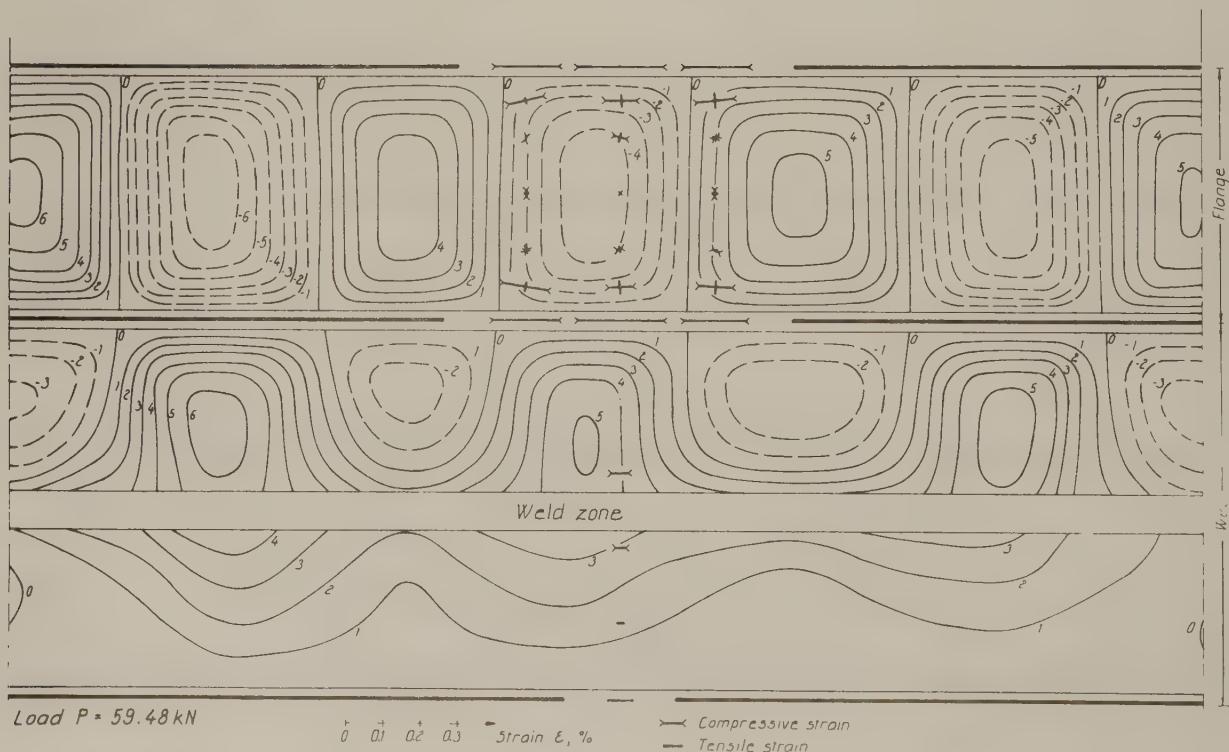


Fig. 3.3.28 a-c. Normal deflections and strains for test beam C5.

Strains.

From data recorded by the three-directional strain gauges in the compression flange, principal strains and principal strain directions were calculated. The results are shown in Figs. 3.3.25-3.3.28 as strain vectors, where the strains in the longitudinal direction of the corners and of the webs are also given. The cross-sectional strain distributions in the longitudinal direction of test beams B4 and B5 are illustrated in Figs. 3.3.29 and 3.3.30 for some load levels. In Figs. 3.3.31 and 3.3.32 principal strains and principal strain directions of some points in the compressed flanges are given as a function of the applied load. Figs. 3.3.33 and 3.3.34 show the variation in the longitudinal strains in the outer and inner surfaces and their mean value with respect to applied load for one point in the compressed flange and one in the web for the test beams B4 and B5 respectively. It must be pointed out that the strains given in Figs. 3.3.25-3.3.34 are in many cases greater than those corresponding to the yield stress.

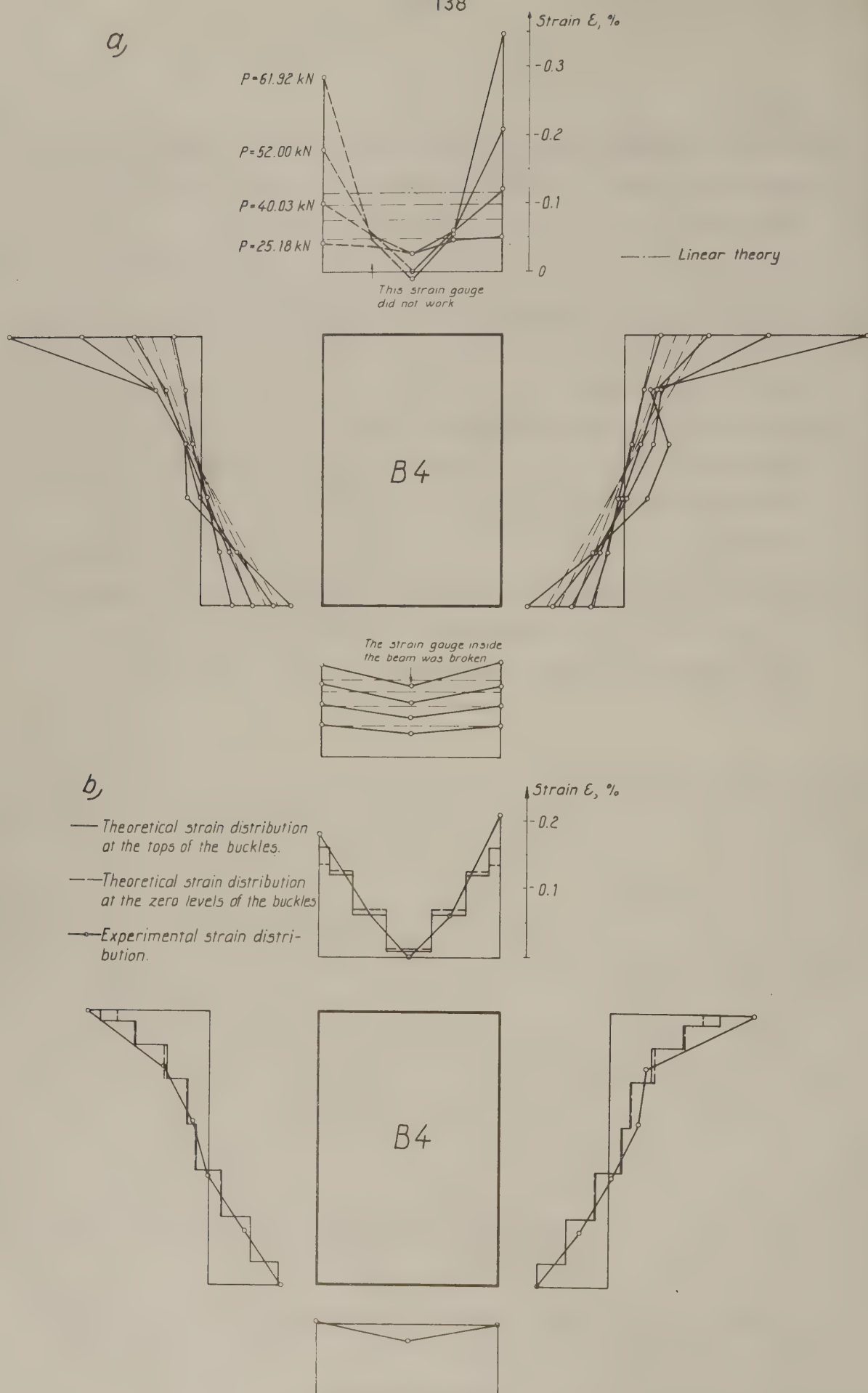
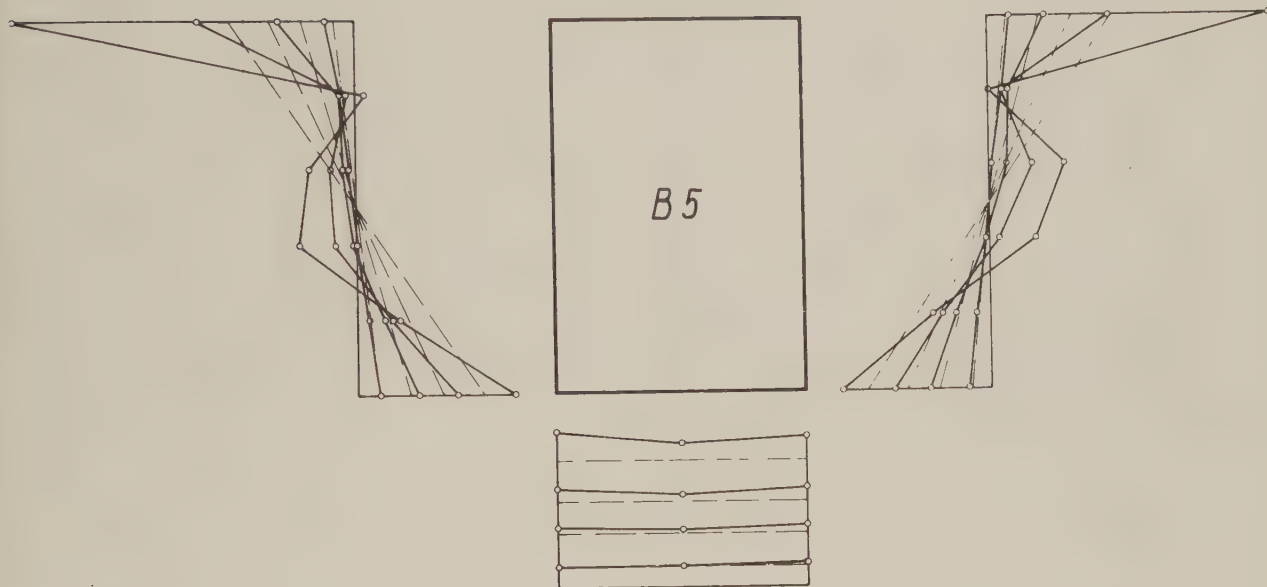
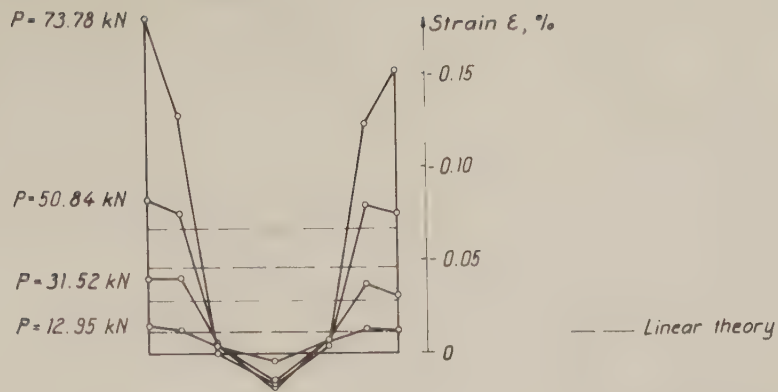


Fig. 3.3.29. a) Distribution of strains in midspan of test beam B4.
 b) Comparison between measured strains and strains obtained with bar model. The load $P = 52.00 \text{ kN}$.

a,



b,

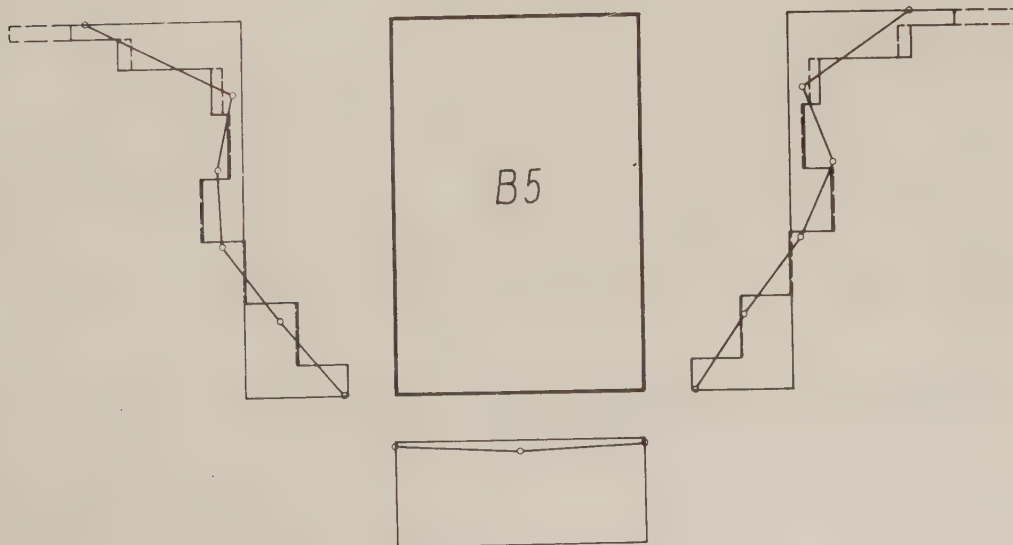
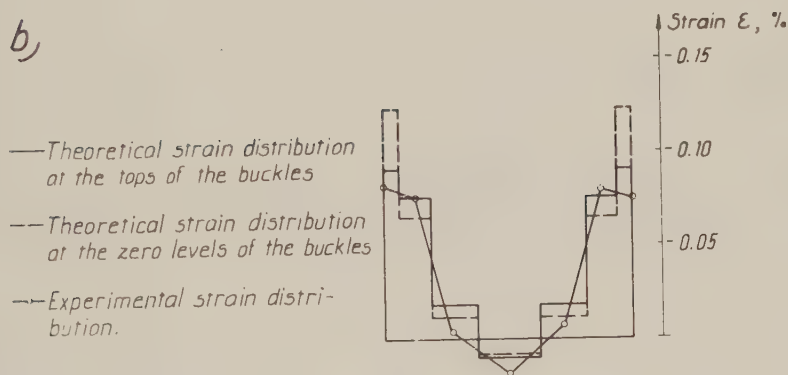


Fig. 3.3.30. a) Distribution of strains in midspan of test beam B5.
 b) Comparison between measured strains and strains obtained with bar model. The load $P = 50.84 \text{ kN}$.

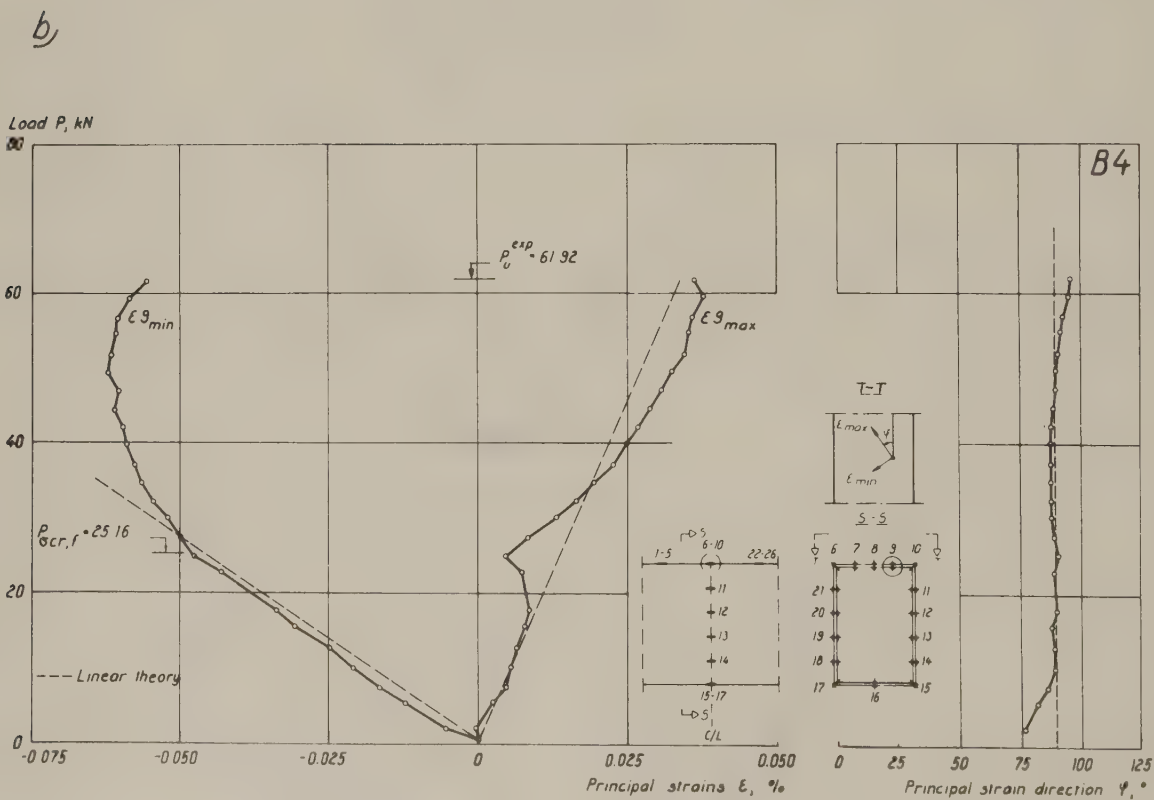
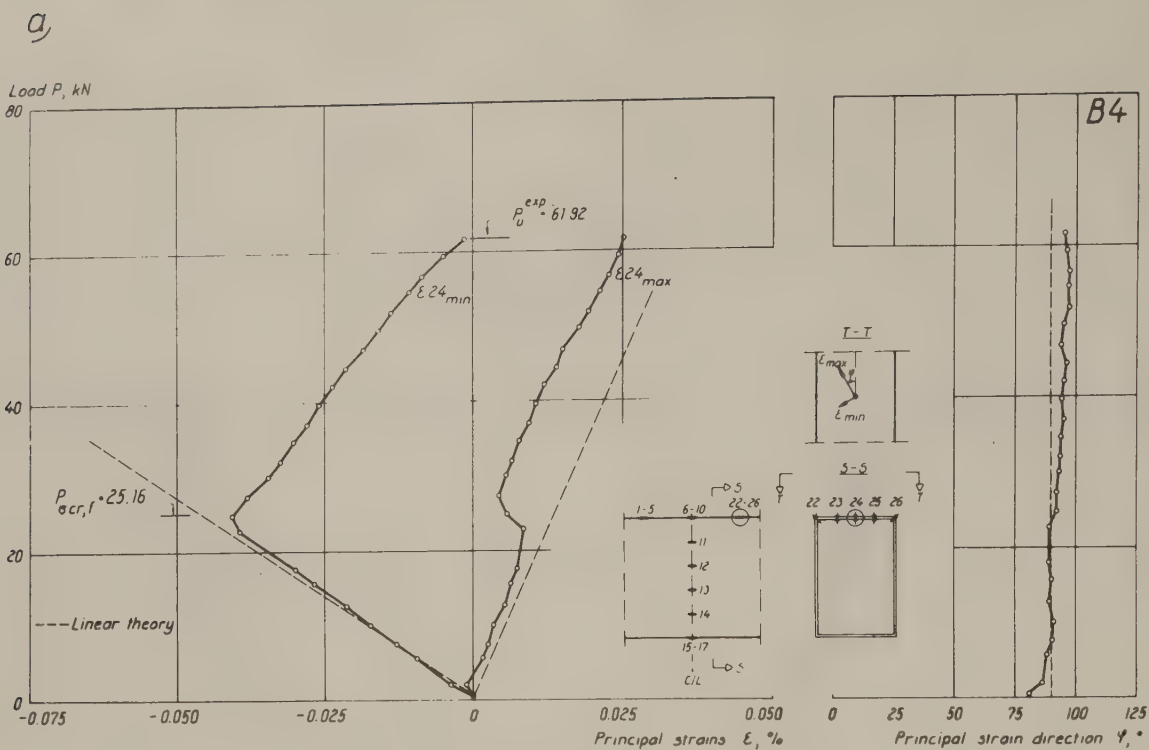
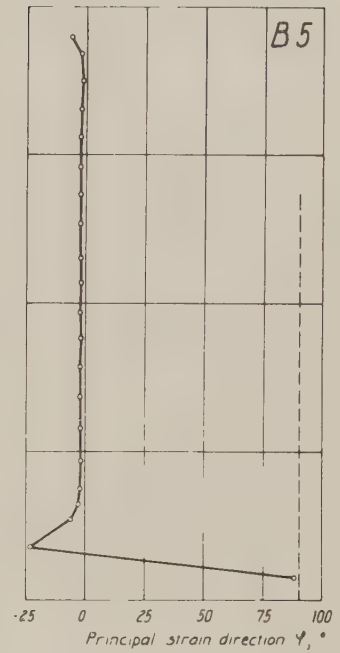
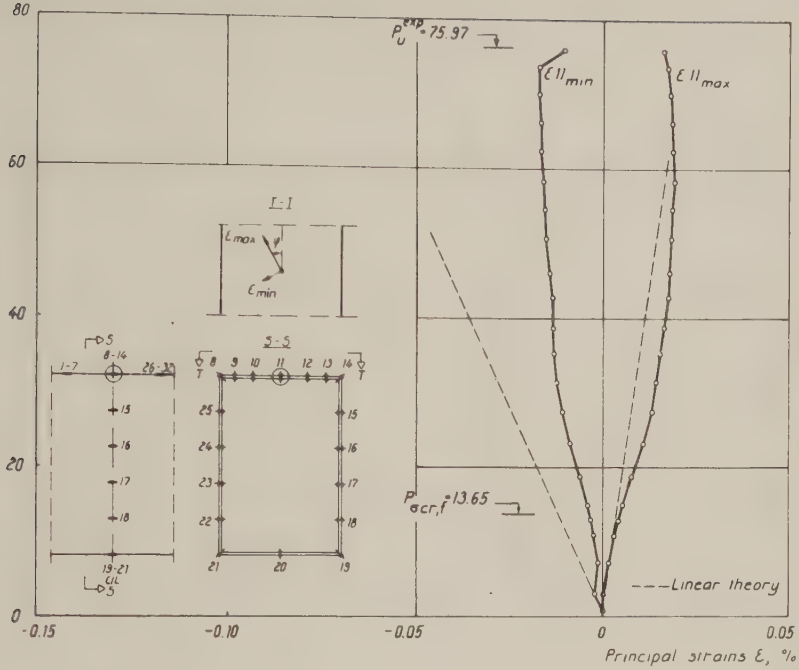
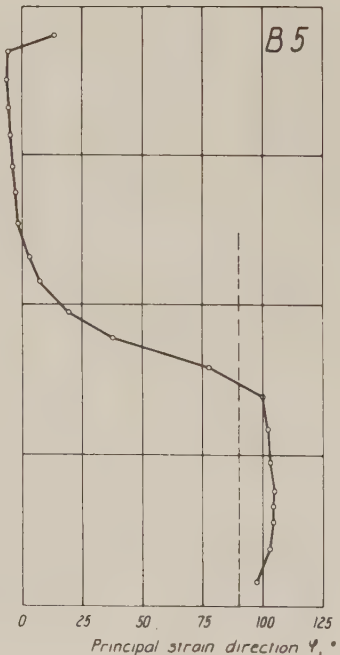
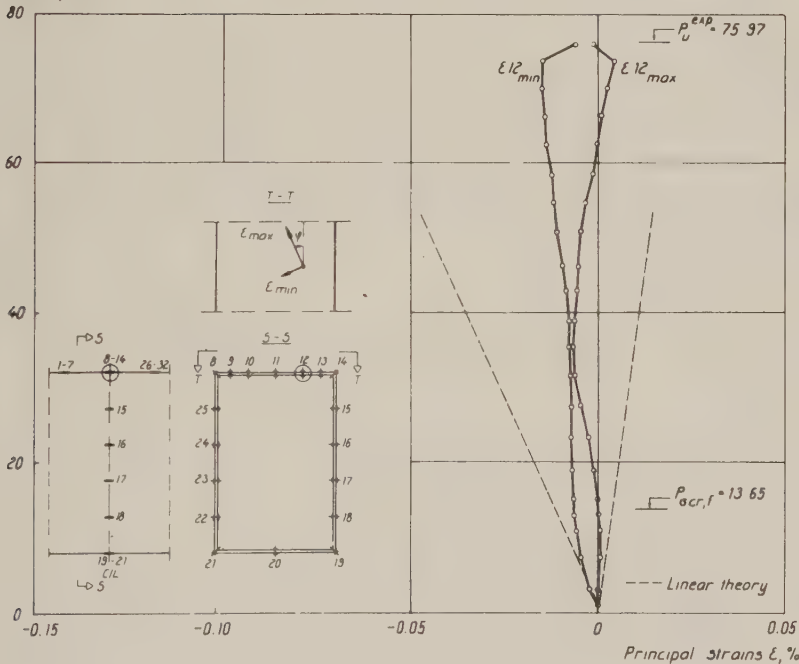


Fig. 3.3.31 a-b. Principal strains in the compression flange of test beam B4.

a,

Load P , kN

b,

Load P , kN

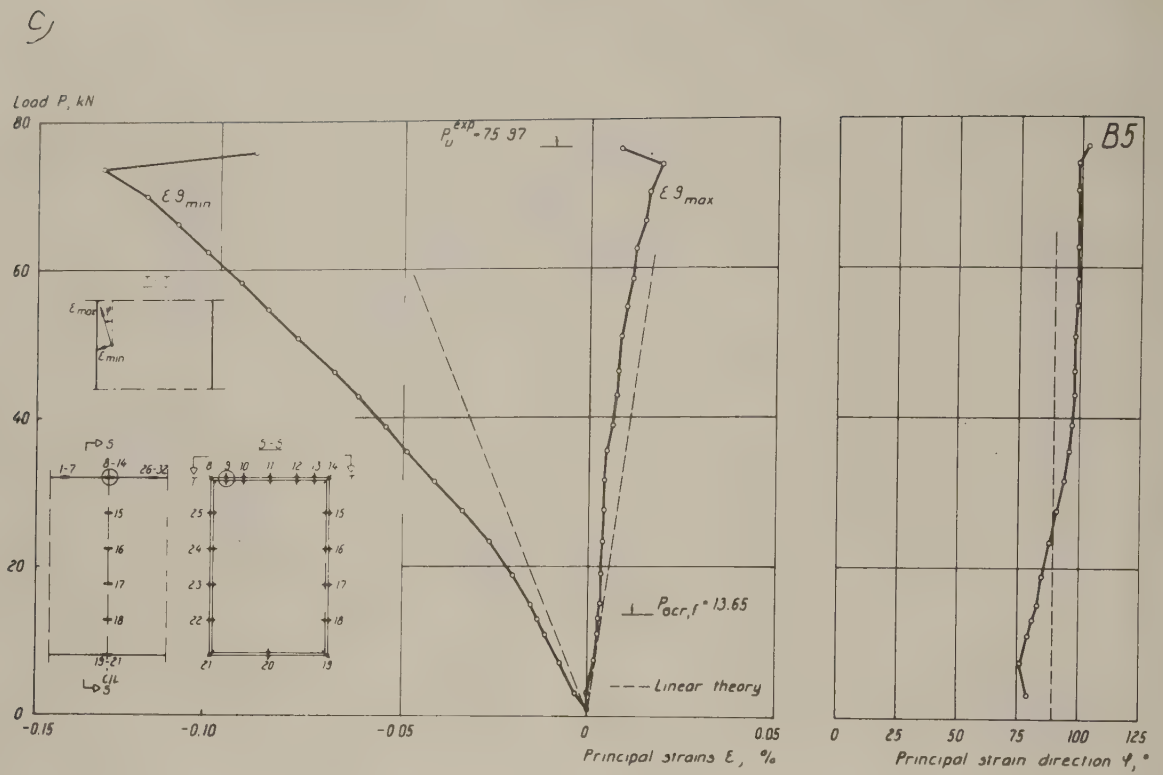
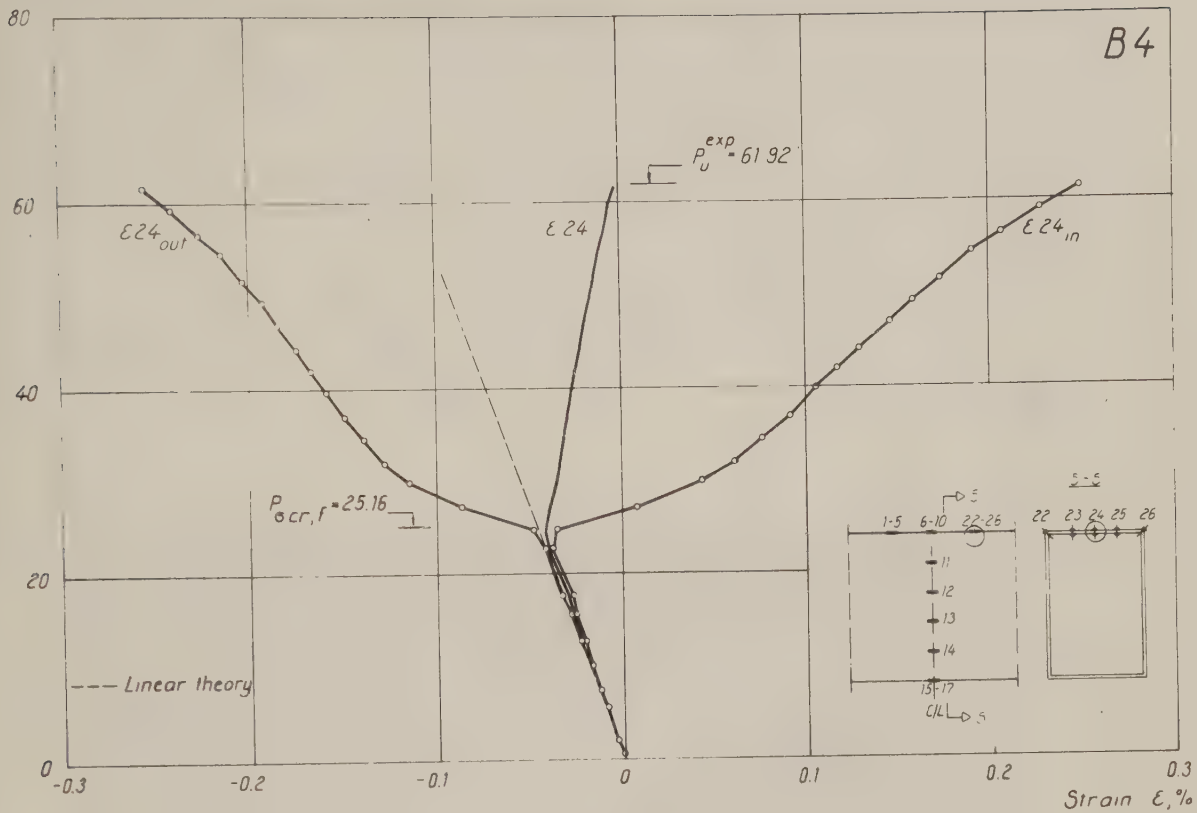


Fig. 3.3.32 a-c. Principal strains in the compression flange of test beam B5.

a,

Load P , kN

b,

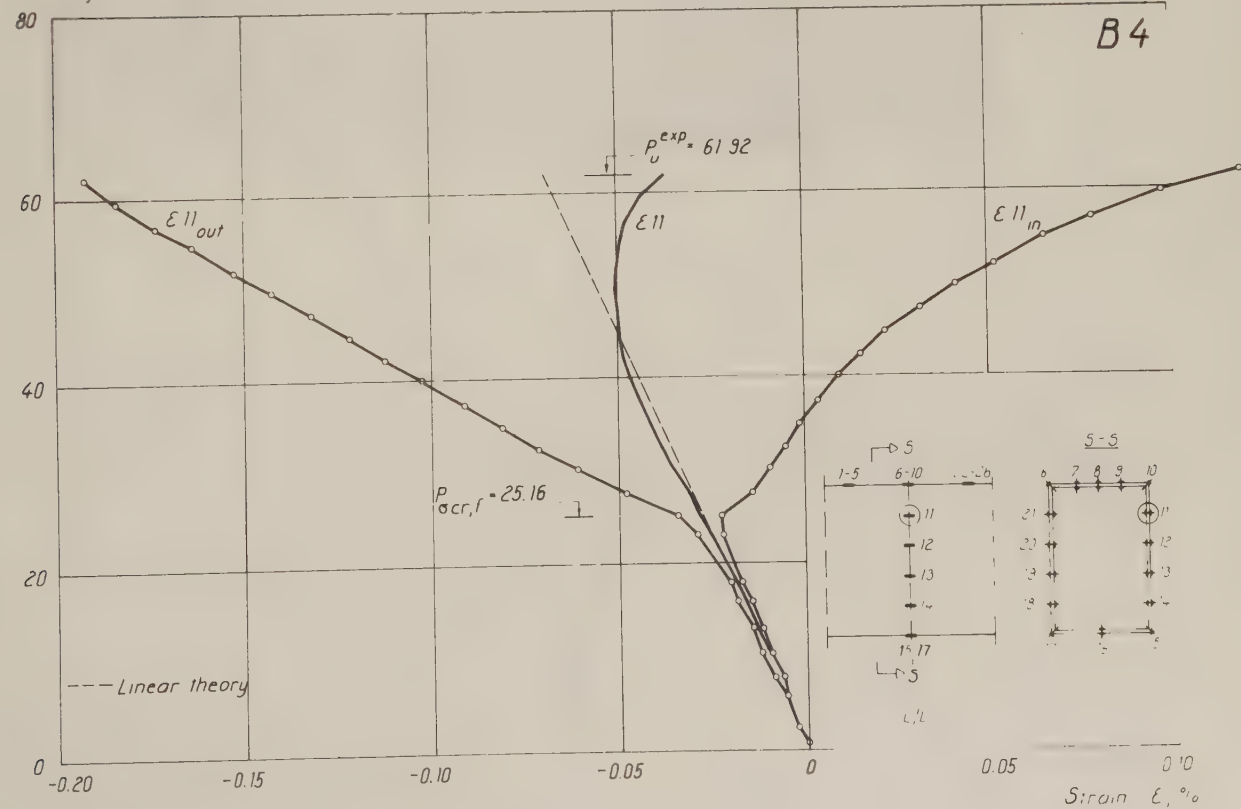
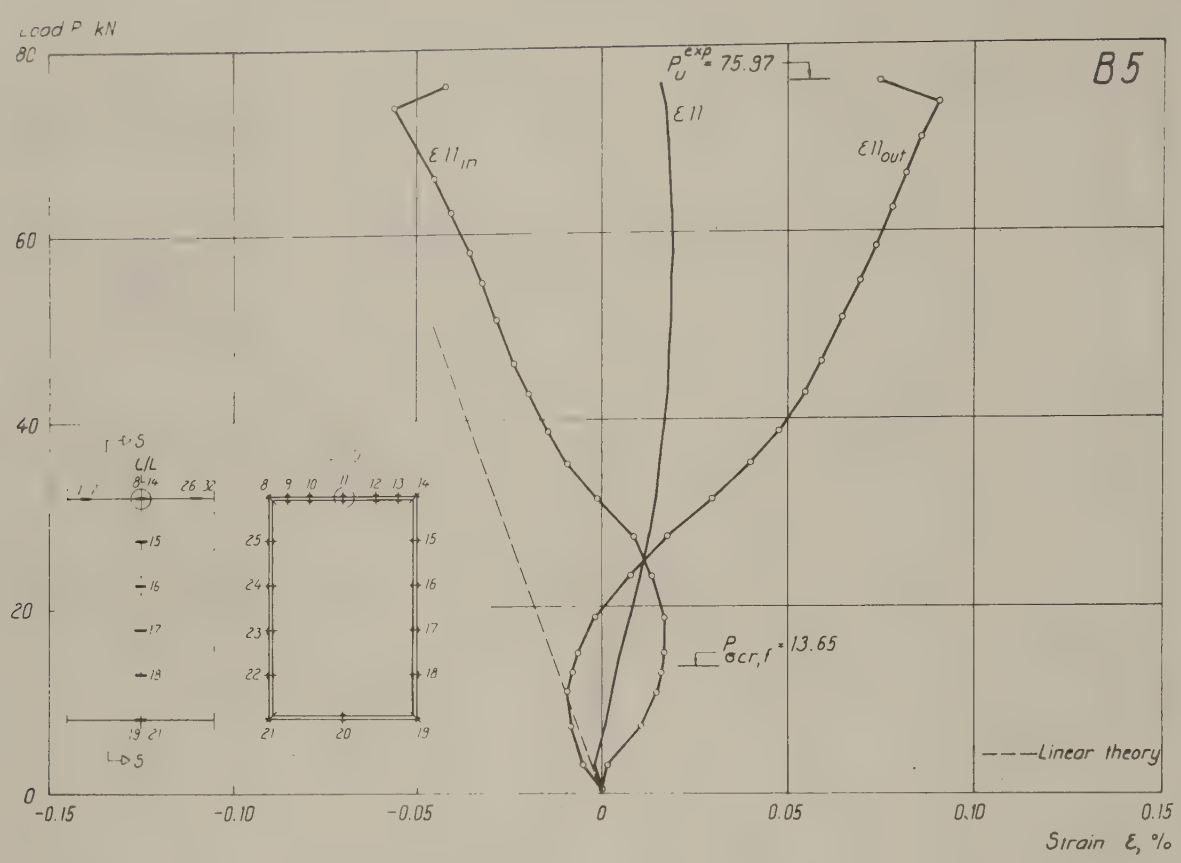
Load P , kN

Fig. 3.3.33 a-b. Longitudinal strains at points on the outer and inner surfaces of test beam B4.

a,



b,

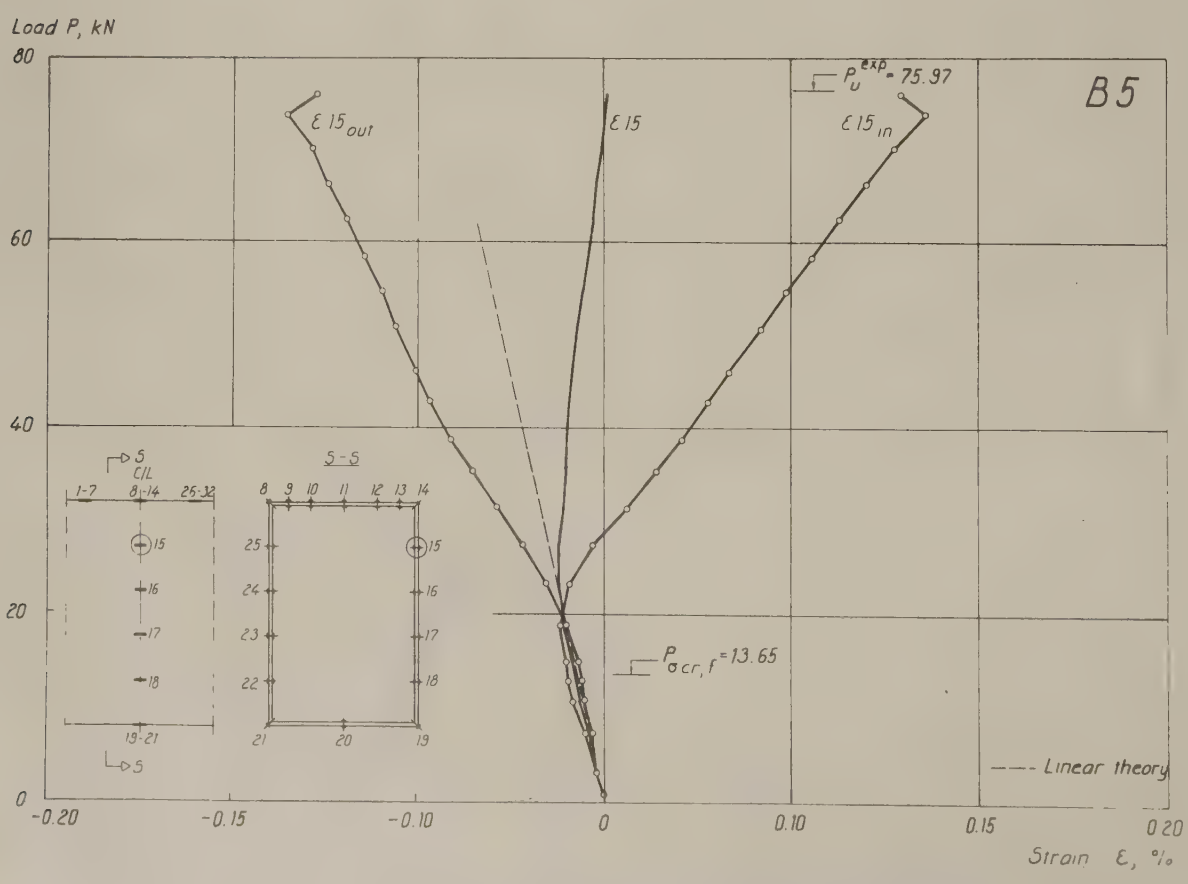


Fig. 3.3.34 a-b. Longitudinal strains at points on the outer and inner surfaces of test beam B5.

Character of failure.

As described earlier in this section, buckles appear in the compressed flange and the adjacent parts of webs in the post-buckling range. When the load is increased the amplitude of the buckles increases and the stresses are concentrated to the corners. Failure is initiated by yielding at the corners in one of the buckles and is propagated by a local folding in the flange and the adjacent parts of webs. These local deformations remain after unloading. A typical example of failure is shown in Fig. 3.3.35.

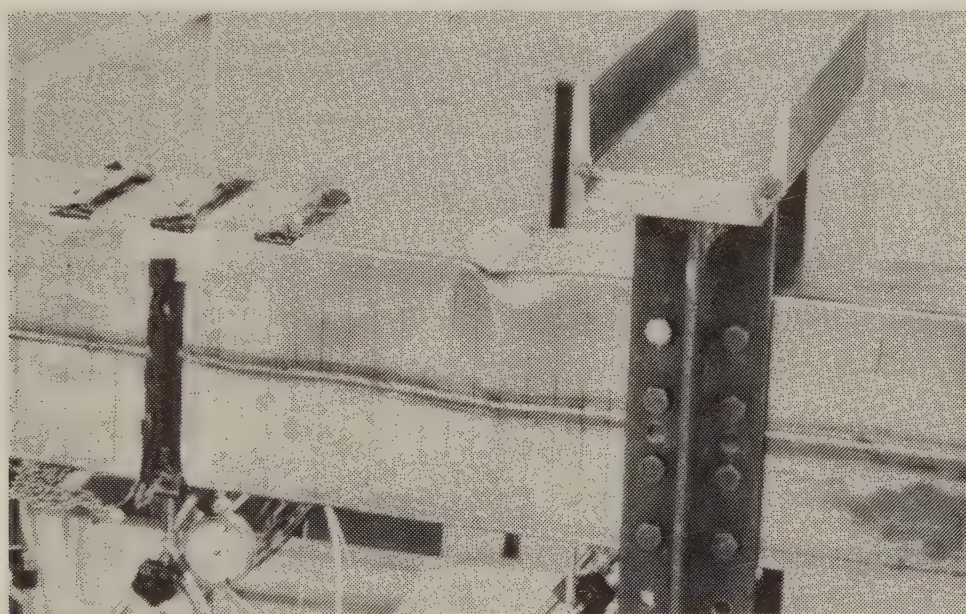


Fig. 3.3.35. Typical example of local failure.

Ultimate loads.

In Table 3.3.13 measured ultimate loads and ultimate moments are given for all the test beams. Note that the load always refers to the recorded maximum load during a certain load step.

Table 3.3.13. Measured ultimate loads and ultimate moments for test beams B1-B7 and C1-C7.

Test beam	p_u^{exp} kN	M_u^{exp} kNm
B1	69.85	24.34
B2	72.94	17.03
B3	82.16	38.12
B4	61.92	14.40
B5	75.97	35.44
B6	104.18	24.38
B7	61.02	28.53
C1	58.37	20.34
C2	51.16	11.92
C3	62.34	29.24
C4	60.68	11.73
C5	59.48	27.74
C6	82.31	19.22
C7	40.51	18.86

3.3.9 Discussion of results.

Deflections.

The deflections in the central section of the test beams were determined theoretically according to the procedure outlined in Section 3.1.14, that is with regard to the diminishing stiffness in the post-buckling range. Further, the extensions of the screw joints were taking into consideration, by assuming that the stiffness was unchanged in these sections. The influence of the shear forces was also included, but only with regard to linear theory. The calculations were performed on a computer by integration of the equation of the elastic line. The results are shown in Figs. 3.3.13-3.3.16 for four test beams. Results from simple linear theory are also shown in the same figures.

For test beams made of material B, the calculated deflections tend

to be greater than the measured ones. The opposite is true of test beams made of material C. These tendencies are confirmed by all the test beams.

The theoretical curvature of the test beams B4, C4, B5 and C5 under constant bending moment has been determined taking into consideration the diminishing stiffness in the post-buckling range. The calculations were performed in accordance with the effective width theory related in Section 3.1.14. The results are shown in Figs. 3.3.17-3.3.20 and are there designated "non-linear theory".

The theoretical curvature of the test beams has also been determined using the bar model described in Section 3.1. The influence of initial deflections and initial stresses was included. The results are shown in Figs. 3.3.17 and 3.3.19 for two length to width ratios λ of the buckles. The deviation between calculated and measured curves may be explained by the influence of initial deflections in the webs. Further, at high load levels the measured curvature may be influenced by the non-linear stress-strain behaviour of the material.

As can be expected in view of the observed deflections, the calculated curvature (non-linear theory) of beams made of material B is greater than the measured curvature, while the contrary is true for beams made of material C. The measured curvatures are not completely reliable since the influence of the screw joints is not entirely negligible. See also Section 3.3.7.

A comparison of the two materials B and C with one another can be summed up in the following way. At low loads, the load-deflection curves of materials B and C follow each other. At high loads, the load-deflection curve of material C is below that of material B. This can be explained by the fact that the yield stress of material C has been reached in some parts of the buckles and creep has started.

Normal deflections.

In Figs. 3.3.25-3.3.28 the initial normal deflections are seen to be decisive for the deflection pattern during the loading. As the

load is increased, the deflection pattern becomes more and more symmetrical. At high loads, the influence of the initial deformations on the size of the buckles will be rather small.

The initial normal deflections of the test beams were mainly introduced during the welding. The magnitude of the normal deflections for the flanges was less than the plate thickness t , whereas for the webs it was up to twice the plate thickness in the areas round the welds.

In Figs. 3.3.21-3.3.24, the critical loads of the compressed flanges have been indicated. At this load the normal deflections have grown considerably. A comparison between the buckles of the flanges and of the webs shows that their normal deflections start to increase at about the same time, but there is a tendency for the buckles in the webs to grow up more slowly.

A comparison between the two materials shows that the buckles in beams of material C tend to propagate faster than those of material B, which is a consequence of the lower yield stress for material C.

It was observed when the test beams were unloaded that beams made of material C exhibited permanent deformations where the buckles had occurred. For beams of material B the permanent deformations were rather small with the exception of the local folding.

Strains.

The theoretical behaviour with regard to strains of a perfect thin-walled box beam subjected to a pure bending moment can be described in the following way: In the sub-critical range the strains are distributed according to simple linear theory. As soon as the critical load is exceeded, buckles will appear in the compression flange and in the compressed parts of the webs. The buckles will give rise to a reduction of the compressive strains at the top of the buckles, and the strains will be concentrated to the areas that are forced to be straight, that is the corners. The behaviour is largely confirmed by the strain distributions for test beams B4 and B5 given in Figs. 3.3.29 a and 3.3.30 a. For test beam B5 the

strains are more concentrated to the corners than for test beam B4 because of the more slender form of that beam. The initial normal deflections of the compression flange of test beam B5 are rather large (cf. Fig. 3.3.23) and the critical load is low. Thus the strains will deviate from linear theory already at very low loads.

The measured and calculated strain distributions for the test beams B4 and B5 are given in Figs. 3.3.29b and 3.3.30b. The calculated values are obtained using the bar model described in Section 3.1 considering initial deflections and initial stresses. The ratio of the length to the width of the buckles in the compressed flange is given by $\lambda = 0.75$. The calculated strain distributions of the beam sections at the top of the buckle as well as at the zero level are shown in the figures. The measured strain distribution of test beam B4 refers to a section near the top of the buckle, whereas the measured strains of test beam B5 refer to a section near zero deflection. The agreement between measured and calculated strains is good.

The theoretical direction of the principal stresses in the compression flange can be roughly analysed as follows: In the sub-critical range the compressive stresses are directed along the box beam and no significant stresses appear in the transverse direction. When buckles arise, transverse tensile stresses will begin in sections of maximum buckle amplitude while transverse compressive stresses will begin in sections of zero amplitude. These transverse stresses will be transferred to the edge zones of the flange, which can be treated as transverse loaded beams. On account of the transverse stresses, the edge zones will be subjected to both longitudinal bending stresses and shear stresses. Superposition of the longitudinal stresses on those caused by the transverse stresses will result in a total stress picture where the trajectories of the total principal compressive stresses will come closer to the centre line of the flange at the points where the amplitude of the buckles is zero. The discussion above is largely confirmed by the principal strains and principal strain directions given in Figs. 4.3.25-4.3.28. In Fig. 3.3.32 (numbering of strain gauges see Fig. 3.3.27a), showing principal strains and principal strain directions of three points

on the compression flange of test beam B5, it can clearly be seen that the strains are concentrated towards the corners with increasing load. Furthermore, since the strain gauges are located in a section where the amplitude of the buckles is almost zero, the points near the centre of the flange will experience a change in principal strain direction with increasing load and the result will be compressive strains transverse the flange. The same tendencies can be seen in Fig. 3.3.31 (numbering of strain gauges see Fig. 3.3.25a) for two points on the compression flange of test beam B4, but, on account of the smaller flange width and web height of test beam B4, the principal strains and principal strain directions are not influenced by buckling phenomena to such an extent.

To determine the load at which buckles appear, it is possible to analyse how the strains on the outer and inner side at a certain point of the buckles change with increasing load. Thus Fig. 3.3.33 shows how the strain curves of test beam B4 distinctly diverge at a load level corresponding to the buckling load. The initial deflections of the compression flange of test beam B4 are rather small (cf. Fig. 3.3.21). The strains diverge simultaneously in the flange and the webs, indicating that buckles arise at the same time. In the beginning the growth of the buckles is more rapid in the flange. Test beam B5, which exhibits rather large initial deflections in the compression flange, shows gradually increasing strains immediately after load application, see Fig. 3.3.34. Depending on the initial deflections, the direction of curvature of the sheet changes for the points shown. As for test beam B4, the strains increase faster in the flange than in the webs at low load levels.

Character of failure.

The test beams made of material B exhibited a more brittle failure than those of material C and the failure was often accompanied by a bang. The load-carrying capacity after failure was about the same for the beams made of the two materials.

The influence of the height of the beam sections can be summed up as follows: Increasing height gave rise to more brittle failure. Increasing height led to a relatively greater reduction in load-carrying capacity after failure.

Ultimate loads.

The ultimate moments have been calculated in accordance with the principles given in Section 3.1.14. To analyse the influence of buckling phenomena in the vertical webs the ultimate moments have been determined with "reduced webs" on the one hand and with "non-reduced webs" on the other. Four different expressions for the effective width have been used, namely those according to Winter, von Karman, Kenedi and Gerard (cf. Eqs. 2.2.9 - 2.2.12). The calculated ultimate moments are given in Table 3.3.14.

Table 3.3.15 presents ratios of measured to calculated ultimate moments. It is interesting to study the variation of the ratio for different test beams. Thus in Table 3.3.15 the respective coefficients of variation (quotient of standard deviation to mean value) are given. It is obvious that the method with "reduced webs" gives rise to more reliable results than the method with "non-reduced webs". The different expressions of effective width result in coefficients of variation of the same magnitude. It must be emphasized that the measured ultimate moments of test beams A1 and A2 have been determined under "constant load" while those of test beams B1-B7 and C1-C7 have been determined under "constant deformation". Consequently the ratios of measured to calculated ultimate moments of test beams A1 and A2 ought to be increased by a few per cent in comparison with the ratios of the other test beams.

Table 3.3.14. Calculated ultimate moments.

Test beam	Calculated ultimate moments in kNm with effective width according to							
	Non-reduced webs				Reduced webs			
	Winter	von Karman	Kenedi	Gerard	Winter	von Karman	Kenedi	Gerard
A1	38.49	41.99	36.66	39.36	33.49	40.70	29.95	35.29
A2	12.22	12.72	12.04	12.40	8.73	9.76	8.35	9.08
B1	37.83	38.62	38.21	38.42	21.69	23.31	22.63	22.98
B2	23.90	24.50	24.30	24.40	16.30	17.73	16.55	17.16
B3	73.82	74.94	74.62	74.78	34.57	36.71	37.96	37.24
B4	18.73	19.51	18.43	19.00	13.45	15.06	12.84	13.98
B5	66.22	67.07	67.78	67.37	31.83	33.60	35.66	34.50
B6	38.39	39.17	38.81	39.00	21.89	23.52	22.91	23.22
B7	50.62	51.43	51.51	51.46	26.51	28.21	28.70	28.41
C1	28.10	28.93	28.03	28.51	18.15	19.87	18.05	19.00
C2	15.22	15.81	15.16	15.50	11.77	13.17	11.34	12.28
C3	44.63	45.73	44.53	45.16	24.93	27.02	25.55	26.30
C4	13.97	14.83	13.53	14.20	11.09	12.84	10.19	11.54
C5	47.20	48.05	47.84	47.95	25.89	27.69	27.50	27.58
C6	28.03	28.86	27.96	28.44	18.12	19.83	18.02	18.96
C7	28.03	28.86	27.96	28.44	18.12	19.83	18.02	18.96

Table 3.3.15. Ratios of measured to calculated ultimate moments, coefficients of variation.

Test beam	The ratio of measured to calculated ultimate moments with effective width according to							
	Non-reduced webs				Reduced webs			
	Winter	von Karman	Kenedi	Gerard	Winter	von Karman	Kenedi	Gerard
A1	0.98	0.90	1.03	0.96	1.13	0.93	1.26	1.07
A2	0.68	0.66	0.69	0.67	0.96	0.86	1.00	0.92
B1	0.64	0.63	0.64	0.63	1.12	1.04	1.08	1.06
B2	0.71	0.70	0.70	0.70	1.04	0.96	1.03	0.99
B3	0.52	0.51	0.51	0.51	1.11	1.04	1.00	1.02
B4	0.77	0.74	0.78	0.76	1.07	0.96	1.12	1.03
B5	0.54	0.53	0.52	0.53	1.11	1.05	0.99	1.03
B6	0.64	0.62	0.63	0.63	1.11	1.04	1.06	1.05
B7	0.56	0.55	0.55	0.55	1.08	1.01	0.99	1.00
C1	0.72	0.70	0.73	0.71	1.12	1.02	1.13	1.07
C2	0.78	0.75	0.79	0.77	1.01	0.91	1.05	0.97
C3	0.66	0.64	0.66	0.65	1.17	1.08	1.14	1.11
C4	0.84	0.79	0.87	0.83	1.06	0.91	1.15	1.02
C5	0.59	0.58	0.58	0.58	1.07	1.00	1.01	1.01
C6	0.69	0.67	0.69	0.68	1.06	0.97	1.07	1.01
C7	0.67	0.65	0.67	0.66	1.04	0.95	1.05	0.99
Coef. of var.	17.1 %	15.2 %	19.4 %	17.1 %	4.8 %	6.3 %	6.9 %	4.4 %

3.4 Two-span beam.

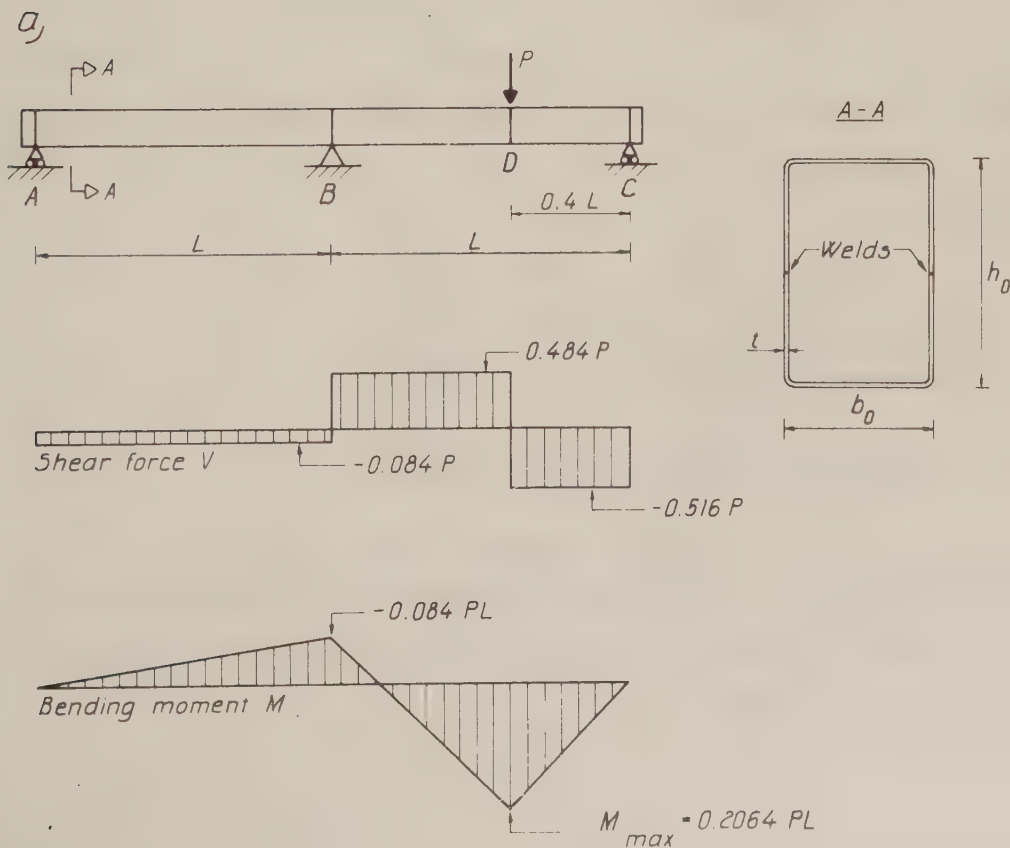
3.4.1 Purpose.

In Sections 3.2 and 3.3 simply supported box beams subjected to pure bending were investigated. A natural experimental continuation was to study statically undetermined two-span beams in bending to determine the possibility of bending moment redistribution and application of yield theory when failure is caused by local buckling.

The experiments were carried out as undergraduate work by the now qualified civil engineers Kyösti Tuutti and Christer Backman with the author as supervisor.

3.4.2 Scope.

The investigation included bending tests on two box beams designated D1 and D2. Distribution of loads and notations for cross section are shown for these test beams in Fig. 3.4.1 a and b. The figures also present the distribution of shear forces and bending moments calculated according to linear theory. Nominal dimensions of test beams D1 and D2 are given in Table 3.4.1.



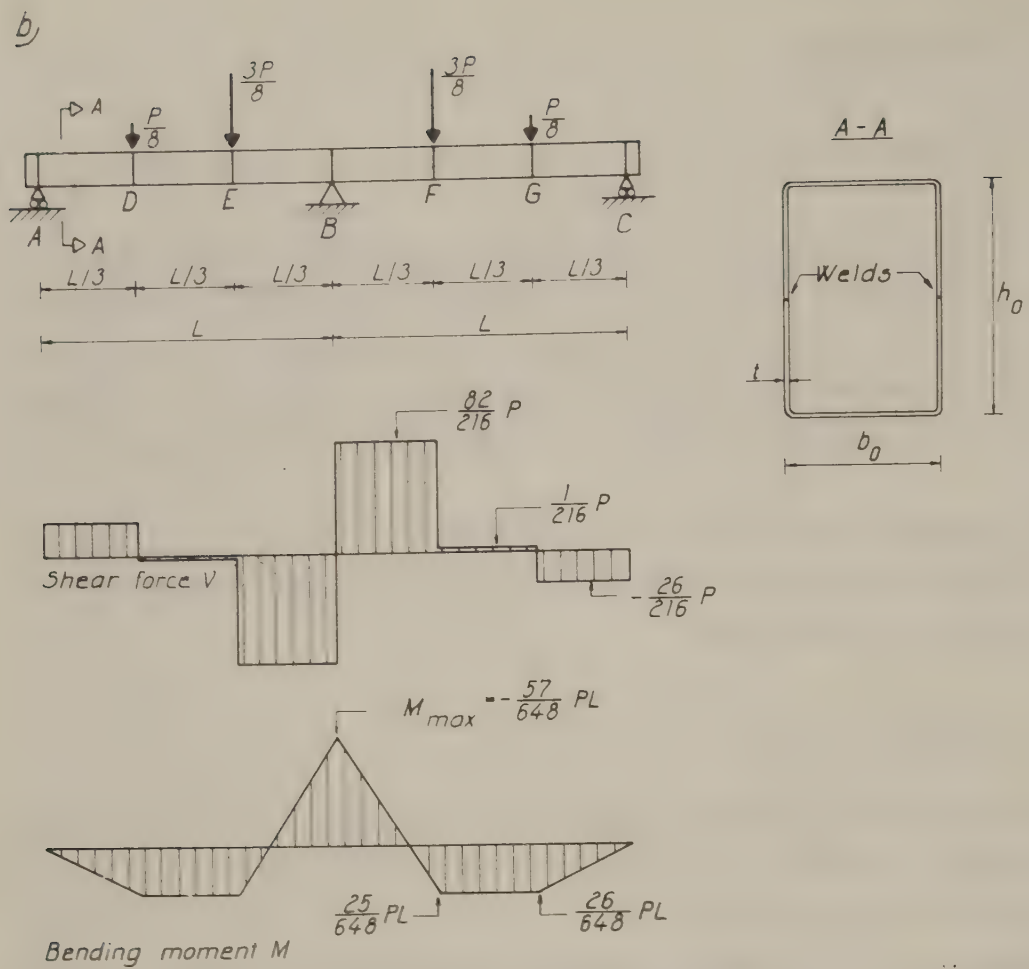


Fig. 3.4.1. Distribution of load. Calculated distribution of shear force and bending moment according to linear theory. Notations for cross section.
a) Test beam D1
b) Test beam D2

Table 3.4.1. Nominal dimensions of test beams D1 and D2.

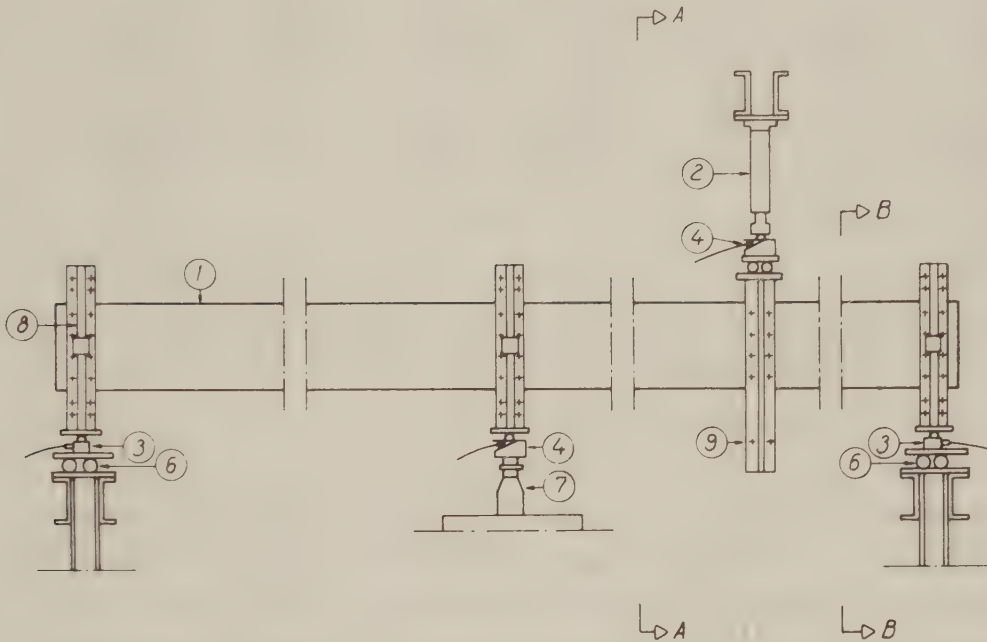
Test beam	b_0 mm	h_0 mm	t mm	L mm
D1, D2	133	200	1	3480

3.4.3 Loading and support arrangements.

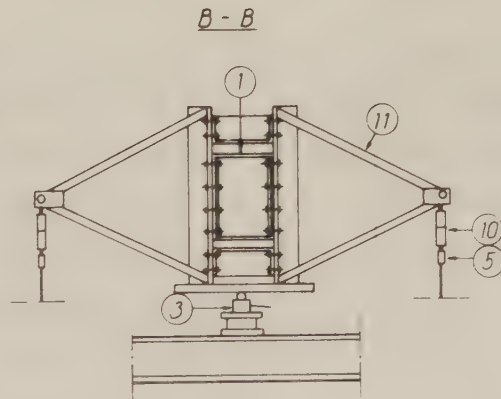
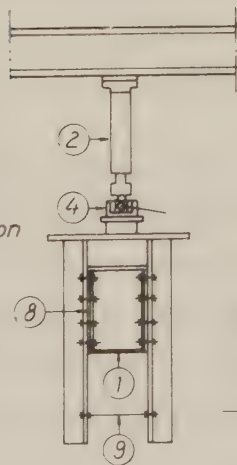
The principal test arrangement for test beam D1 is shown in Fig. 3.4.2 a. The load was applied by a hydraulic jack and transferred through a load cell and a roller bearing via screw joints into the webs of the test beam.

Fig. 3.4.2b shows the principal test arrangement for test beam D2. In this case the load was applied by two hydraulic jacks which were coupled together. The load was distributed into the test beam by two I-beams.

a,



- A - A
- ① Test beam
 - ② Hydraulic jack
 - ③ Load cell for compression
 - ④ Built in load cell for compression
 - ⑤ Load cell for tension
 - ⑥ Roller bearing
 - ⑦ Vertically adjustable fixed support
 - ⑧ Screw joint
 - ⑨ Spacer screw
 - ⑩ Joint hinged in two directions
 - ⑪ Anti-torque frame



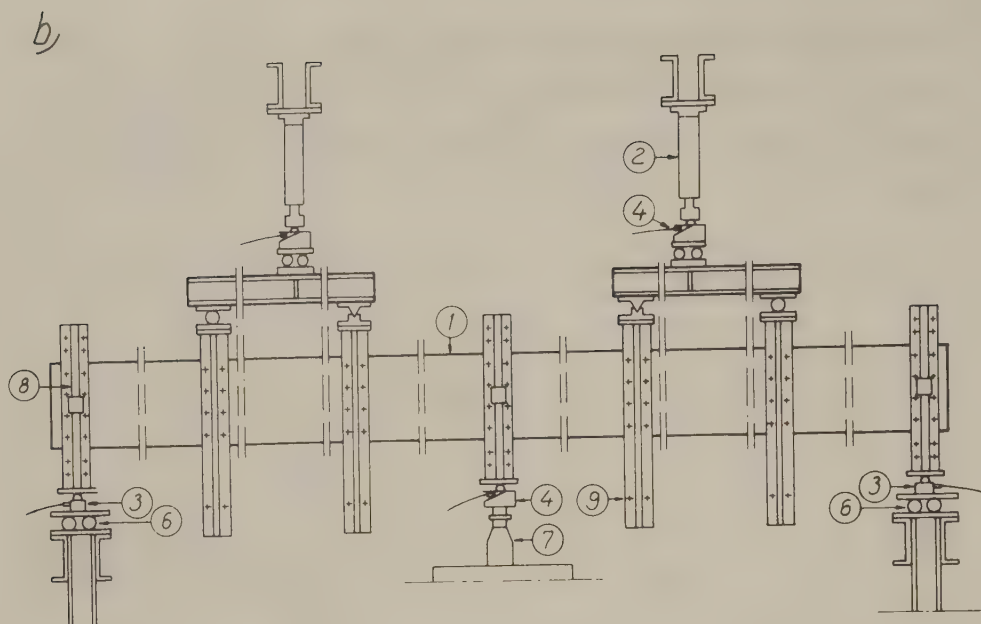


Fig. 3.4.2. Principal arrangement of loading and support devices.
 a) Test beam D1.
 b) test beam D2.

The supports consisted of one vertically adjustable fixed support at the centre of the test beams and of roller supports at the ends. By the use of prestressed anti-torque frames, the supports were made to prevent elevation and unintentional torsion. The flanges were free at the supports. The performance of the screw joints under the point loads is identical to that described in Section 3.3.3.

3.4.4 Dimensions. Material properties.

The process of manufacture of the test beams was the same as described in Section 3.3.4 with one improvement, namely that the welding was done in an automatic welding machine. Since the test beams D1-D2 as well as F1-F6 (described in Section 4.3), G1-G4 (described in Section 5.1) and H1-H5 (described in Section 5.2) were made from the same cold-rolled steel sheet, their material properties will be summarized at the same time.

The material consisted of cold-rolled sheet COR-TEN A. The properties of the material are given in further detail in Section 3.3.4. Results of chemical analysis carried out by the manufacturer of the cold-rolled sheet are shown in Table 3.4.2.

Table 3.4.2. Results of chemical analysis carried out by the manufacturer of the cold-rolled sheet.

Test beam	C	Si	Mn	P	S	N	Cr	Cu	Ni
	%	%	%	%	%	%	%	%	%
D1, D2	0.09	0.37	0.45	0.093	0.020	0.010	0.97	0.37	0.01
F1-F6									
G1-G4									
H1-H5									

Tensile test pieces were taken from all the test beams D1-D2, F1-F6, G1-G4 and H1-H5, and were tested according to the procedure outlined in Appendix B. One tensile test piece was cut from each of the two flanges, except for the test beam D1 where five tensile test pieces were cut from the lower flange. The tensile test pieces were located at the areas near the beam ends to ensure against effects of previous loading history. Results of the tensile tests are given in Table 3.4.3. As there are only small deviations from the mean values these will be used hereafter. A typical stress-strain diagram is shown in Fig. 3.4.3. Results of tensile tests carried out by the manufacturer of the cold-rolled sheet are shown in Table 3.4.4. The results are in rather good agreement with those given in Table 3.4.3.

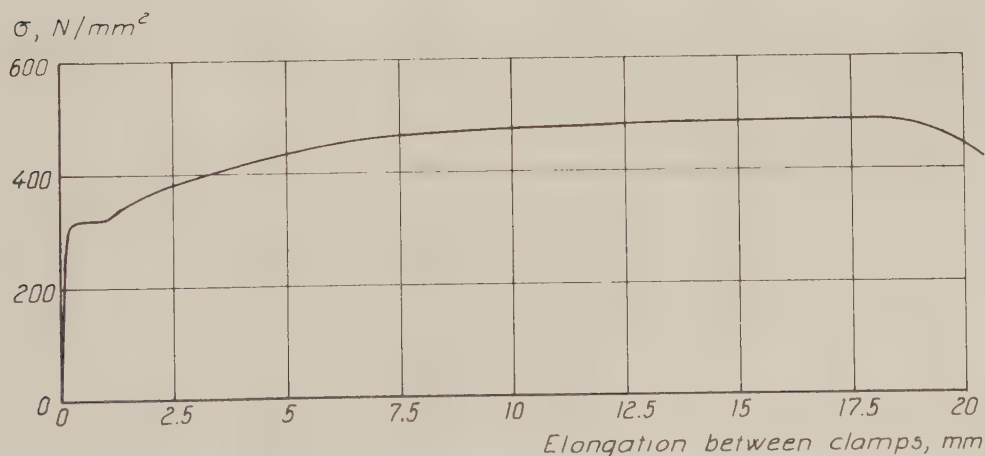


Fig. 3.4.3. Typical stress-strain curve. Tensile test piece H5:u.

Table 3.4.3. Results of tensile tests carried out on pieces from the test beams D1-D2, F1-F6, G1-G4 and H1-H5.
u = up d = down MV = mean value

Test beam	Tensile test piece	t mm	σ_y N/mm ²	σ_u N/mm ²	δ_5 %
D1	:d1	1.04	309	486	34
	:d2	1.04	313	489	36
	:d3	1.05	310	483	37
	:d4	1.05	311	485	39
	:d5	1.05	312	481	40
	:MV		311	485	37
	:u	1.07	310	481	39
D2	:d	1.04	324	485	37
	:u	1.05	335	491	38
F1	:d	1.07	337	491	39
	:u	1.07	327	481	39
F2	:d	1.09	324	488	37
	:u	1.09	332	488	37
F3	:d	1.06	320	491	38
	:u	1.04	317	488	39
F4	:d	1.07	337	486	39
	:u	1.05	330	485	39
F5, F6	:d	1.06	328	485	37
	:u	1.07	329	487	40
G1	:d	1.10	329	484	36
	:u	1.09	317	488	39
G2	:d	1.07	319	490	38
	:u	1.07	319	485	38
G3	:d	1.10	336	489	39
	:u	1.09	311	484	39
G4	:d	1.10	333	486	39
	:u	1.08	318	491	39
H1	:d	1.03	336	489	37
	:u	1.05	316	487	37
H2	:d	1.02	321	487	42
	:u	1.05	325	487	38
H3	:d	1.05	322	488	36
	:u	1.05	325	487	39
H4	:d	1.06	331	491	38
	:u	1.06	334	491	40
H5	:d	1.05	322	489	38
	:u	1.06	315	486	37
MV	:		325	487	38

Table 3.4.4. Results of tensile test carried out by the manufacturer of the cold-rolled sheet.

Test beam	Rolling direction	σ_y N/mm ²	σ_u N/mm ²	δ_5 %
D1-D2	//	310	496	33
F1-F6	⊥	349	496	37
G1-G4	//	321	491	37
H1-H5	⊥	341	495	37

From the test beams D1-D2, F1-F6, G1-G4 and H1-H5, bending test pieces were cut out and tested according to the procedure described in Appendix C. Results from the vibration tests are given in Table 3.4.5.

The measured lengths of the test beams D1 and D2 and the positions where the loads acted were in very good agreement with the nominal ones given in Section 3.4.2, with one exception, namely for test beam D1 where the distances 0.6 L (= 2088 mm) and 0.4 L (= 1392 mm) were instead 2150 and 1330 mm respectively. Measured cross-sectional dimensions are given in Table 3.4.6 as mean values. Cross-sectional constants and critical stresses, defined according to Appendix A, are given in Table 3.4.7.

Table 3.4.5. Results of vibration tests carried out on bending test pieces from the test beams D1-D2, F1-F6, G1-G4 and H1-H5. u = up d = down MV = mean value

Test beam	Bending test piece	t mm	b mm	L mm	m g	f Hz	$E \cdot 10^{-5}$ N/mm ²
D1	: bu	1.07	20.04	204.0	33.87	135.5	2.02
L2	: bd	1.04	20.04	204.1	32.89	131.4	2.02
	: bu	1.05	20.05	204.1	33.23	132.4	2.02
F1	: bd	1.07	20.16	203.6	34.15	137.0	2.06
	: bu	1.07	20.15	202.7	33.82	137.6	2.04
F2	: bd	1.09	20.14	203.6	34.53	139.3	2.07
	: bu	1.09	20.13	202.8	34.44	139.9	2.04
F3	: bd	1.07	20.10	203.6	33.73	136.1	2.04
	: bu	1.04	20.11	202.8	32.63	133.3	2.04
F4	: bd	1.07	20.07	203.6	33.86	135.4	2.02
	: bu	1.05	20.11	202.8	33.03	134.2	2.02
F5,F6	: bd	1.06	20.04	204.1	33.44	133.5	2.01
	: bu	1.07	20.05	204.1	33.99	135.5	2.02
G1	: bd	1.10	20.02	203.6	34.63	139.5	2.02
	: bu	1.09	20.15	202.8	34.39	139.8	2.03
G2	: bd	1.07	20.05	203.6	33.80	136.1	2.03
	: bu	1.07	20.11	202.8	33.73	136.8	2.01
G3	: bd	1.10	20.02	203.6	34.56	139.0	2.00
	: bu	1.09	20.14	202.8	34.44	139.4	2.02
G4	: bd	1.10	20.01	203.6	34.55	139.8	2.02
	: bu	1.08	20.15	202.8	34.25	139.1	2.03
H1	: bd	1.03	19.94	203.6	32.18	130.8	2.01
	: bu	1.05	20.15	202.7	33.20	134.3	2.00
H2	: bd	1.02	19.97	203.6	31.95	129.4	2.02
	: bu	1.05	20.15	202.8	32.94	133.8	2.01
H3	: bd	1.05	20.05	203.9	33.20	132.3	2.01
	: bu	1.05	20.04	204.1	33.07	132.0	1.99
H4	: bd	1.07	20.06	204.1	33.77	134.7	2.02
	: bu	1.06	20.04	204.2	33.61	134.0	2.04
H5	: bd	1.05	20.05	204.1	33.20	133.0	2.02
	: bu	1.06	20.04	204.1	33.39	133.6	2.02
MV	:						2.02

Table 3.4.6. Measured dimensions of the test beams D1 and D2.

Test beam	b_o mm	h_o mm	t mm	R mm
D1	136	198	1.05	3
D2	134	199	1.05	3

Table 3.4.7. Cross-sectional constants and critical stresses for the test beams D1 and D2.

Test beam	b/t	h/t	$I_o \cdot 10^{-4}$ mm^4	$\sigma_{cr,f}$ N/mm^2	$\sigma_{cr,w}$ N/mm^2	σ_{cr} N/mm^2
D1	128.5	187.6	409	45.1	126.5	59
D2	126.6	188.5	409	46.5	125.2	60

3.4.5 Measuring devices and measurement procedure.

The computer controlled data acquisition system described in Appendix D was used. All measured data were punched on paper tape by means of a high speed punch.

The measurements comprised the determination of applied loads and support reactions, the measurement of the deflection of the test beams and the recording of the size of normal deflections at separate points on the compressed flanges.

Load and support reaction measurement.

The load cells for the measurement of compression forces were of the Bofors LSK-2 type while the load cells for tension forces were fabricated at the Department. During every load step an attempt was made to keep the deformations of the test beams constant. To study the influence of "creep", the loads and support reactions were recorded three times in every load step, immediately after application of the load, about ten seconds after load application and just before the next load step. Furthermore, the applied loads

were recorded as continuous functions of time.

Fig. 3.4.4 shows the positions and numbering of load cells for test beam D1.

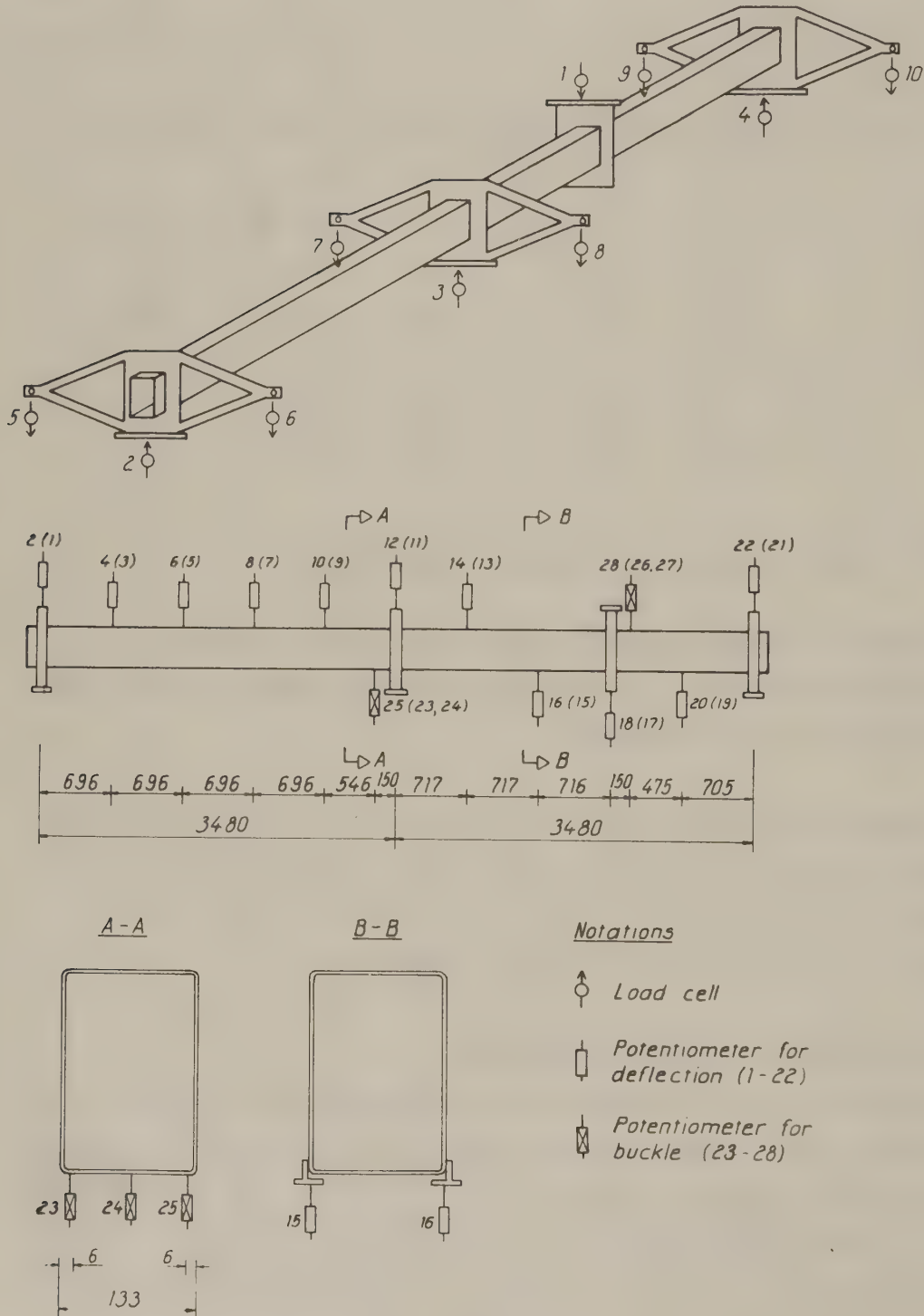


Fig. 3.4.4. Positions and numbering of load cells and potentiometers for test beam D1.

Deflection measurement.

The deflections of the test beams were measured by means of linear motion potentiometers with a resolution of 0.03 mm. To determine the deflections of a certain section two potentiometers, one in the extension of each web, were used. The points of the potentiometers measured against pieces of perspex which were fastened to the tension side of the webs. At the supports and at the points of load application the pieces of perspex were fastened to the screw joints.

Fig. 3.4.4 shows the positions and numbering of potentiometers for test beam D1.

Measurement of normal deflections.

The normal deflections of the test beams were measured by means of linear motion potentiometers with a resolution of 0.03 mm. The normal deflections were only measured at a few points. To determine the size of a buckle at a single point three potentiometers were needed (see Fig. 3.4.4 section A-A). The two outer potentiometers were used to obtain a reference line.

3.4.6 Results.

In all figures and tables of Sections 3.4.6 and 3.4.7 the loads refer to those measured immediately after the actual load application.

Deflections.

A load-deflection curve for test beam D1 is shown in Fig. 3.4.5 where also failure, appearing as local folding of the compressed flanges, is indicated by circles. Fig. 3.4.6 shows the corresponding curves for test beam D2.

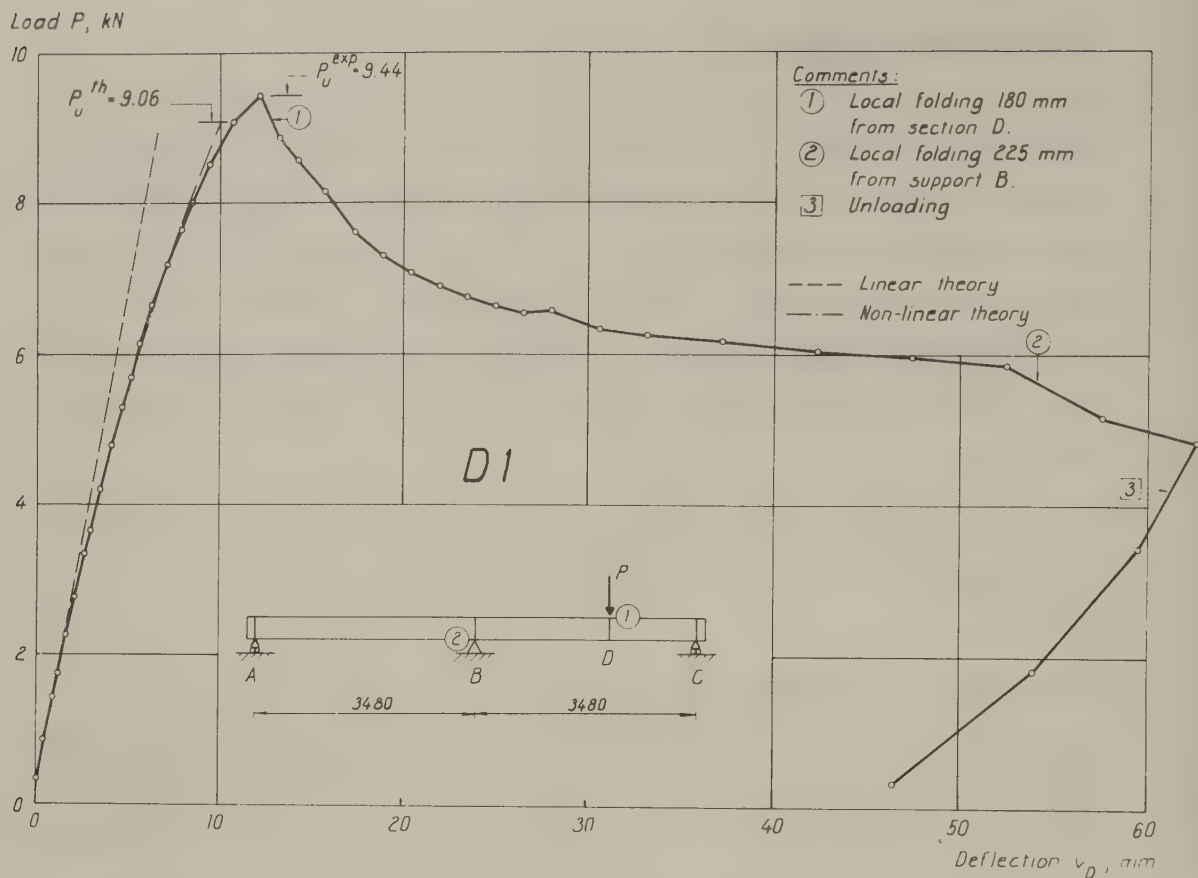
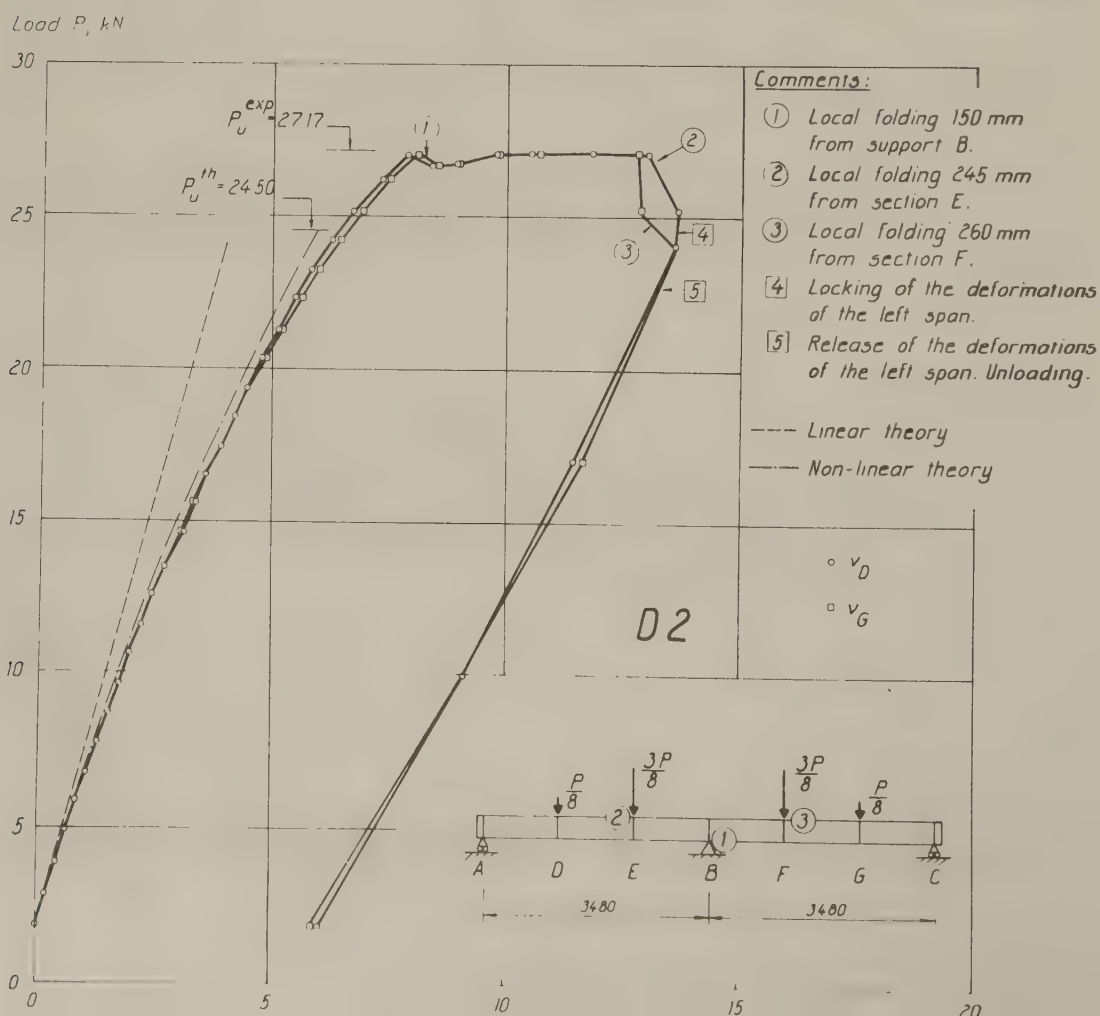


Fig. 3.4.5. Load-deflection curve for test beam D1.

a)



b,

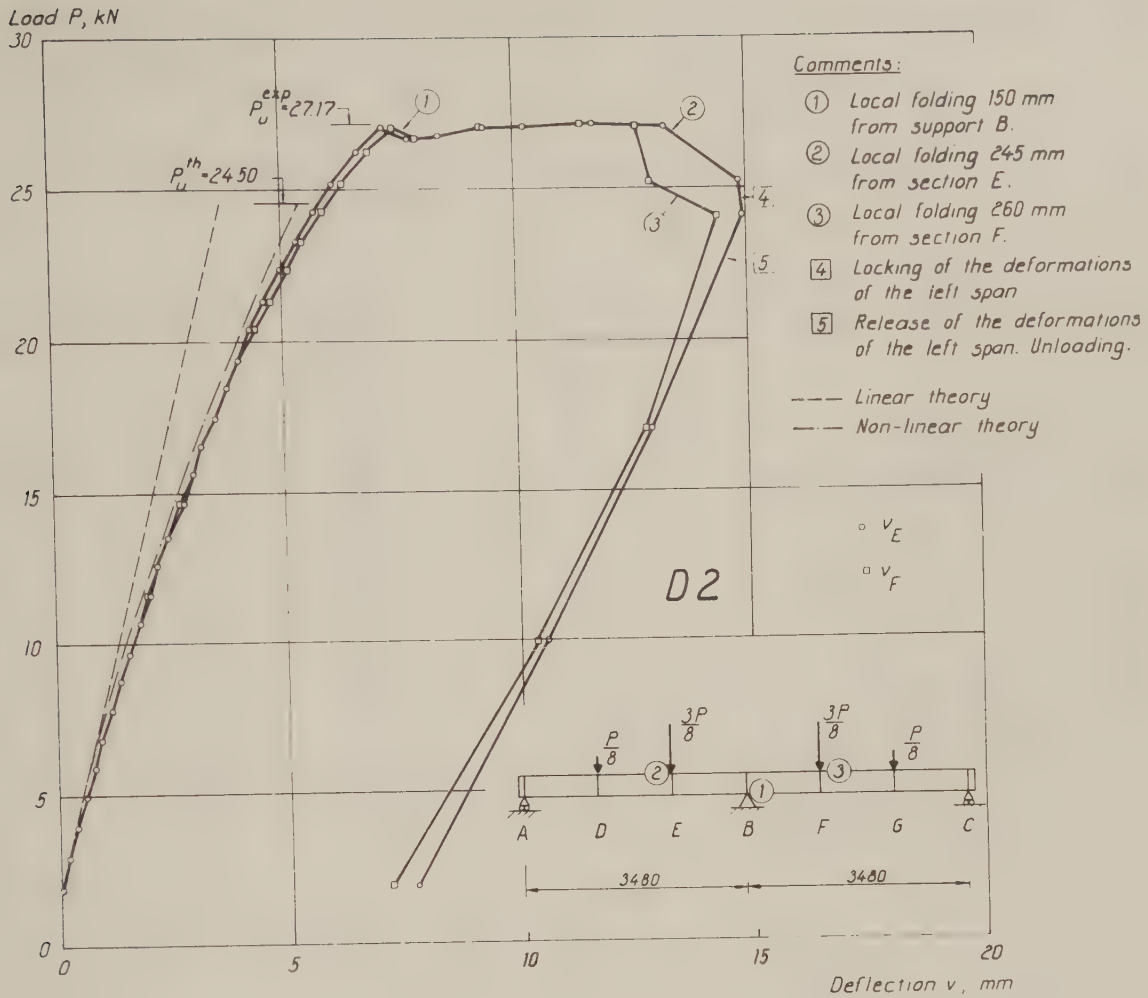


Fig. 3.4.6 a-b. Load-deflection curves for test beam D2.

Normal deflections.

Fig. 3.4.7 shows the normal deflections of four points on the flanges of test beam D2. The initial buckles were not measured and they have therefore been given zero values.

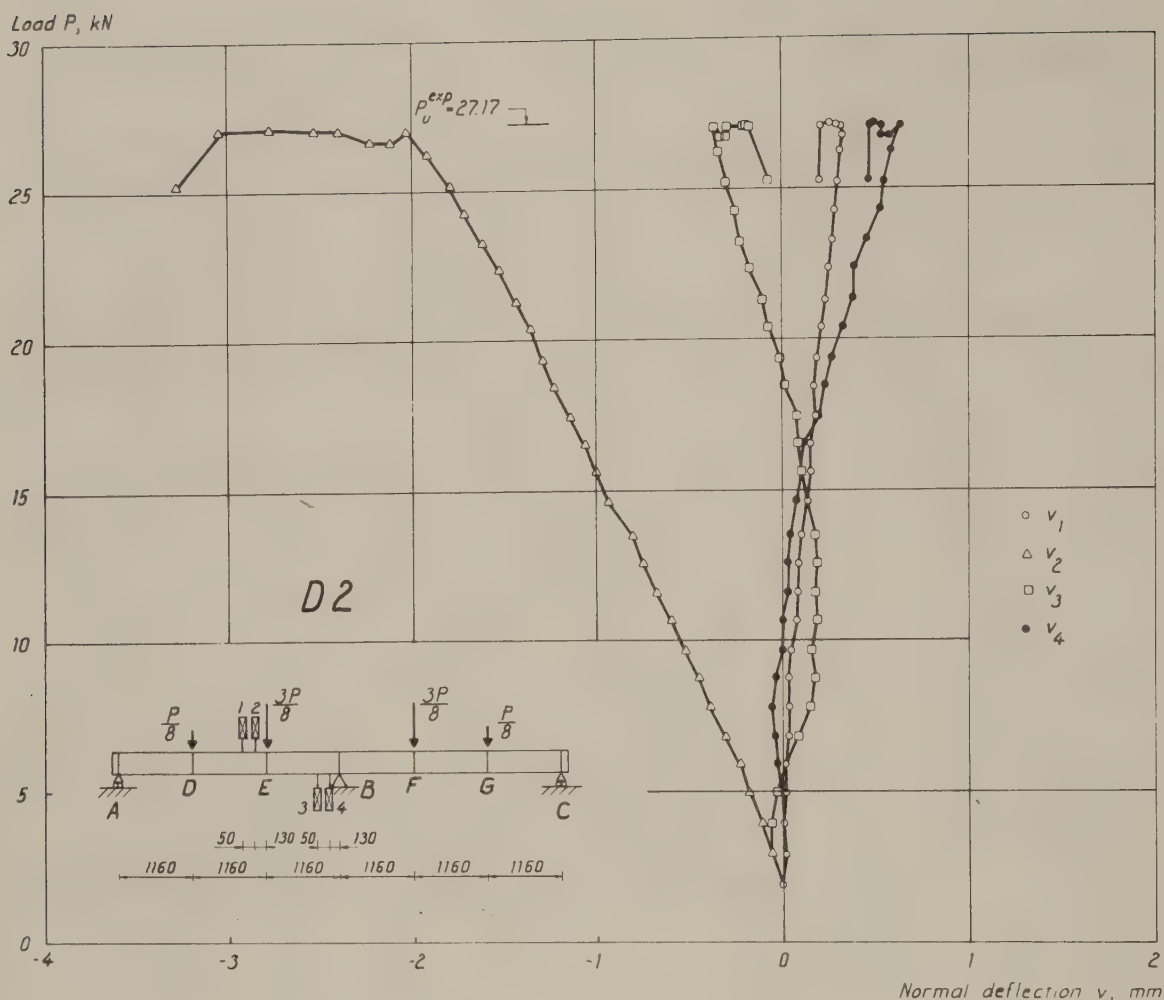


Fig. 3.4.7. Load versus normal deflection for some points on the compressed parts of the flanges. Test beam D2.

Ultimate loads.

Table 3.4.8 gives measured ultimate loads and ultimate moments for test beams D1 and D2. The ultimate moments have been calculated from applied loads and measured support reactions, and refer to those sections where the local folding has occurred (cf. Figs. 3.4.5 and 3.4.6). As the screw joints prevent failure occurring at sections of maximum moment, it has been assumed that the local folding occurs at a distance of half the height of the beam from the edges of the screw joints, that is in this case 150 mm from the centre of the screw joints.

The ultimate moments of test beam D1 have been determined from the reactions of the end supports. The ultimate moments of test

beam D2 have been obtained by putting the moment of the inner support equal to the mean value of the calculated left and right hand moments of support B.

Table 3.4.8. Measured ultimate loads and ultimate moments for the test beams D1 and D2.

Test beam	p_u^{exp} kN	M_u^{exp} kNm	Comment
D1	9.44	5.82	Local folding right of D
		7.10	" " left of B
D2	27.17	5.80	" " right of B
		5.79	" " left of E
		5.75	" " left of F

3.4.7 Discussion of results.

Deflections.

The deflections were determined theoretically according to the principles given in Section 3.1.14 implying a reduction of bending rigidity in the post-buckling range. The calculations were based on the assumption that the effective width was given according to Winter and that the area of the webs were reduced. The influence of vertical support displacements was included. The elasticities of the supports, obtained during the tests, were given the values shown in Table 3.4.9. The influence of the shear forces and the extension of the screw joints were neglected. The calculation of deflections was performed on a computer. The problem was solved iteratively in the following way: All external loads are given. We assume a value of the vertical support reaction at B, V_B . Now the end support reactions can be calculated. The distribution of bending moments and consequently the moments of inertia will then be known. By expressing the equation of the elastic line in finite differences we obtain a system of equations where the deflections are unknown. A solution by the Gauss elimination method will give

Table 3.4.9. The elasticity of the supports of test beams D1 and D2.

Test beam	k_A kN/mm	k_B kN/mm	k_C kN/mm
D1	23	20	15
D2	15	20	15

a value of the vertical deflection at B, v_B . If v_B is equal to V_B/k_B the assumed value of V_B was the right one, otherwise we have to assume a new value of R_B (greater if v_B was too great and vice versa) and continue the calculations.

The results of the calculations are shown in Figs. 3.4.5 and 3.4.6. Results from simple linear theory are also given in the figures. The load-deflection curves in Figs. 3.4.5 and 3.4.6 are of somewhat different characters. For test beam D1 a considerable decrease in strength is obtained after the first local failure has taken place. For test beam D2, on the other hand, the load is broadly maintained at the same level after the first failure has occurred until the second failure has taken place. From Sections 3.2 and 3.3 we know that there is marked decrease in the ability to transfer a bending moment in a section where a local failure has occurred. The different behaviours of the beams D1 and D2 can be explained as follows: When the first failure has taken place in test beam D1, the ability to resist a bending moment is decreased at section D. Instead the load has to be taken up by cantilever action from the left span. However, this latter effect will require rather great additional deflections and will not increase the load-carrying capacity as fast as the strength is reduced by the local failure at section D. For test beam D2 the ability to transfer a bending moment at section B will be reduced after the first local failure has occurred. The load will instead be taken up by increasing span moments. In this case the load-carrying capacity of the beam will be nearly constant as the influence of the first failure will be neutralized by increasing span moments.

Ultimate loads.

In Table 3.4.10 calculated ultimate loads and ultimate moments are given. The same computer programme that was described under the heading of deflections was used. As in Section 3.4.6, the failures were assumed to occur at a certain distance from the screw joints.

Table 3.4.10. Calculated ultimate loads and ultimate bending moments for the test beams D1 and D2.

Test beam	P_u^{th}	M_u^{th}
	kN	kNm
D1	9.06	5.61
D2	24.5	5.63

The agreement between measured (Table 3.4.8) and calculated ultimate loads is rather good. For the measured ultimate moments one value deviates from the others, namely the one left of support B for test beam D1. This probably depends on the poor accuracy of the load cells for tension forces. Support A was the only one subjected to tension.

Bending moment ratios.

In Fig. 3.4.8 the measured ratio of moment at point load to moment at support is shown as a function of deflection at point load. The figure also includes two theoretical curves namely one calculated according to linear theory and one by the non-linear theory described earlier in this section. The results obtained from the non-linear theory are in better agreement with the measured curve than those from linear theory. The tendency of the theoretical curve to reach a constant level before failure is not confirmed by the test. This may be an effect of a plastification of sections when they are almost at failure.

Fig. 3.4.9 shows some bending moment ratios of test beam D2. The moment at support B, M_B , is calculated from the loads and support reaction to the left of B when compared with moments left of B

and vice versa. In Fig. 3.4.9 the linear theory has resulted in a non-linear curve. This depends on the fact that the load of the loading devices is included in the determination of the bending moments. The same conclusions as for test beam D1 can be drawn.

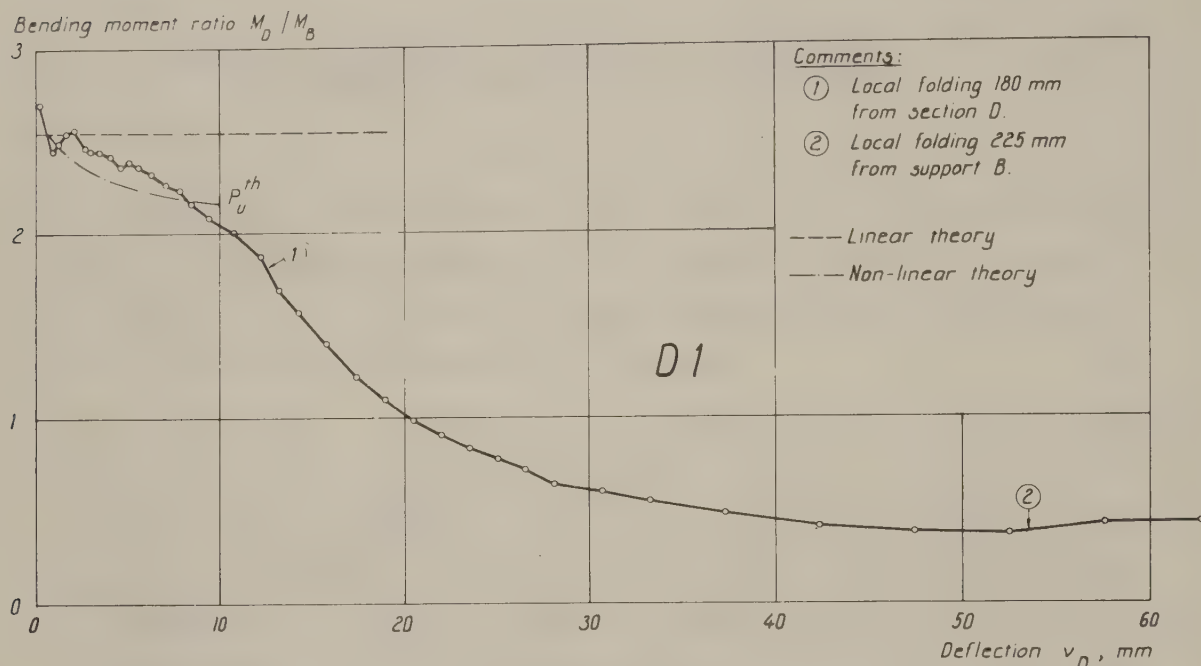
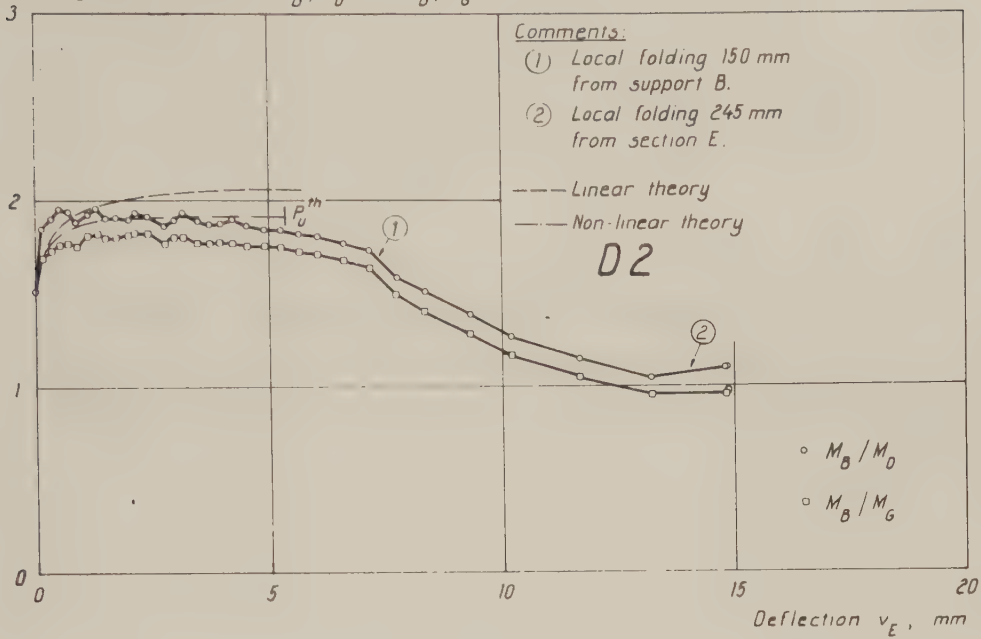


Fig. 3.4.8. Bending moment ratio versus deflection for test beam D1.

From Sections 3.1 and 3.2 we know that the bending strength of a box beam is markedly reduced when local failure occurs. As was seen from the Figs. 3.4.5 and 3.4.6, the first local failure in practice will mean, also for a statically undetermined beam, that the maximum load has been attained. Consequently the redistribution of bending moment that can be read from the Figs. 3.4.8 and 3.4.9 is in practice only interesting as long as the first local failure has not taken place.

Observe that the actual bending moments in this case have been calculated at the centre of the screw joints instead of at a distance of 150 mm from them.

a,

Bending moment ratios M_B/M_D and M_B/M_G 

b,

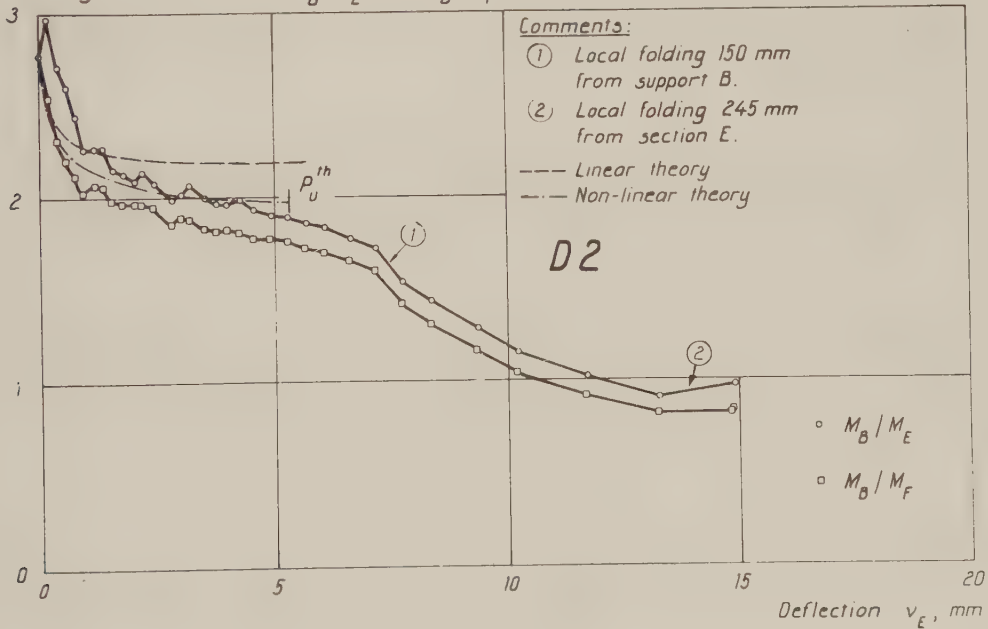
Bending moment ratios M_B/M_E and M_B/M_F 

Fig. 3.4.9 a-b. Bending moment ratios versus deflection for test beam D2.

4 TORSION OF BOX BEAMS

4.1 Theory

4.1.1 Warp stresses in box beams subjected to torsion.

General.

The purpose of this section is to analyse the magnitude of longitudinal warp stresses and briefly study the suitability of different theories to describe the torsion of elastic box beams. For a more detailed study dealing with torsion of box beams, Johansson (1976) is recommended.

In connection with the experiments performed, two different loading cases have been found interesting to analyse, namely

- A box beam subjected to a point torque at one end. Completely restrained warping at the ends.
- A box beam subjected to a point torque at one end. Free warping at the ends.

The clamping devices used in the experiments have mainly led to the beams being completely restrained against warping.

Three different theories have been used to analyse the behaviour of a box beam, namely

- a theory based on the assumption of a deformable cross-sectional shape
- a theory based on the assumption of a rigid cross-sectional shape
- a theory based on the assumption of hinged connected plate elements.

Deformable cross-sectional shape.

The method is based on a general theory presented by Vlasov (1961). The following basic assumptions are made

- For each of the narrow rectangular plates forming the thin-walled beam the hypothesis that plane sections remain plane is applied separately.
- The shear displacements are calculated as a mean value for each plate element.
- The changes in length in the cross-sectional direction of the plates are neglected.

The geometry of the box beam and the choice of coordinate axes are given in Fig. 4.1.1. In studying box beams subjected to torsional moment three generalized displacements are of interest, namely the angle of twist $\theta(z)$, the generalized contour deformation $\kappa(z)$ and the generalized warping $u(z)$. The generalized displacements are defined according to Fig. 4.1.2 and are merely functions of the longitudinal coordinate z .

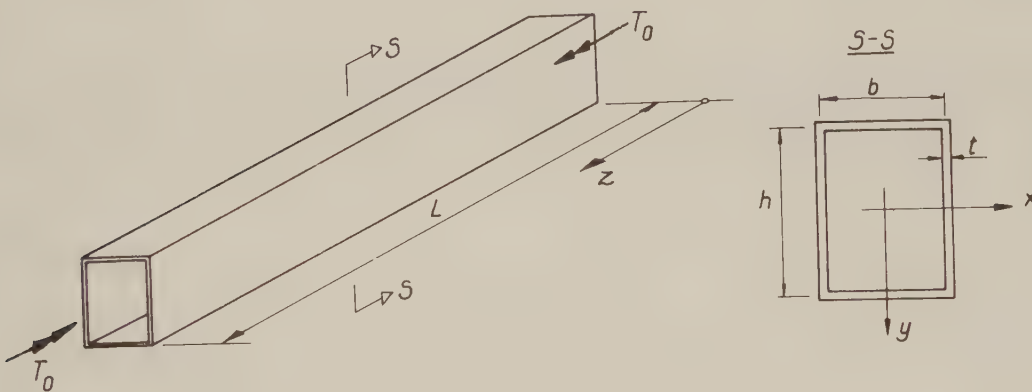


Fig. 4.1.1. Box beam subjected to torsion. Definition of geometry and coordinate axes.

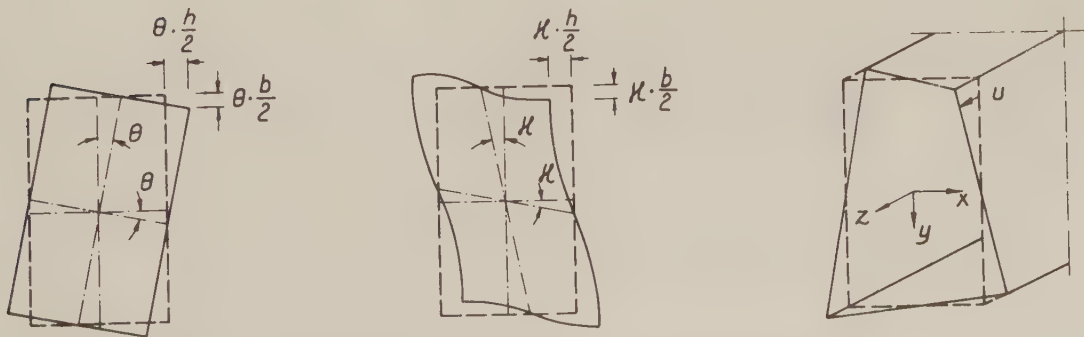


Fig. 4.1.2. Definition of generalized displacements.

We also need to define three generalized forces namely the torsional moment T , the longitudinal bimoment B , and the transverse bimoment Q . The torsional moment T is defined as being positive when the shear forces V_b and V_h have directions according to Fig. 4.1.3a and we obtain

$$T = V_b \cdot h + V_h \cdot b \quad (4.1.1)$$

The longitudinal bimoment B is defined as a pair of equal but opposite bending moments acting in two parallel planes. The numerical value of the longitudinal bimoment is given by the product of the distance between these parallel planes and the moment in one of them. In our sign convention a bimoment will be assumed positive when the directions agree with Fig. 4.1.3b. Thus the longitudinal bimoment can be expressed as

$$B = M_b \cdot h + M_h \cdot b \quad (4.1.2)$$

The transverse bimoment Q is defined as a group of shear forces which are in equilibrium. By using the directions of the shear forces given in Fig. 4.1.3a we define Q as

$$Q = -V_b \cdot h + V_h \cdot b \quad (4.1.3)$$

The transverse bimoment Q corresponds to the deformation of the contour.

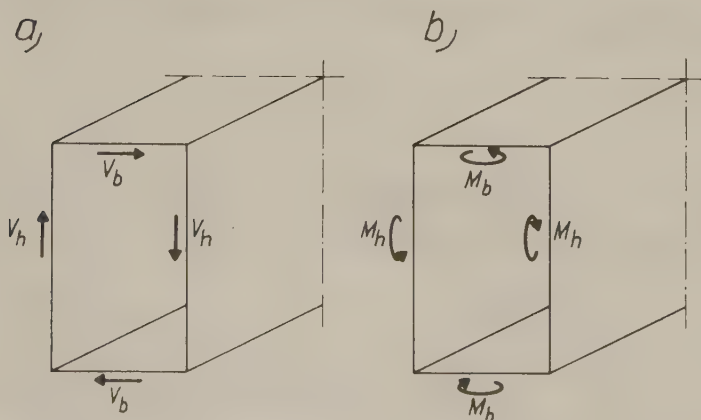


Fig. 4.1.3. a) Box beam section subjected to shear forces.
b) Box beam section subjected to bending moments (bimoment).

First we regard a box beam subjected to a point torque T_0 at one end which is free to rotate. Both ends are free from longitudinal restraints but completely restrained with respect to cross-sectional shape. The method proposed by Vlasov is used introducing initial parameters. This means that a number of generalized displacements u_0 , θ_0 and κ_0 and generalized forces B_0 , T_0 and Q_0 at the initial end $z = 0$ are taken as initial parameters. From the boundary conditions these initial parameters are determined and the generalized displacements and forces can then be expressed as functions of the initial parameters. In the actual case the boundary conditions will be:

$$\kappa(0) = 0$$

$$B(0) = 0$$

$$\theta(0) = 0$$

$$\kappa(L) = 0$$

$$B(L) = 0$$

$$T(L) = T_0$$

Applying these conditions in the Vlasov solution (table 29 page 240-241) we obtain

$$\kappa(z) = 0$$

$$u(z) = \frac{h - b}{2G b^2 h^2 t} T_0$$

$$\theta(z) = \frac{b + h}{2G b^2 h^2 t} T_0 z$$

$$T(z) = T_0$$

$$B(z) = 0$$

$$Q(z) = 0$$

This solution agrees with the solution for pure Saint-Venant torsion. The angle of twist at $z = L$ thus can be expressed as

$$\theta_L = \frac{T_0 L}{G I_T} \quad (4.1.4)$$

where the quantity

$$I_T = \frac{2b^2 h^2 t}{b + h} \quad (4.1.5)$$

is the torsional constant.

We now regard the loading case of a box beam subjected to a point torque T_0 at one end which is free to rotate. Both ends are completely restrained against warping and cross-sectional distortion. The boundary conditions will have the form

$$\kappa(0) = 0$$

$$u(0) = 0$$

$$\theta(0) = 0$$

$$\kappa(L) = 0$$

$$u(L) = 0$$

$$T(L) = T_0$$

The problem has been solved by means of a computer programme where the input parameters have been the height, the width, the thickness and the length of the beam. In connection with the performed experiments the following geometrical values have been chosen as input parameters

$$b = 133 \text{ mm}$$

$$h = 200 \text{ mm}$$

$$t = 1 \text{ mm}$$

In Fig. 4.1.4 the ratio of the warp stress σ_w to the Saint Venant shear stress τ_s is given along the beam for some span lengths L . For the actual beam section and not too long span widths ($L \leq 2500$ mm) the results are in very good agreement with the results obtained from the theory based on the assumption of hinged connected plate elements (see below).

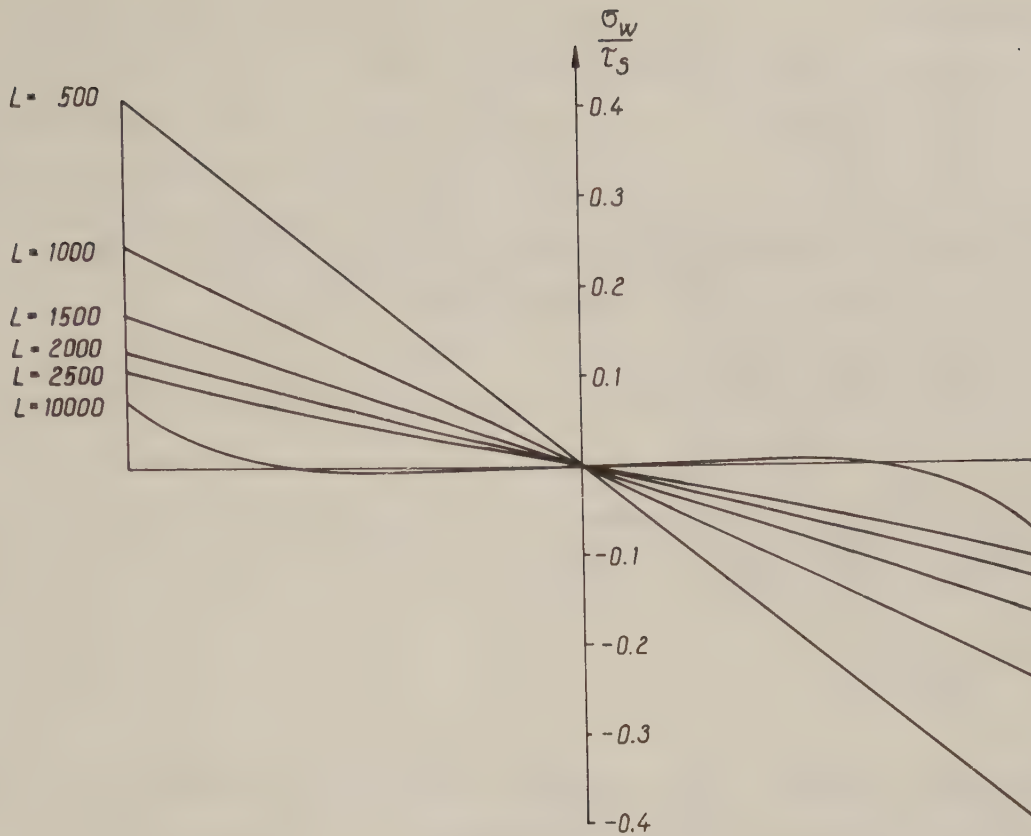


Fig. 4.1.4 Ratio of warp stress σ_w to Saint Venant shear stress τ_s along box beam subjected to constant torsional moment T_s and with both ends restrained against warping.

Rigid cross-sectional shape.

The method of solution given in Vlasov (1961) is used (page 249-251). In the case of a box beam subjected to a constant torsional moment T_0 where both ends are completely restrained against warping, the boundary conditions will have the form

$$\begin{aligned}\theta(0) &= 0 \\ u(0) &= 0 \\ u(L) &= 0 \\ T(L) &= T_0\end{aligned}$$

After some calculation, we obtain the expression for the bimoment B as

$$B(z) = -\frac{b-h}{b+h} \frac{L}{k} \left(\cosh \frac{k}{L} z \frac{1 - \cosh k}{\sinh k} + \sinh \frac{k}{L} z \right) T_0 \quad (4.1.6)$$

where

$$k = \sqrt{\frac{48 G}{E} \cdot \frac{L}{b + h}} \quad (4.1.7)$$

The bimoment B will adopt its maximum absolute values in the sections $z = 0$ and $z = L$. For $z = 0$ we obtain

$$B_0 = - \frac{b - h}{b + h} \frac{L}{k} \frac{1 - \cosh k}{\sinh k} T_0 \quad (4.1.8)$$

The relation between the bimoment B and the warp stress σ_w (in the right-hand lower corner) is derived as:

$$\sigma_w = - \frac{6B}{hbt(h + b)} \quad (4.1.9)$$

Inserting B_0 from Eq. (4.1.8) into Eq. (4.1.9) and assuming $E = 2.6 G$ and the span width L to be long we obtain

$$\sigma_w = 2.79 \frac{h - b}{h + b} \frac{T_0}{2bht} \quad (4.1.10)$$

The last quotient in Eq. (4.1.10) denotes the Saint Venant shear stress τ_s and consequently we finally obtain

$$\frac{\sigma_w}{\tau_s} = 2.79 \frac{h - b}{h + b} \quad (4.1.11)$$

For finite L -values, the ratio of σ_w to τ_s will always adopt larger absolute values than those given by Eq. (4.1.11). The same expression is given in Johansson (1976).

Hinged connected plate elements.

The theory is based on the assumption that the different plate elements are connected to each other by longitudinal hinges at the corners of the box beam. The shear deformations of the plate elements are calculated by means of the mean shear stresses.

We study the loading case of a box beam subjected to a constant torsional moment T_0 . Both ends are completely restrained against warping and cross-sectional distortion. Fig. 4.1.5 a shows the bottom flange of the box beam given in Fig. 4.1.1. The flange is regarded from above. Fig. 4.1.5 b shows the web on the right-hand side of the same box beam. The web is regarded from the left. The deflections are caused on the one hand by the pure Saint Venant shear stresses and on the other hand by the warping moments (bi-moments). The deflection of the flange caused by the Saint Venant shear stresses is

$$\delta_{b1} = \frac{\tau_s}{G} L \quad (4.1.12)$$

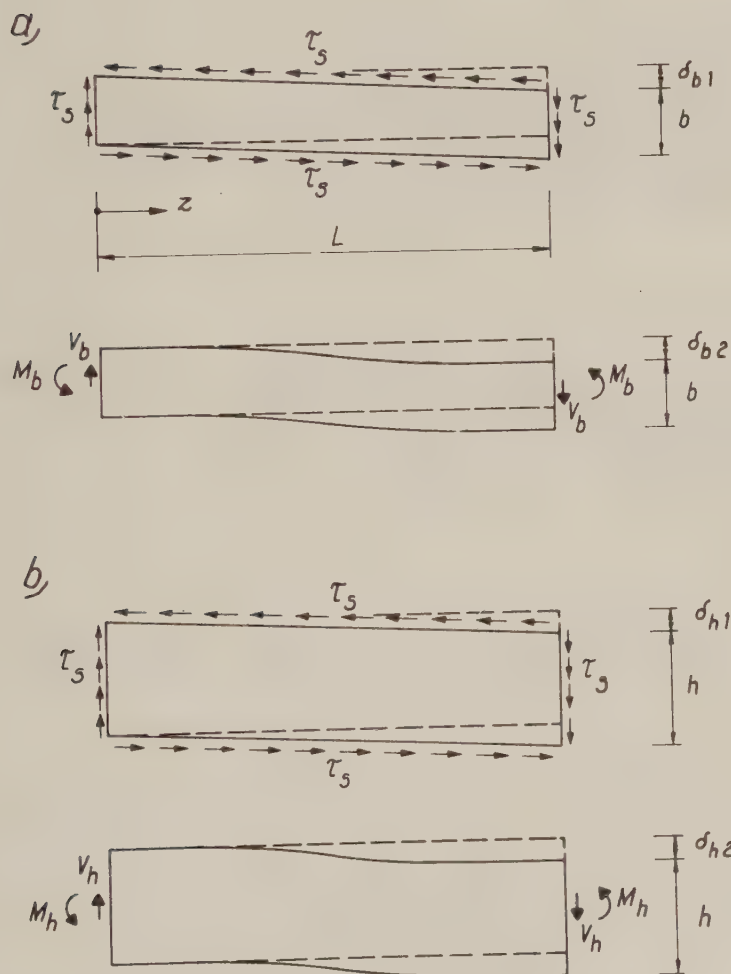


Fig. 4.1.5 Box beam subjected to constant torsional moment and with both ends restrained against warping.
a) Forces acting on the bottom flange.
b) Forces acting on the right-hand web.

The deflection of the flange caused by the warping moment and the related shear forces can be given as

$$\delta_{b2} = \frac{M_b L^2}{6EI_b} + \frac{V_b L}{bt G} \quad (4.1.13)$$

where

$$I_b = \frac{tb^3}{12}$$

An equation of equilibrium for the bottom flange gives

$$2 M_b = V_b L \quad (4.1.14)$$

Under the assumption of a linear longitudinal stress distribution across every plate element, where the warp stress in section $z = L$ is equal to σ_w in the left-hand corner of the bottom flange (Fig. 4.1.1) and $-\sigma_w$ in the right-hand corner, we obtain

$$M_b = \sigma_w \frac{t b^2}{6} \quad (4.1.15)$$

The deflection of the flange in section $z = L$ in the case of rigidly fixed ends can be expressed as

$$\theta_L \frac{h}{2} = \delta_{b1} + \delta_{b2} \quad (4.1.16)$$

Insertion of Eqs. (4.1.12), (4.1.13), (4.1.14) and (4.1.15) into Eq. (4.1.16) gives

$$\theta_L \frac{h}{2} = \frac{\tau_s}{G} L + \frac{\sigma_w L^2}{3Eb} \left(1 + \frac{b^2}{L^2} \frac{E}{G}\right) \quad (4.1.17)$$

In the same way we derive for the web the equation

$$\theta_L \frac{b}{2} = \frac{\tau_s}{G} L - \frac{\sigma_w L^2}{3Eh} \left(1 + \frac{h^2}{L^2} \frac{E}{G}\right) \quad (4.1.18)$$

From the Eqs. (4.1.17) and (4.1.18) we obtain

$$\frac{\sigma_w}{\tau_s} = \frac{3E}{2G} \frac{h-b}{L} \frac{1}{1 + \frac{b^2+h^2}{L^2} \frac{E}{2G}} \quad (4.1.19)$$

If $E = 2.6 G$ and the span length is assumed to be large in relation to b and h , Eq. (4.1.19) will have the form

$$\frac{\sigma_w}{\tau_s} = 3.90 \frac{h-b}{L} \quad (4.1.20)$$

A comparison between σ_w/τ_s obtained by Vlasov's theory based on the assumption of a deformable cross-sectional shape and σ_w/τ_s obtained by the theory based on hinged connected plate elements shows that for the dimensions of the actual test beams there is a close agreement. Thus Eq. (4.1.19) will give results which deviate at the most by 2 per cent (for $L \leq 2500$ mm) from the corresponding results obtained by Vlasov's theory. This fact shows that, for the actual beam dimensions, the bending rigidity in the transverse direction of the beams can to a very good approximation be neglected when the warp stresses are calculated.

A comparison of Eq. (4.1.20) and Eq. (4.1.11) clearly shows that the theory based on a rigid cross section is not suitable for the analysis of beams with the actual dimensions. This result is also valid for considerably larger plate thicknesses (e.g. 10 times larger).

The torsional moment T_0 can be expressed as

$$T_0 = 2 b h t \tau_s + V_b h + V_h b \quad (4.1.21)$$

By insertion of Eqs. (4.1.14), (4.1.15) and (4.1.21) into Eqs. (4.1.17) and (4.1.18), the angle of twist θ_L can be determined:

$$\theta_L = \frac{T_0 L(b+h)}{2b^2 h^2 t G} \left[1 - \frac{\left(\frac{h-b}{L}\right)^2}{4 \frac{G}{E} + \left(\frac{h+b}{L}\right)^2} \right] \quad (4.1.22)$$

If $h = b$, i.e. with a square-shaped section, θ_L agrees with the case of pure Saint Venant torsion. For the previous example, the expression within square brackets in Eq. (4.1.22) will deviate at the most by 0.9 per cent (for $L \geq 500$ mm) from unity.

Discussion and conclusions.

From the different theories described in this section we establish that cross-sectional deformation occurs in spite of the fact that no external cross-sectional deforming forces are acting along the box beams. The theory based on the assumption of a rigid cross section gives magnitudes of the warp stresses which agree poorly with reality for the actual beam dimensions. Even in the case of considerably greater plate thickness (10 times), the theory based on a deformable cross-sectional shape ought to be used.

In the sub-critical range, the warp stresses of the actual test beams seem to be rather small in relation to the shear stresses, Fig. 4.1.4.

In the post-critical range, the influence of local shear buckling might be interpreted as a decrease of the shear modulus G . The bending stiffness of the plate members seems, however, to be rather unaffected by the buckling phenomena, implying that E does not decrease appreciably. Since σ_w/τ_s is proportional to E/G (see Eq. (4.1.19)), the influence of the warp stresses ought to increase in the post-critical range.

4.1.2 Determination of the ultimate torsional moment.

Höglund (1971) gives a model for the determination of the maximum shear force that an I-girder can resist. The distance between web stiffeners is assumed to be long. In the model the web is replaced with a system of intersecting tensile and compressive bars. On the basis of experimental results and calculations Höglund proposes the following expressions for the maximum average stress:

For rigid web stiffeners at the supports
(two stiffeners at each support):

$$\tau_{\max} = \frac{1.8}{\alpha+1} \tau_y \quad \text{when} \quad \alpha < 2.75 \quad (4.1.23)$$

$$\tau_{\max} = \frac{1.32}{\alpha} \tau_y \quad \text{when} \quad \alpha > 2.75 \quad (4.1.24)$$

For flexible web stiffeners at the supports
(one stiffener at each support):

$$\tau_{\max} = \frac{1}{\alpha} \tau_y \quad \text{when} \quad \alpha > 1.25 \quad (4.1.25)$$

where

$$\alpha = \sqrt{\frac{\tau_y}{\tau_{cr}}} \quad (4.1.26)$$

$$\tau_y = \frac{\sigma_y}{\sqrt{3}} \quad (4.1.27)$$

$$\tau_{cr} = [5.34 + 4.00 \left(\frac{h}{a}\right)^2] \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{h}\right)^2 \quad (4.1.28)$$

a = distance between web stiffeners

h = height of web

How well are these expressions suited to the determination of the load-carrying capacity of box beams in torsion? To obtain an answer to this question, a calculation was carried out using the dimensions of the square-shaped box beams in Sections 4.2 and 4.3. From this follows that the load-carrying capacity is overestimated by 19 to 35 % even if flexible web stiffeners are assumed. This is probably a consequence of the favourable influence of the flanges in the case of an I-beam subjected to shear. Firstly, the flanges lead to a certain degree of restraint resulting in an increase of the critical load. Secondly, the flanges of an I-beam in the post-buckling range contribute to taking up the longitudinal compressive stresses along the boundaries of the webs. These stresses are caused by the local buckles appearing in the webs. Thirdly, for an I-beam the upper and lower parts of the webs will interact with the flanges in taking up the varying tensile and compressive forces

acting in the transverse direction along the beam. The same favourable conditions cannot be found in connection with the torsion of square-shaped box beams.

As no simple theoretical method of calculating the ultimate torsional moment of a square-shaped box beam seems to be available, the following empirical expression for the maximum average stress is proposed:

$$\tau_{\max} = \sqrt[3]{\tau_y \tau_{cr}^2} = 0.833 \sqrt[3]{\sigma_y \tau_{cr}^2} \quad (4.1.29)$$

In the case of flanges and webs of different dimensions the maximum average stress is proposed as:

$$\tau_{\max} = \sqrt[3]{\tau_y \tau_{cr,f} \tau_{cr,w}} = 0.833 \sqrt[3]{\sigma_y \tau_{cr,f} \tau_{cr,w}} \quad (4.1.30)$$

where $\tau_{cr,f}$ and $\tau_{cr,w}$ are the critical loads of the flanges and the webs respectively according to Eq. (4.1.28). When using Eq. (4.1.28), the second term within square brackets can be neglected for not too short beams. This has been done for the actual test beams.

Eqs. (4.1.29) and (4.1.30) are based on experimental results given in Sections 4.2 and 4.3 with width to thickness ratios (b/t or h/t) within the interval 125-190 and b/h values between 0.68 and 1.00. As the intervals are rather narrow, the proposed expressions cannot be extended to include other b/t (or h/t) and b/h values without complementary research. Particular caution must be observed for low width-to-thickness ratios where a transition curve to the plastic range must no doubt be used.

4.2 Beam with one end restrained from twisting. Introductory experiment.

4.2.1 Purpose.

The main purpose of the introductory experiments was to determine the function of thin-walled square-shaped box beams subjected to pure torsion.

The aim of the experiments was to study the influence of buckling phenomena, to investigate different relations between load and deformation and to determine the nature of failure.

The experiments were performed as an undergraduate work by the now qualified civil engineer Bo Sjödaahl with the author as supervisor.

4.2.2 Scope.

The experimental studies comprised two square-shaped test beams E1 and E2. The test beams were loaded by a torque at one end and were restrained from twisting at the other. FIG 4.2.1 shows distribution of load and torsional moment. Nominal dimensions of the test beams E1 and E2 are given in Table 4.2.1.

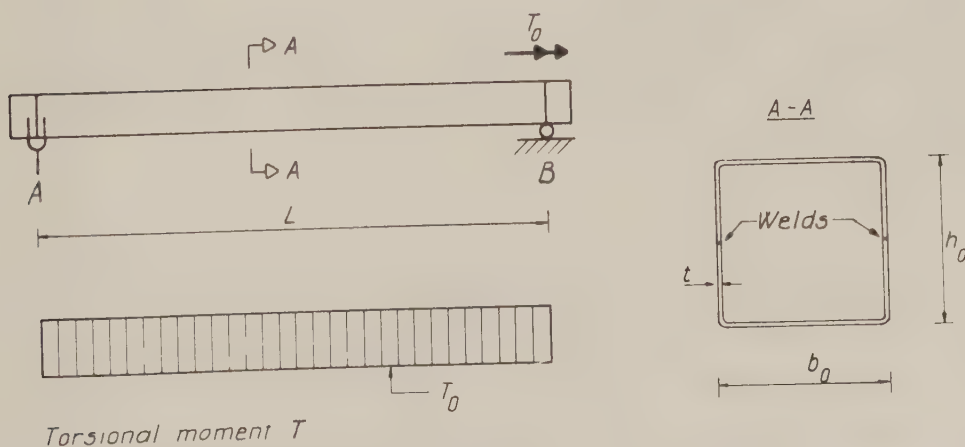


Fig. 4.2.1. Distribution of load and torsional moment. Notations for cross section.

Table 4.2.1. Nominal dimensions of the test beams E1 and E2.

Test beam	L mm	b_o mm	h_o mm	t mm
E1	1155	200	200	1.5
E2	2561	200	200	1.5

4.2.3 Loading and support arrangements.

To reduce to a minimum the work of making a test rig, a rig for testing concrete beams in pure torsion was used. Only new clamp devices for the test beams had to be made. The principal test arrangement is shown in Fig. 4.2.2. The test rig used was designed to satisfy, as closely as possible, the following boundary conditions for the box beams, namely that

- 1) one end was restrained from twisting,
- 2) the box beams were simply supported with regard to vertical bending.

The torque was created by a jack supported on the floor and was transferred to the test beams by two wires and a torque frame.

The left-hand support of Fig. 4.2.2 consisted of an anti-torque frame resting on a knife-edge bearing. The right-hand support was made as a longitudinal axis through a spherical ball-bearing provided with roller bearings.

The clamp devices for the test beams consisted of steel boxes provided with stiffeners. These clamp boxes were attached to the test beams by two screws on each side. Inside the test beams, resisting pieces of flat bar with dimensions 12 x 70 mm were used. The distance between the screws was 125 mm. In practice the screw joints worked as high tensile bolt connections.

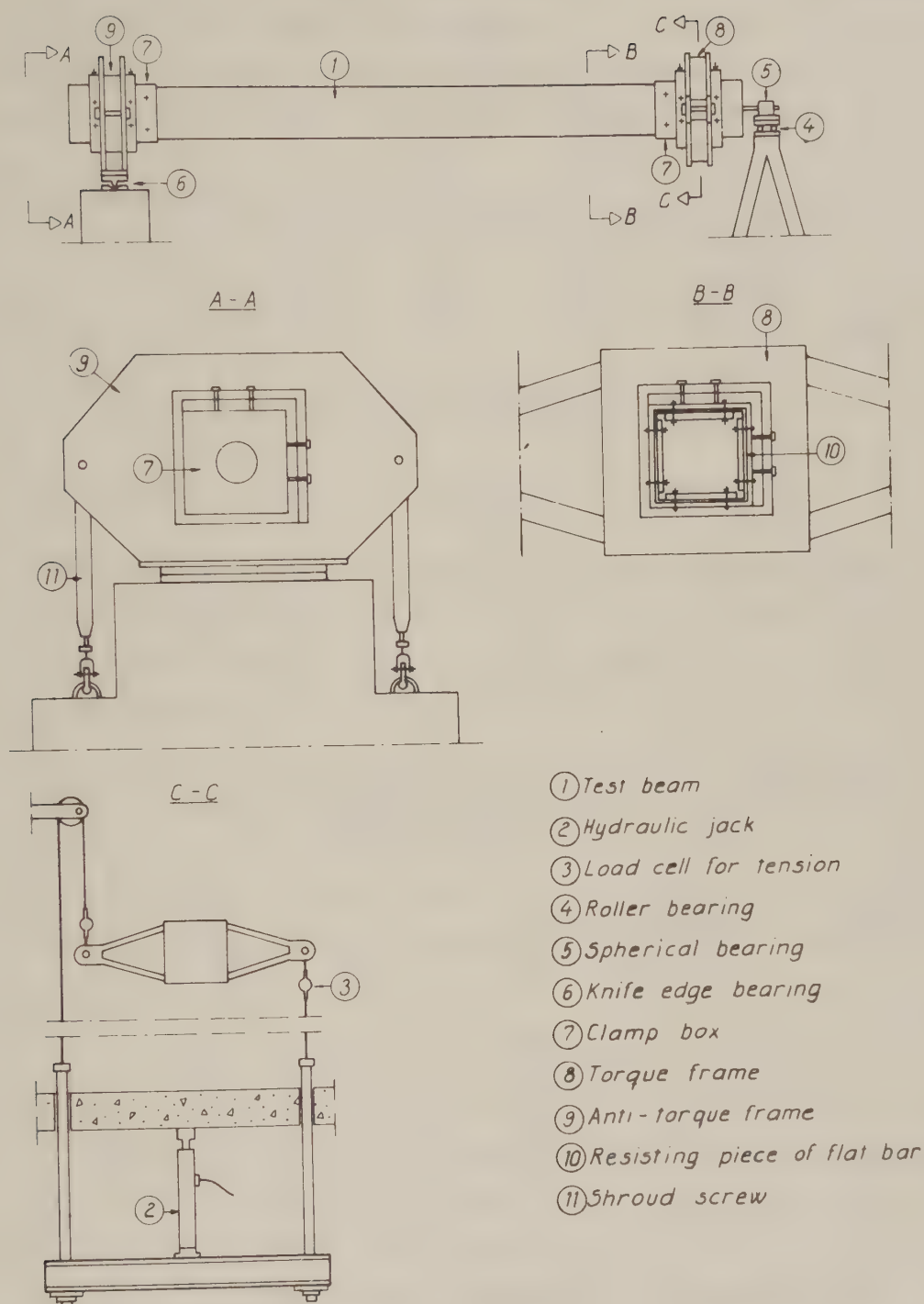


Fig. 4.2.2. Principal arrangement of loading and support devices for test beams E1 and E2.

4.2.4 Dimensions. Material properties.

The process of manufacture of the test beams E1 and E2 was identical to that described in Section 3.2.4, with the exception that all welds were ground outside the test beams to reduce their stiffening effect.

The material consisted of cold-rolled structural steel COR-TEN A. The general properties of the material are described in Section 3.3.4. Results of a chemical analysis carried out by the manufacturer of the cold-rolled sheet are given in Table 4.2.2.

Table 4.2.2. Results of a chemical analysis carried out by the manufacturer of the cold-rolled sheet.

Test beam	C	Si	Mn	P	S	N	Cr	Cu	Ni
	%	%	%	%	%	%	%	%	%
E1, E2	0.11	0.34	0.30	0.080	0.021	0.010	0.61	0.43	0.02

Tensile test pieces were cut out from test beams E1 and E2. The results of these tensile tests were in good agreement with results obtained from tensile tests carried out by the manufacturer of the cold-rolled sheet. The latter results are shown in Table 4.2.3.

Table 4.2.3. Results of tensile tests carried out by the manufacturer of the cold-rolled sheet.

Test beam	Rolling direction	σ_y N/mm ²	σ_u N/mm ²	δ_5 %
E1, E2	//	309	486	37
	⊥	305	487	40

Measured lengths and cross-sectional data are shown in Table 4.2.4. The lengths of the test beams refer to the distances between the clamp boxes. Cross-sectional constants and critical stresses, defined according to Appendix A, are given in Table 4.2.5.

Table 4.2.4. Measured dimensions of test beams E1 and E2.

Test beam	L	b_o	h_o	t	R
	mm	mm	mm	mm	mm
E1	1155	201	201.5	1.55	3
E2	2561	202	202	1.55	3

Table 4.2.5. Cross-sectional constants and critical stresses for test beams E1 and E2.

Test beam	b/t	h/t	$I_T \cdot 10^{-4}$ mm ⁴	$\tau_{cr,f}$ N/mm ²	$\tau_{cr,w}$ N/mm ²
E1	128.7	129.0	1234	60.0	59.7
E2	129.3	129.3	1248	59.4	59.4

4.2.5 Measuring devices and measurement procedure.

The measurements comprised determination of applied torque, measurement of angles of twist, measurement of strains and the recording of shape and size of normal deflections. All measurements were performed manually.

Load measurement.

Due to "creep" at high load levels, the loads were applied so as to give a constant angle of twist during each load step. The two load cells used were of the BOFORS KRG-4 type and the loads were recorded by means of two BOFORS BKI-1 load cell indicators. The loads were recorded when they achieved their maximum value during the actual load step.

On account of frictional forces in the loading device, the two point loads were not of the same size. Thus some bending was introduced to the test beams. The influence can, however, be regarded as negligible.

When test beam E1 was tested, the two point loads were recorded as continuous functions of time.

Angle of twist measurement.

To measure the angle of twist, four dial gauges graduated in units of 0.01 mm were used. Positions and numbering of dial gauges are shown in Fig. 4.2.3. The dial gauges measured against pieces of glass which were fastened to the clamp boxes. If the distance between two dial gauges on the same beam section is known, the angle

of twist can easily be calculated.

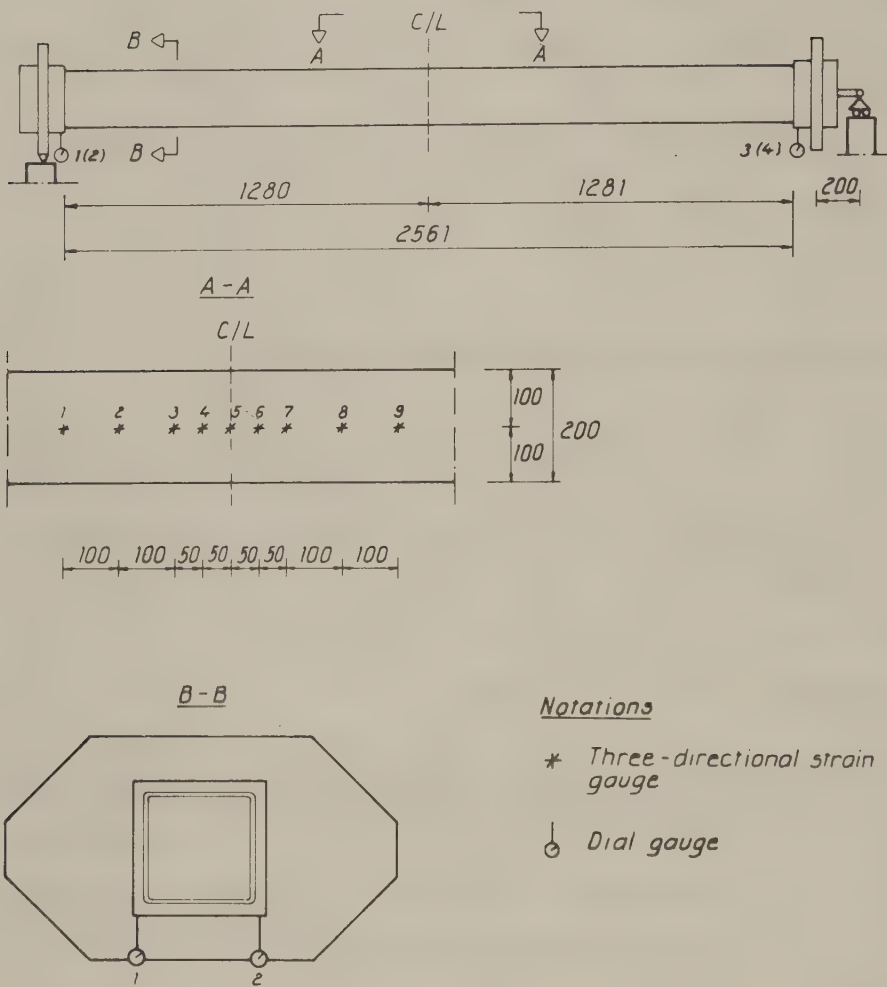


Fig. 4.2.3. Positions and numbering of dial gauges and strain gauges for test beam E2.

Measurement of normal deflections.

The buckles were recorded on the upper flange and on one of the webs in the same way as was described for the manual measurements in Section 3.3.7. The only difference was that the guides were independent of the motion of the test beams.

Strain measurement.

The strains were measured by means of three-directional foil strain gauges with a measuring length of 10 mm. To obtain the mean strains of the sheet, the strain gauges were placed opposite each other at the inner and outer surfaces. The strains were recorded by means of

a Peekel B 105 strain gauge indicator. For test beam E2, positions and numbering of the strain gauges are shown in Fig. 4.2.3.

4.2.6 Results.

Angle of twist.

The relationship between the applied torque and the angle of twist of test beam E2 is shown in Fig. 4.2.4.

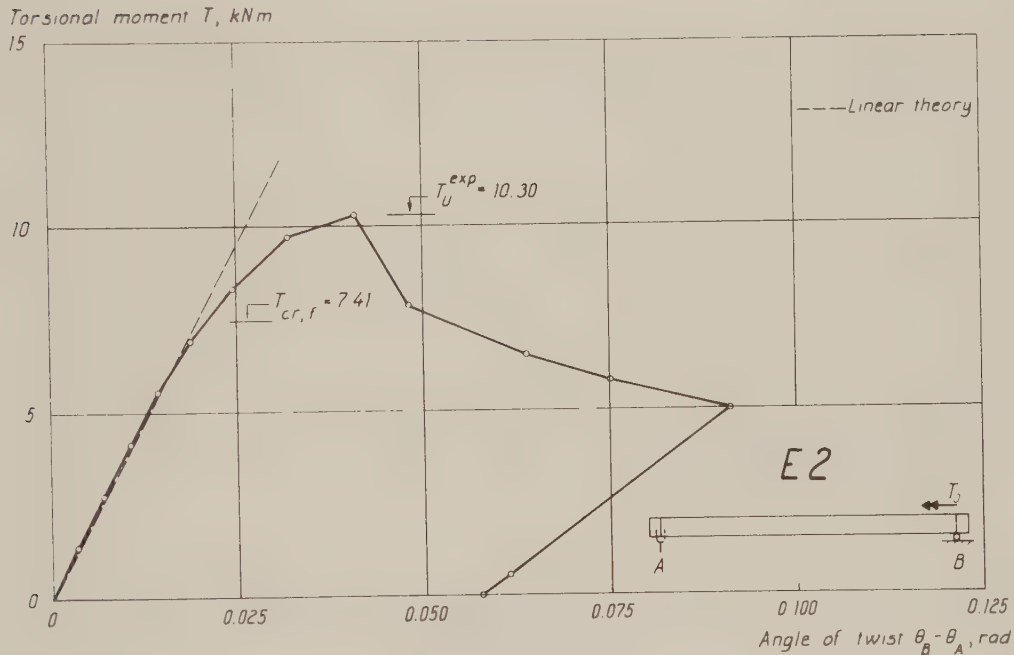


Fig. 4.2.4. Relationship between torsional moment and angle of twist for test beam E2.

Normal deflections.

Deflection surfaces of the upper flange of test beam E1 at three different load levels, namely at zero load, somewhat below the theoretical critical load and immediately before failure are shown in Fig. 4.2.5.

For test beam E2 no deflection surfaces were measured but only the normal deflections of a longitudinal line 20 mm from the centre of the upper flange, see Fig. 4.2.6a. The normal deflections of four points on this line are shown as a function of the applied torque in Fig. 4.2.7.

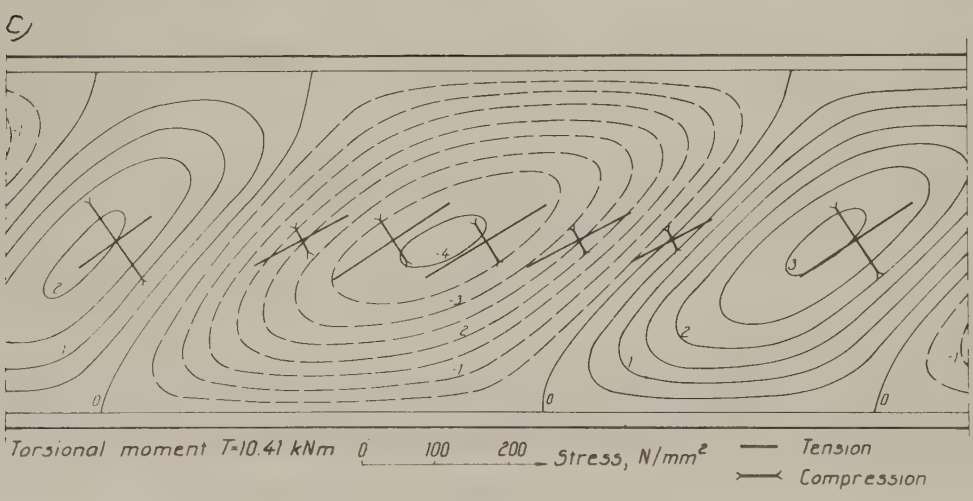
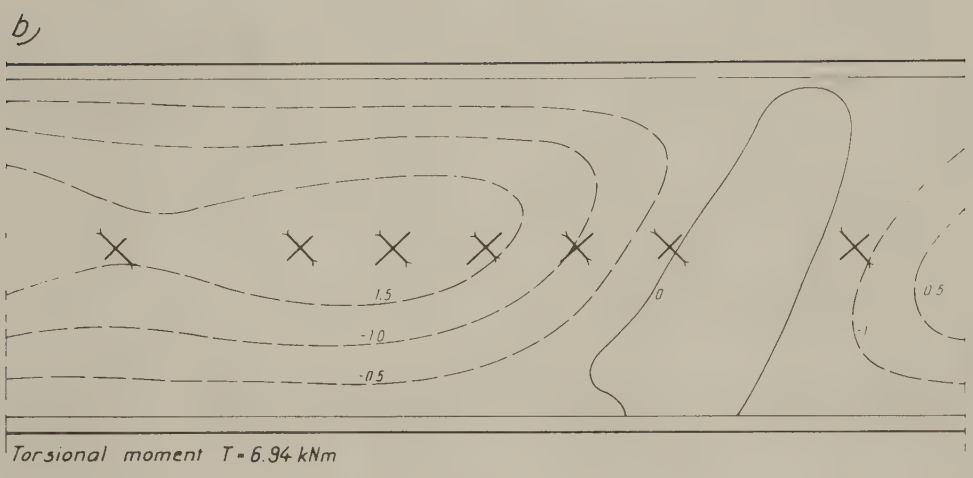
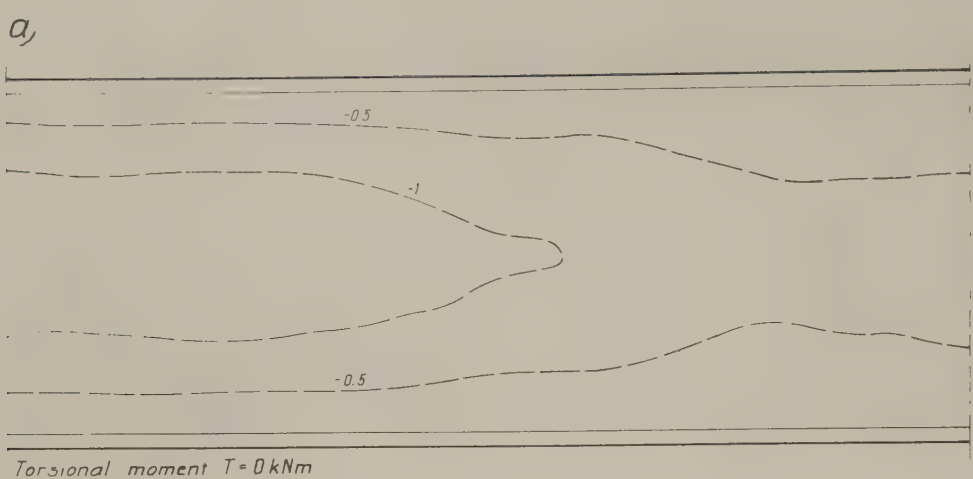


Fig. 4.2.5 a-c. Normal deflections and principal stresses for test beam E2.

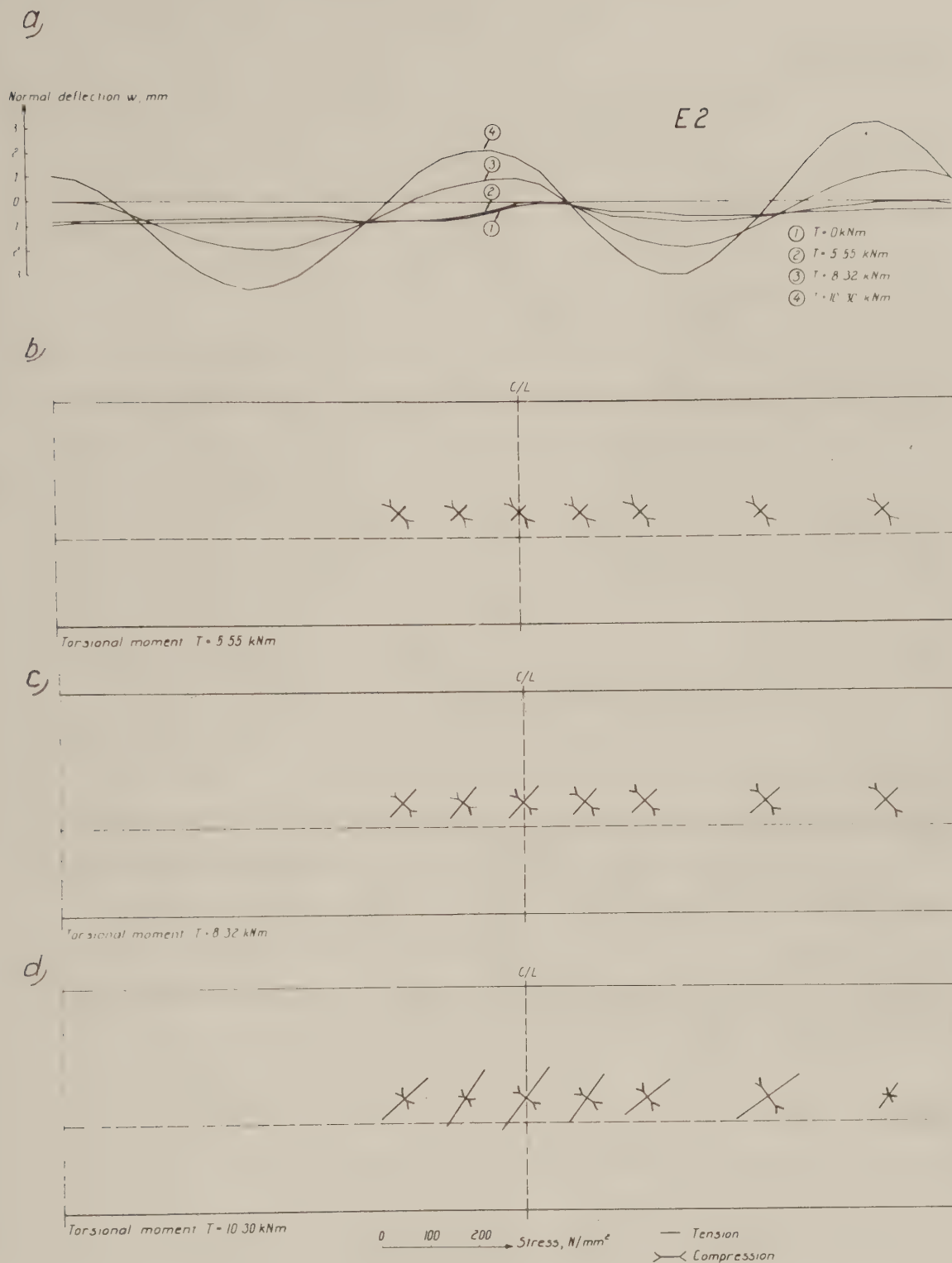


Fig. 4.2.6 a. Normal deflections for a line 20 mm from the centre of the upper flange of test beam E2.

b-d. Principal stresses in the same flange.

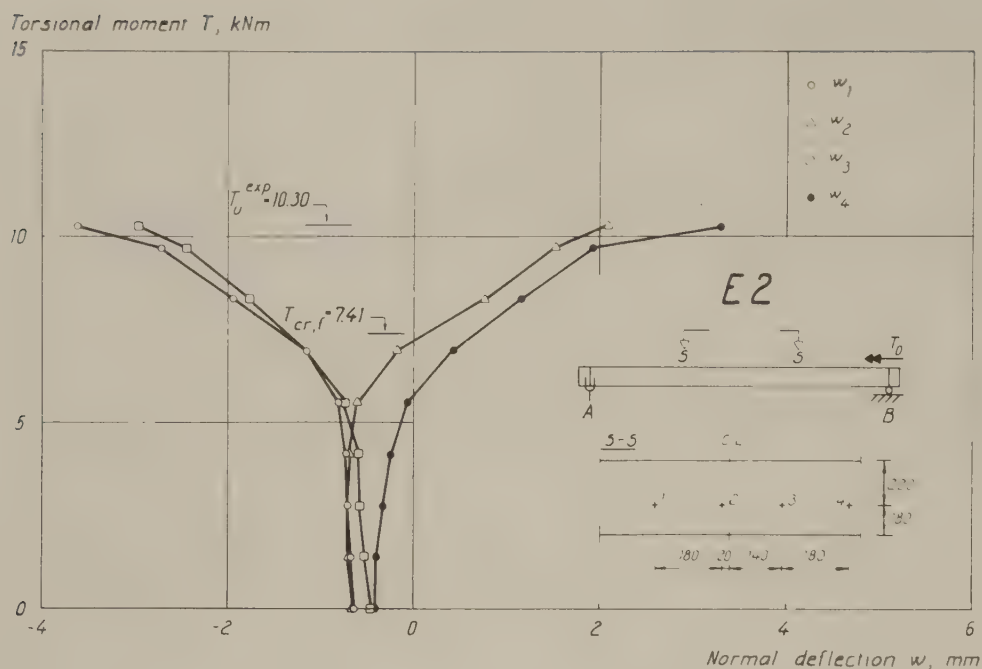


Fig. 4.2.7. Torsional moment versus normal deflection for some points on the upper flange of test beam E2.

Strains.

Principal stresses and principal stress directions for test beam E1 are also given in Fig. 4.2.5. The corresponding quantities for test beam E2 are shown in Fig. 4.2.6 b-d. Notice that the normal deflections of test beam E2 are not given in the same longitudinal section as the strain gauges. The position of the deflection curve is marked by longitudinal dotted lines in Fig. 4.2.6.

Ultimate loads.

Measured ultimate torsional moments are given in Table 4.2.6.

Table 4.2.6. Measured ultimate torsional moments of test beams E1 and E2.

Test beam	T_u^{exp} kNm
E1	10.75
E2	10.30

Character of failure.

Failure occurred as a local folding of one of the corners. The failure developed slowly but there was a marked drop in the load-carrying capacity just after the maximum load had been reached.

4.2.7 Discussion of results.

Angle of twist.

A theoretical curve, describing the relationship between torsional moment and angle of twist, calculated according to the theory of pure Saint Venant torsion (Eq. (4.1.4)) is shown in Fig. 4.2.4. The length L in this case has been increased by 30 mm owing to the fact that the screw joints cannot work fully at the edges of the clamp boxes. The theoretical critical torsional moment has also been indicated in the figure.

Normal deflections.

In the same way as for buckling due to normal stresses, the initial normal deflections have a great influence on the size and position of the subsequent normal deflections (cf. Fig. 4.2.5). Further it can be seen how the buckles are diagonally directed before failure, which is typical for shear buckling.

From Figs. 4.2.6 and 4.2.7 it is evident that the normal deflections are almost constant at low loads and that they increase considerably just below the critical load.

Strains.

In the sub-critical range, the shear stresses τ of a perfect plate will be uniformly distributed over the whole section. This is equivalent to a stress condition where the principal stresses are equal to τ and $-\tau$ and the principal stress directions are at an angle of 45° to the longitudinal corners of the test beams. In the post-critical range, the principal tensile stresses at the centre of the plate will adopt a more longitudinal direction. Further, the principal compressive stresses will remain at a fairly constant value while the principal tensile stresses will increase considerably.

This behaviour is confirmed by the principal stresses and principal stress directions shown in Fig. 4.2.5 and Fig. 4.2.6. In the calculations, an ideally elastic stress-strain diagram has been assumed. Notice that the measured stresses in many cases exceed the yield stress. The results must thus be interpreted with caution.

Ultimate loads.

A theoretical determination of the ultimate torsional moments using Eq. (4.1.30) has been carried out. The results of the calculations are given in Table 4.2.7 where a comparison with measured values is also made. If the influence of span length according to Eq. (4.1.28) is included, the theoretical ultimate torsional moments will increase by 1.5 % for test beam E1 and by 0.3 % for E2.

Table 4.2.7. Theoretical ultimate torsional moments. Ratio of measured to theoretical ultimate torsional moments.

Test beam	T_u^{th} kNm	$\frac{T_u^{exp}}{T_u^{th}}$
E1	10.66	1.01
E2	10.68	0.96

4.3 Beam with both ends restrained from twisting.

4.3.1 Purpose.

In Section 4.2, test beams having one end restrained from twisting were investigated. A logical extension of the investigation was to study beams with two ends restrained from twisting, to find out whether torsional moment redistribution can occur when a beam is subjected to a torque applied at different positions along the beam.

4.3.2 Scope.

The investigation included four square shaped box beams, F1-F4, and two rectangular box beams, F5-F6, subjected to torsion. Fig. 4.3.1 shows the distribution of load and the calculated distribution of torsional moment according to pure Saint Venant torsion theory. Table 4.3.1 shows load positions and nominal dimensions of the test beams F1-F6.

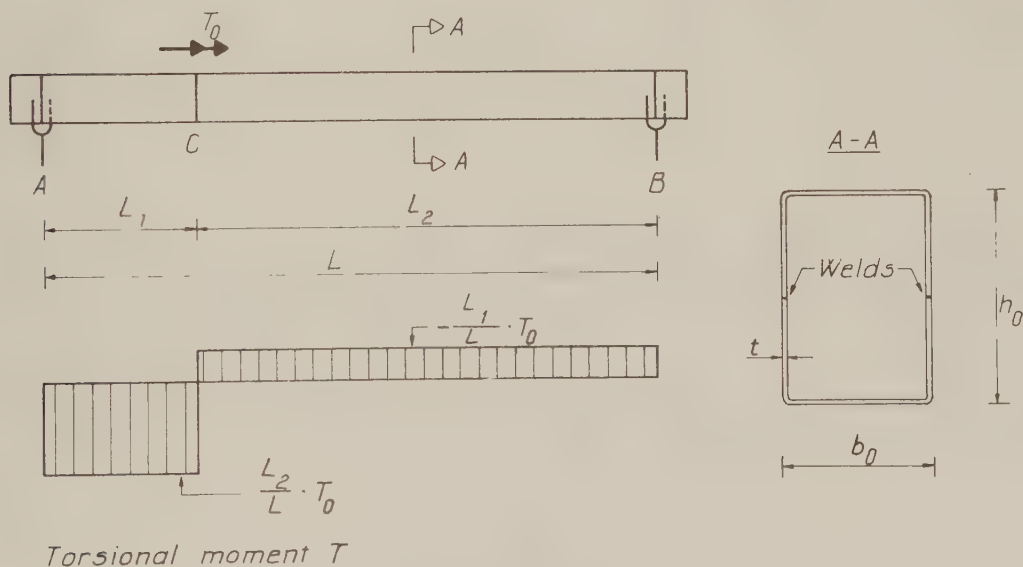


Fig. 4.3.1. Distribution of load and calculated distribution of torsional moment according to linear theory. Notations for cross section.

Table 4.3.1. Load positions and nominal dimensions of test beams F1-F6.

Test beam	L_1/L	L_2/L	L mm	b_0 mm	h_0 mm	t mm
F1	0.45	0.55	4800	200	200	1
F2	0.35	0.65	4800	200	200	1
F3	0.25	0.75	4800	200	200	1
F4	0.15	0.85	4800	200	200	1
F5	0.40	0.60	3480	133	200	1
F6	0.20	0.80	3480	133	200	1

4.3.3 Loading and support arrangements.

The principal load and support arrangements are shown in Fig. 4.3.2. The test rig had been designed to satisfy, as closely as possible, the boundary conditions that

- 1) both ends were restrained from twisting,
- 2) the box beams were simply supported with regard to vertical bending.

The torque was created by a hydraulic jack and applied to the test beams by steel wires fastened to a torque frame. To eliminate effects of bending one steel wire was attached to the centre of the torque frame. This wire was connected to a water tank in which the quantity of water was regulated by a data processing unit.

The supports consisted of anti-torque frames resting on roller bearings and were restrained from twisting by pre-stressed vertical bars.

The torque and anti-torque frames transferred the torque to the test beams by screw joints. Inside the test beams, resisting pieces of flat bar with dimensions 10 x 100 mm were used. For test beams F1-F4, each side of the screw joints was provided with eight screws, while for test beams F5 and F6 the flanges were provided with only

four screws on account of their smaller dimensions.

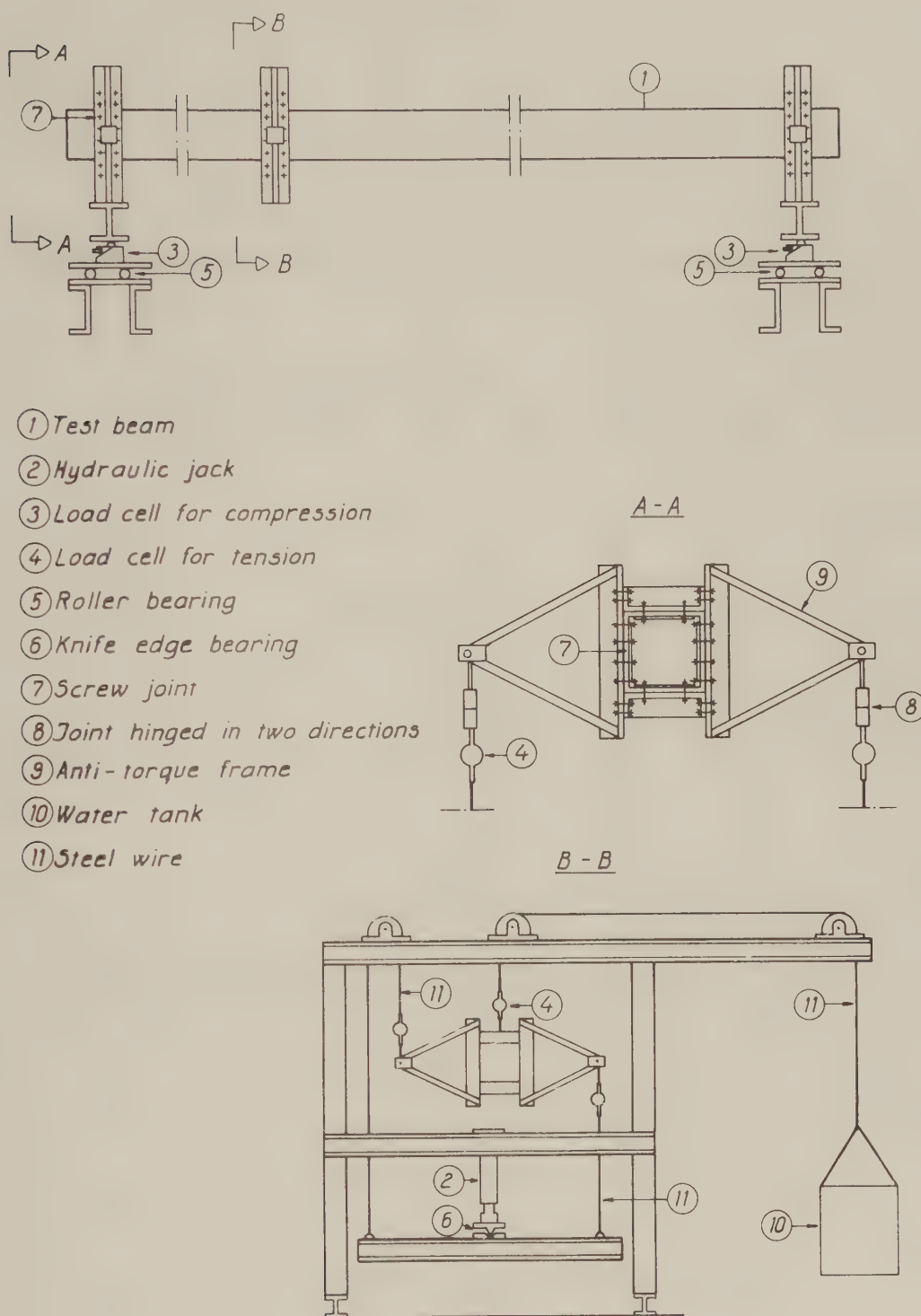


Fig. 4.3.2. Principal load and support devices for test beams F1-F6.

4.3.4 Dimensions. Material properties.

The process of manufacture and the material properties of the test beams F1-F6 were described in Section 3.4.4.

Mean values of measured cross-sectional dimensions of the test beams are given in Table 4.3.2. The lengths of the beams were in very good agreement with the nominal lengths. Cross-sectional constants and critical stresses, defined according to Appendix A, are given in Table 4.3.3.

Table 4.3.2. Measured dimensions of test beams F1-F6.

Test beam	b_o mm	h_o mm	t mm	R mm
F1	198.8	200.9	1.074	3
F2	198.6	201.3	1.078	3
F3	200.2	199.7	1.036	3
F4	199.2	200.5	1.047	3
F5	135.7	197.1	1.058	3
F6	135.0	198.3	1.060	3

Table 4.3.3. Cross-sectional constants and critical stresses for test beams F1-F6.

Test beam	b/t	h/t	$I_T \cdot 10^{-4}$ mm^4	$\tau_{cr,f}$ N/mm^2	$\tau_{cr,w}$ N/mm^2
F1	184.1	186.1	843	29.3	28.7
F2	183.2	185.7	848	29.6	28.8
F3	192.2	191.8	815	26.9	27.0
F4	189.3	190.5	823	27.8	27.4
F5	127.3	185.3	446	61.4	29.0
F6	126.4	186.1	447	62.3	28.7

4.3.5 Measuring devices and measurement procedure.

A computer controlled data acquisition system was used for the measurements. The measurements comprised measurement of loads, determination of angles of twist and the recording of normal deflections.

Due to "creep" at high loads the deformations of the test beams were kept as constant as possible during each load step. In this case deformation refers to the angle of twist at the section of the applied torque. The measurements were performed three times for every load step namely immediately after load application, ten seconds after load application and just before the next load step.

Load measurement.

The load cells for the measurement of compression forces were of the BOFORS LSK-2 type while the load cells for tension forces were of the BOFORS KRG-4 type. The load cells for tension forces used for the measurements on test beam F1 had been fabricated at the Department. The accuracy of these was, however, insufficient, and they were therefore not used subsequently.

Positions and numbering of load cells for test beam F3 are shown in Fig 4.3.3. The forces on the two load cells numbered 2 and 3 were recorded as continuous functions of time.

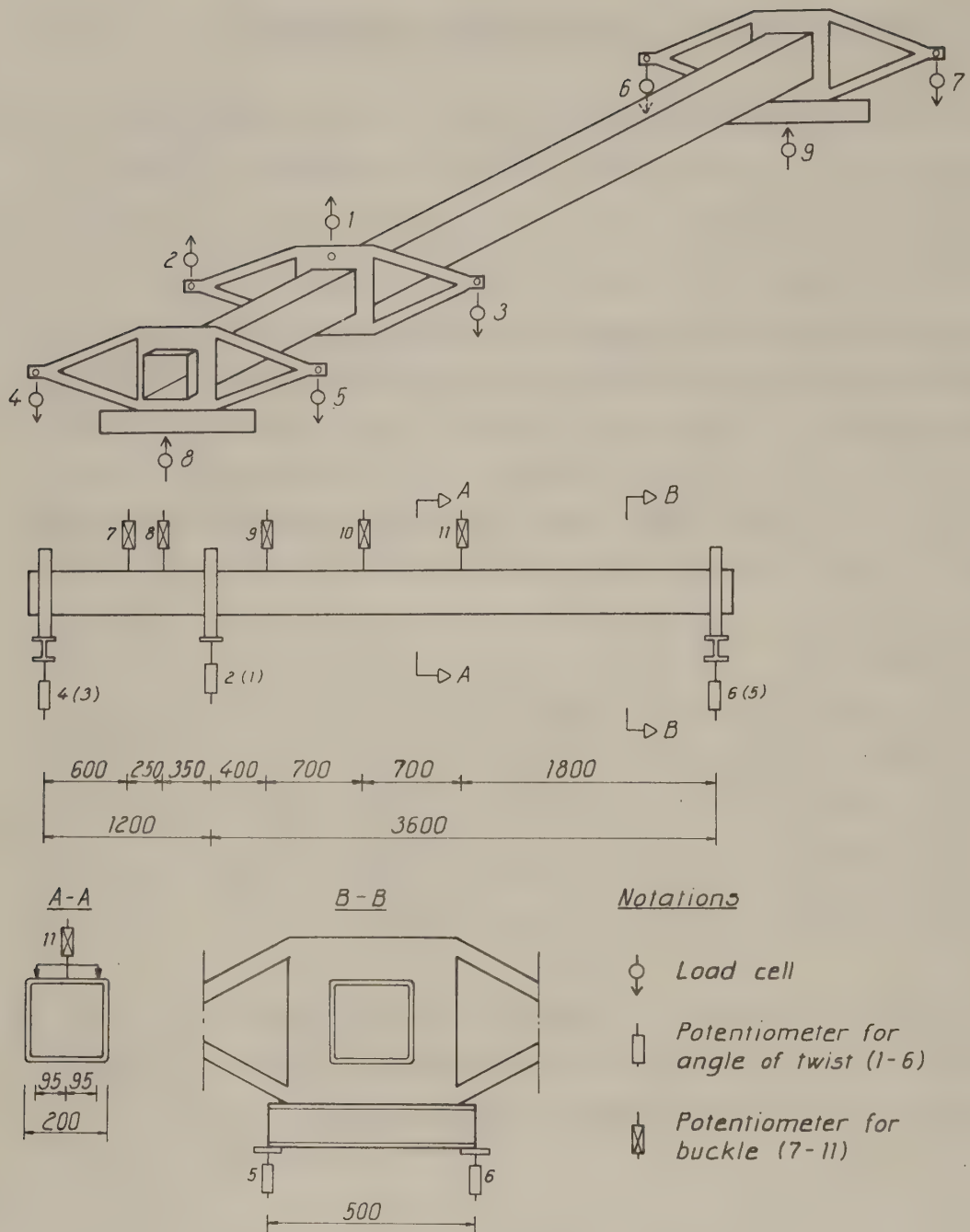


Fig. 4.3.3. Positions and numbering of load cells and potentiometers for test beam F3.

Angle of twist measurement.

The angles of twist were determined by means of potentiometers (resolution 0.03 mm) which were placed in pairs at the actual beam sections. If the distance between two potentiometers is known, the angle of twist can be calculated. The principle is shown for test beam F3 in Fig. 4.3.3 (section B-B). Besides the angles of

twist, the deflections of the test beams were also calculated for the beam sections concerned.

Measurement of normal deflections.

A device for measurement of normal deflection is shown in Fig. 4.3.4. A potentiometer acts against the centre of the flange while two screws rest near the corners of the flange. The aim of the device is not to determine the local buckling load but rather to obtain an idea of the size of the buckles and to find out whether there are any radical changes in the normal deflection pattern. Positions and numbering of the potentiometers are shown in Fig. 4.3.3.

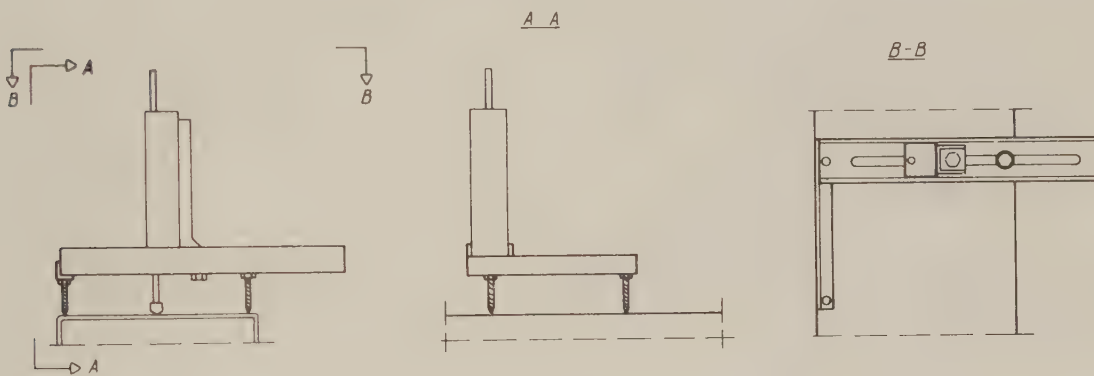


Fig. 4.3.4. Device for buckle measurement.

Computer controlled data acquisition system.

The computer controlled data acquisition system is described in Appendix D. A principal flow chart of the measurement programme is shown in Fig. 4.3.5. All measured data were punched on paper tape by means of a high speed punch.

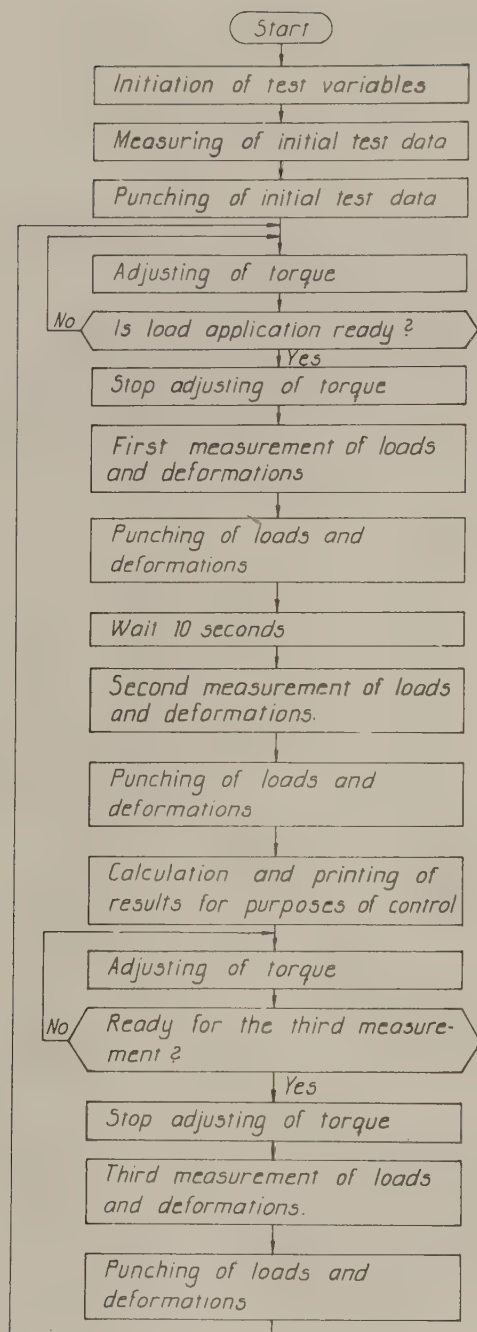


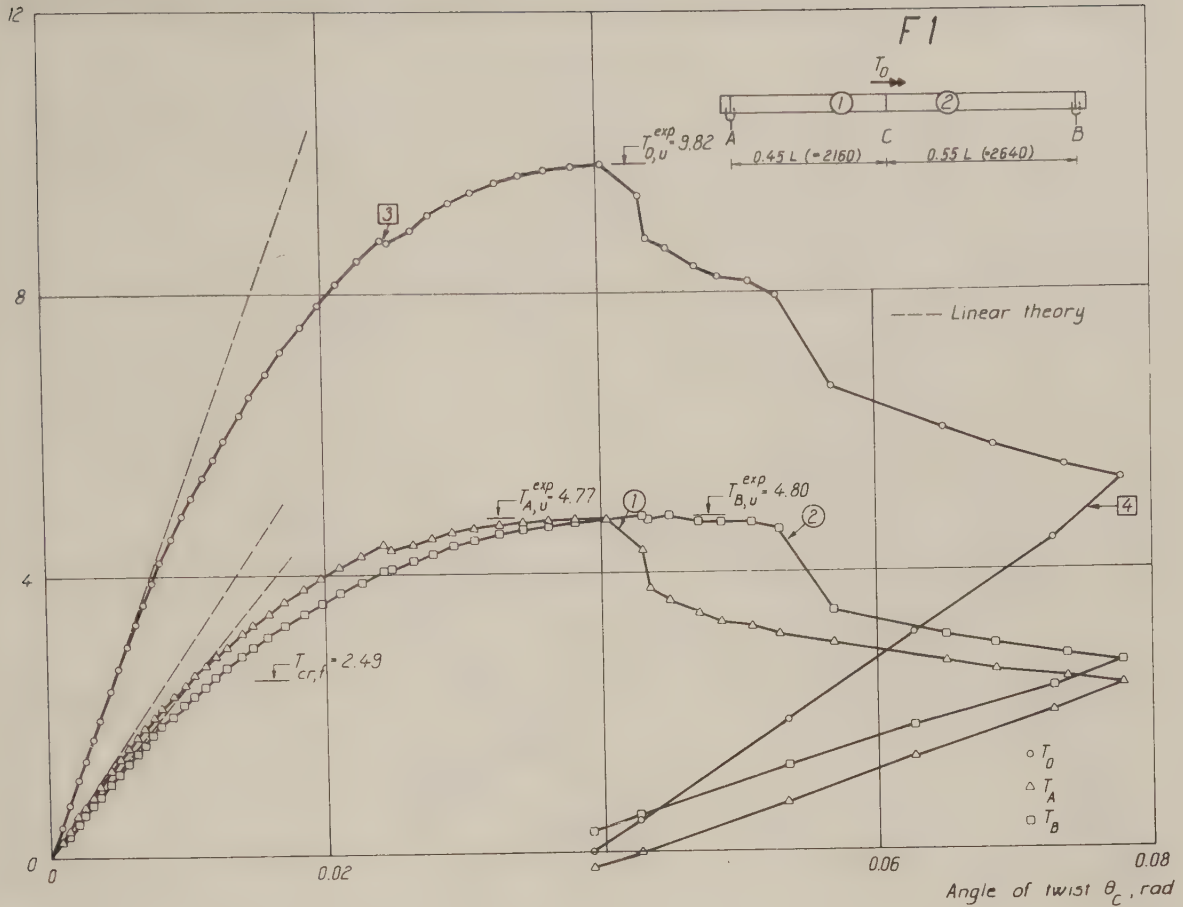
Fig. 4.3.5. Principal flow chart of measurement programme for test beams F1-F6.

4.3.6 Results.

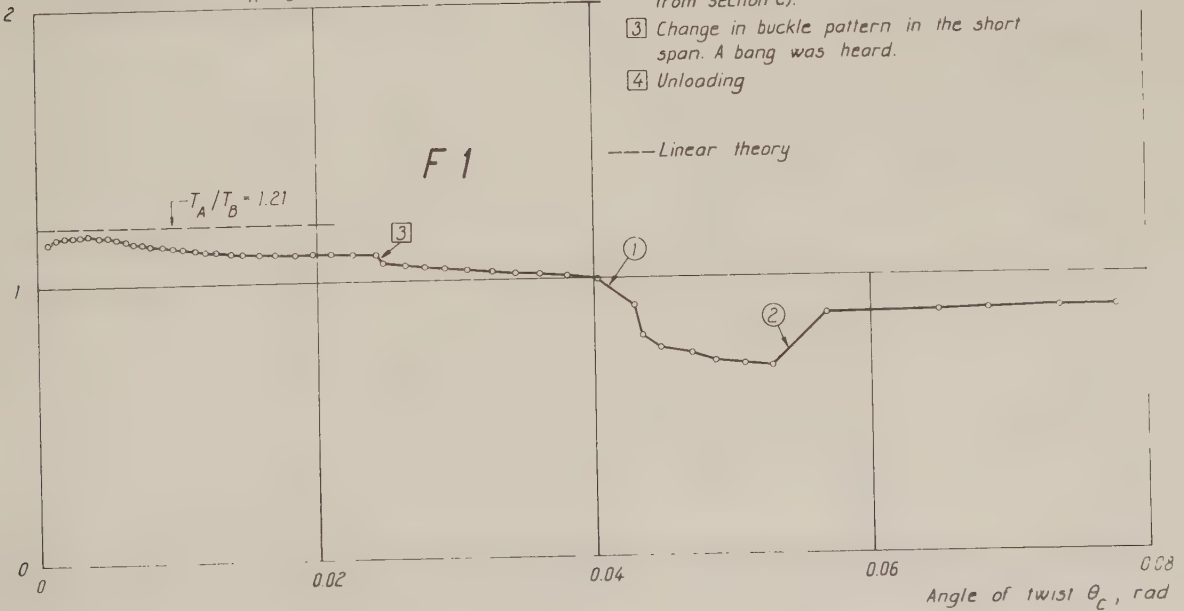
Angle of twist.

Figs. 4.3.6a-4.3.11a show the external torsional moments acting on the test beams F1-F6 as a function of the angle of twist θ_c . No correction has been made for errors in measurement values. Thus for

a

Torsional moment T , kNm

b

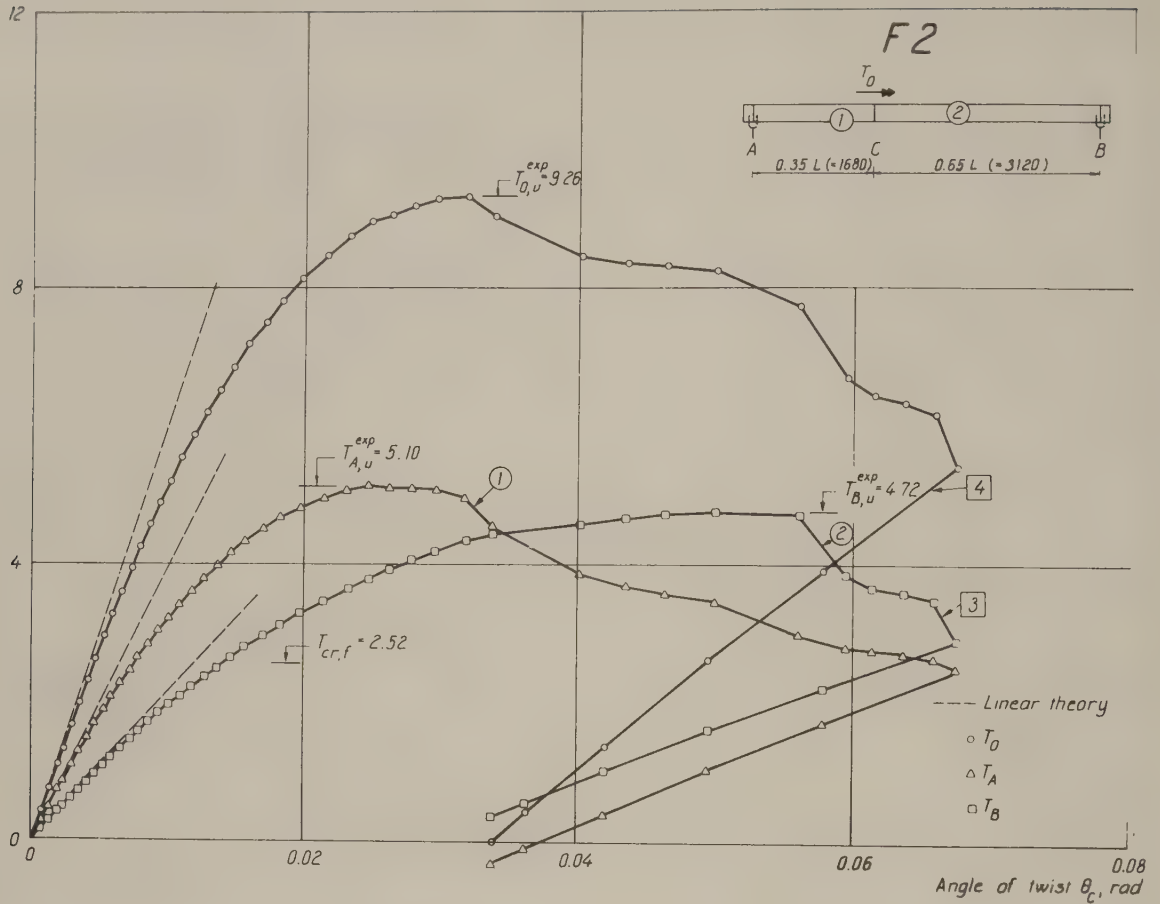
Torsional moment ratio T_A/T_B 

Comments

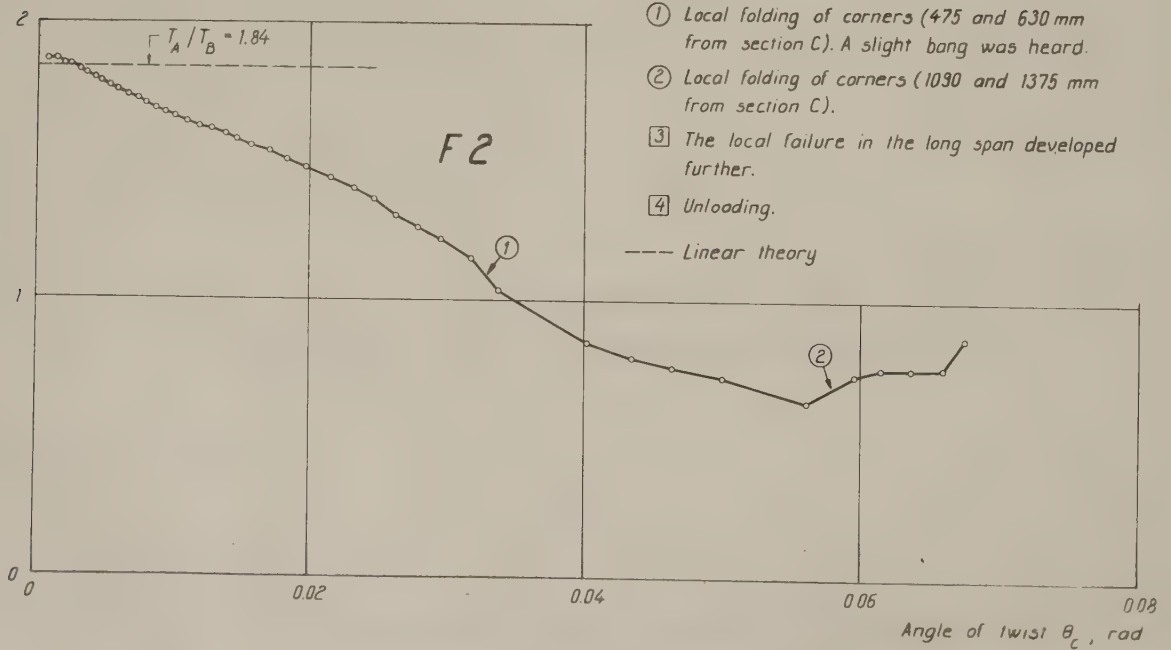
- ① Local folding of corners (580 and 805 mm from section C).
- ② Local folding of corners (720 and 1000 mm from section C).
- ③ Change in buckle pattern in the short span. A bang was heard.
- ④ Unloading

Fig. 4.3.6 a. Relationships between torsional moments and angle of twist for test beam F1.
b. Torsional moment ratio versus angle of twist.

a

Torsional moment T , kNm

b

Torsional moment ratio T_A / T_B Comments:

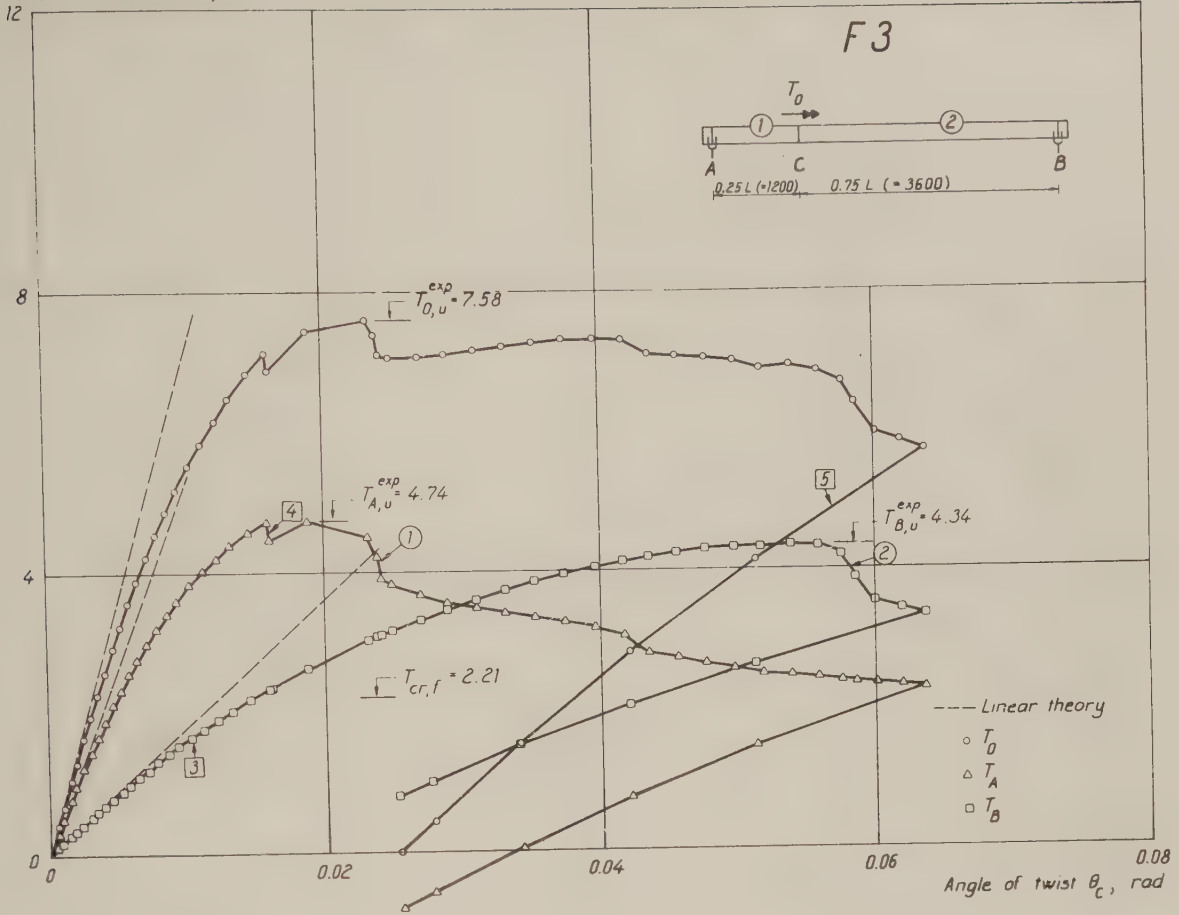
- ① Local folding of corners (475 and 630 mm from section C). A slight bang was heard.
- ② Local folding of corners (1030 and 1375 mm from section C).
- ③ The local failure in the long span developed further.
- ④ Unloading.

--- Linear theory

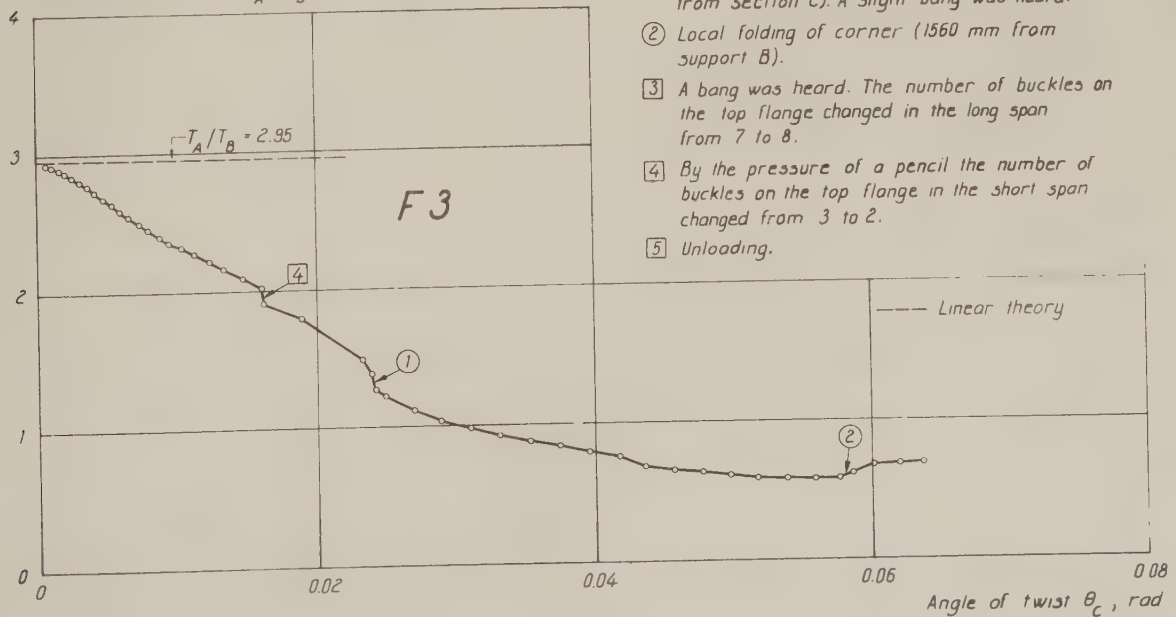
Fig. 4.3.7 a. Relationships between torsional moments and angle of twist for test beam F2.

b. Torsional moment ratio versus angle of twist.

a,

Torsional moment T , kNm

b,

Torsional moment ratio T_A / T_B Comments:

- ① Local folding of corners (550 and 600 mm from section C). A slight bang was heard.
- ② Local folding of corner (1560 mm from support B).
- ③ A bang was heard. The number of buckles on the top flange changed in the long span from 7 to 8.
- ④ By the pressure of a pencil the number of buckles on the top flange in the short span changed from 3 to 2.
- ⑤ Unloading.

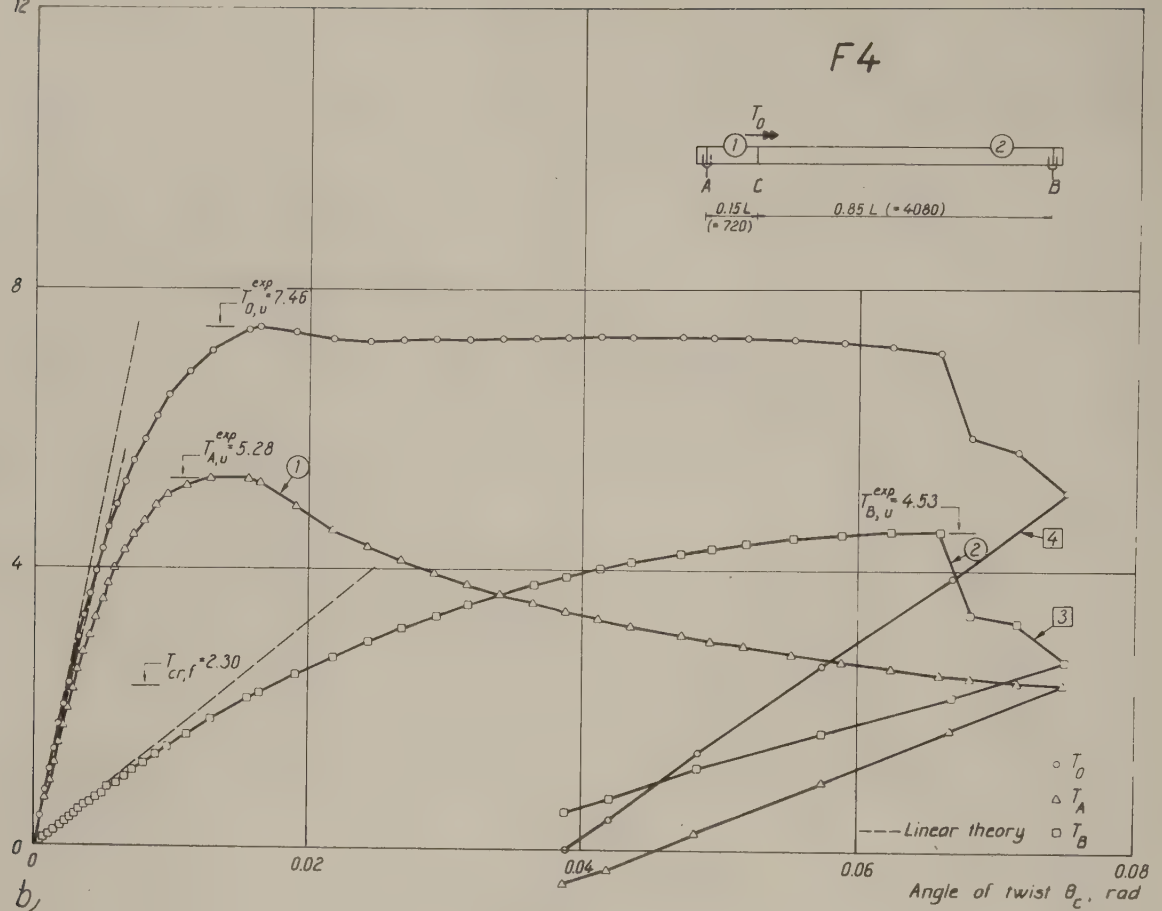
Fig. 4.3.8 a. Relationships between torsional moments and angle of twist for test beam F3.

b. Torsional moment ratio versus angle of twist.

a)

Torsional moment T , kNm

12



b)

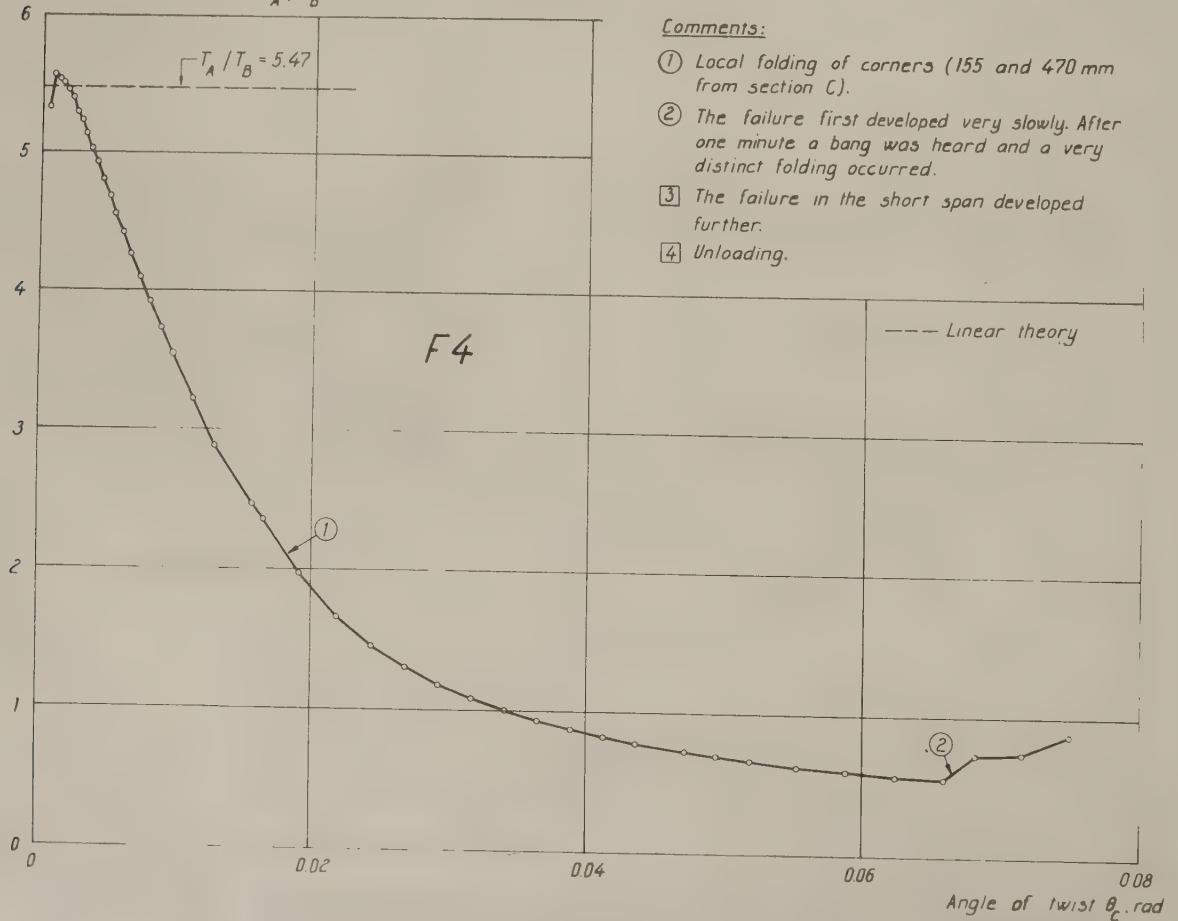
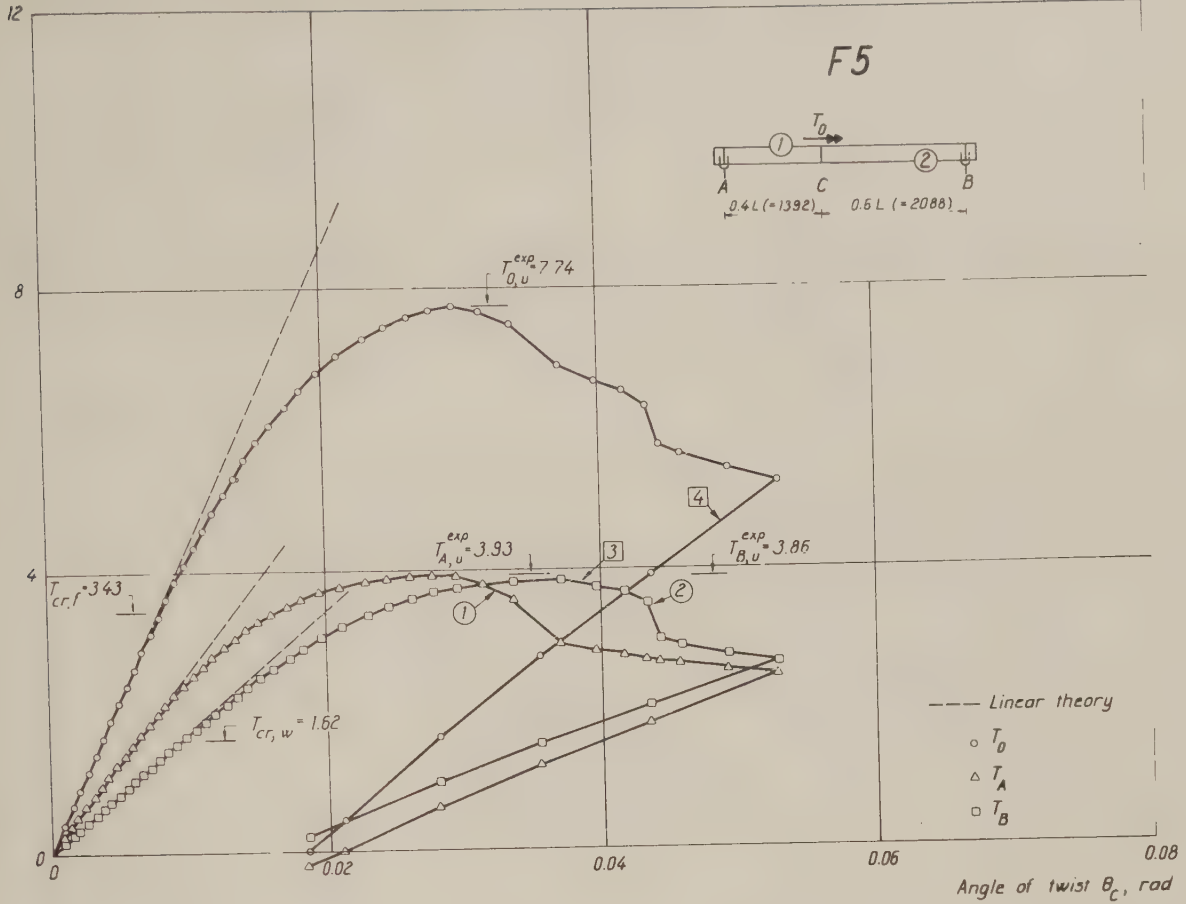
Torsional moment ratio T_A / T_B 

Fig. 4.3.9 a. Relationships between torsional moments and angle of twist for test beam F4.

b. Torsional moment ratio versus angle of twist.

a,

Torsional moment T , kNm

b,

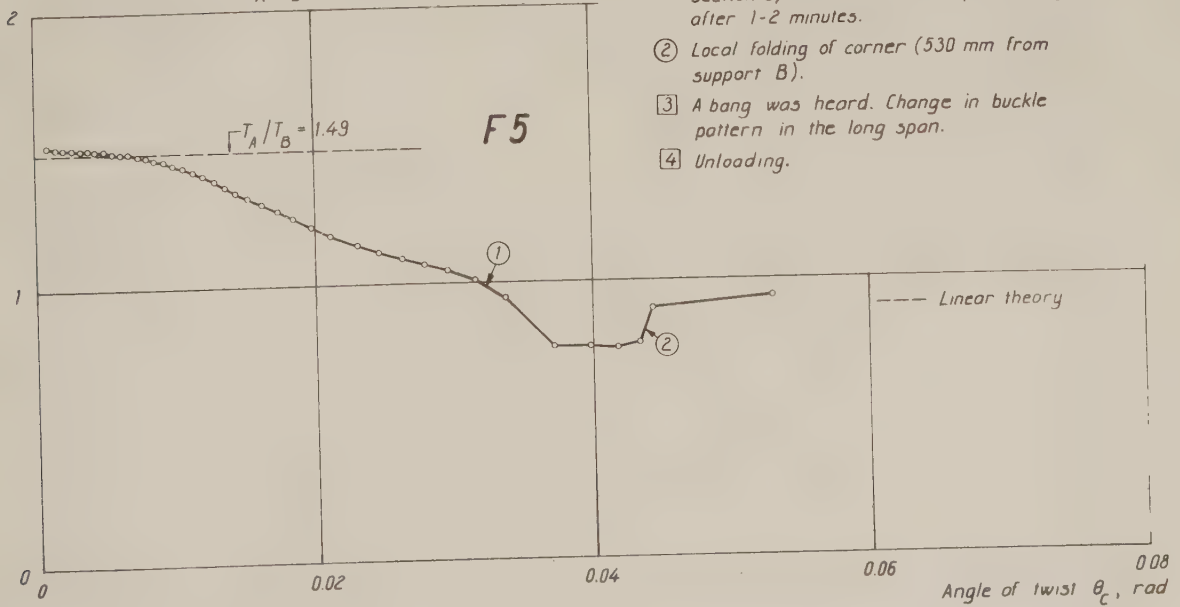
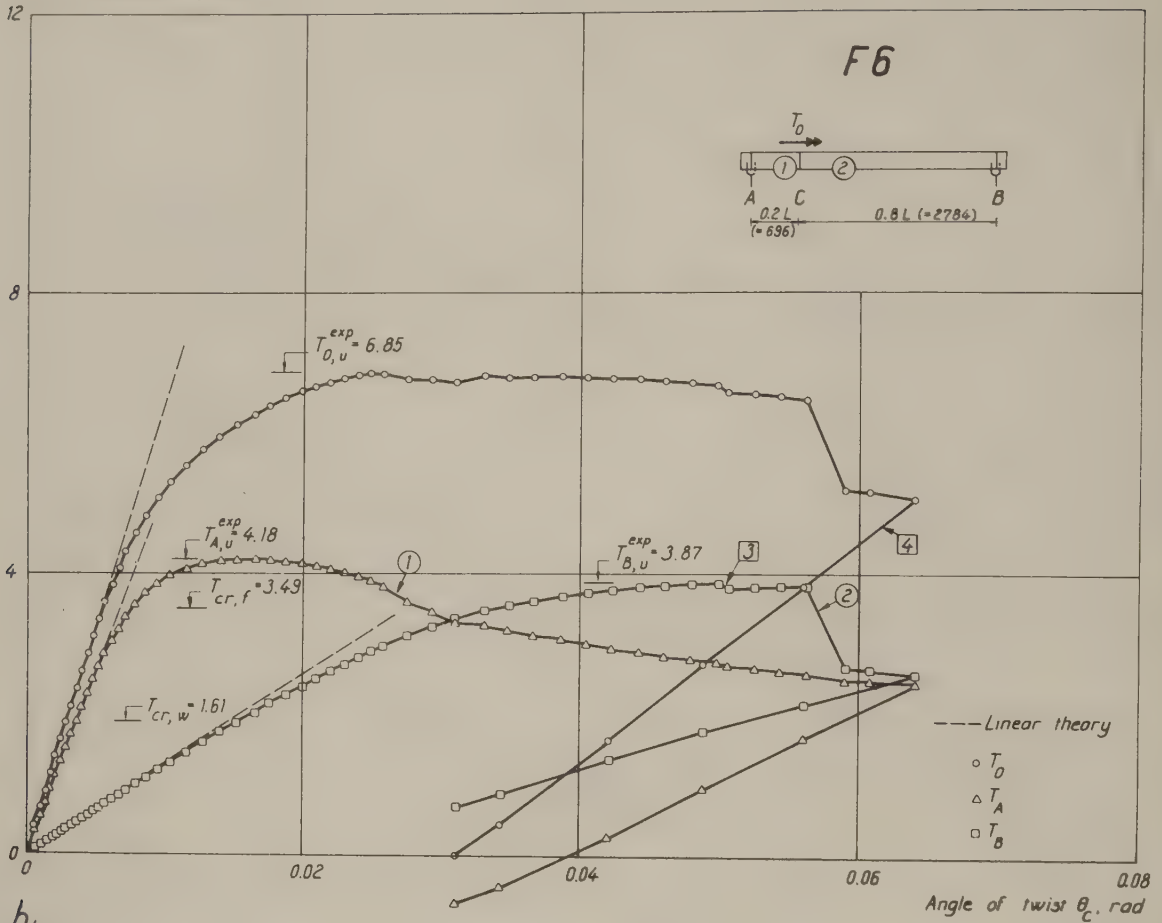
Torsional moment ratio T_A/T_B 

Fig. 4.3.10 a. Relationships between torsional moments and angle of twist for test beam F5.

b. Torsional moment ratio versus angle of twist.

a,

Torsional moment T , kNm

b,

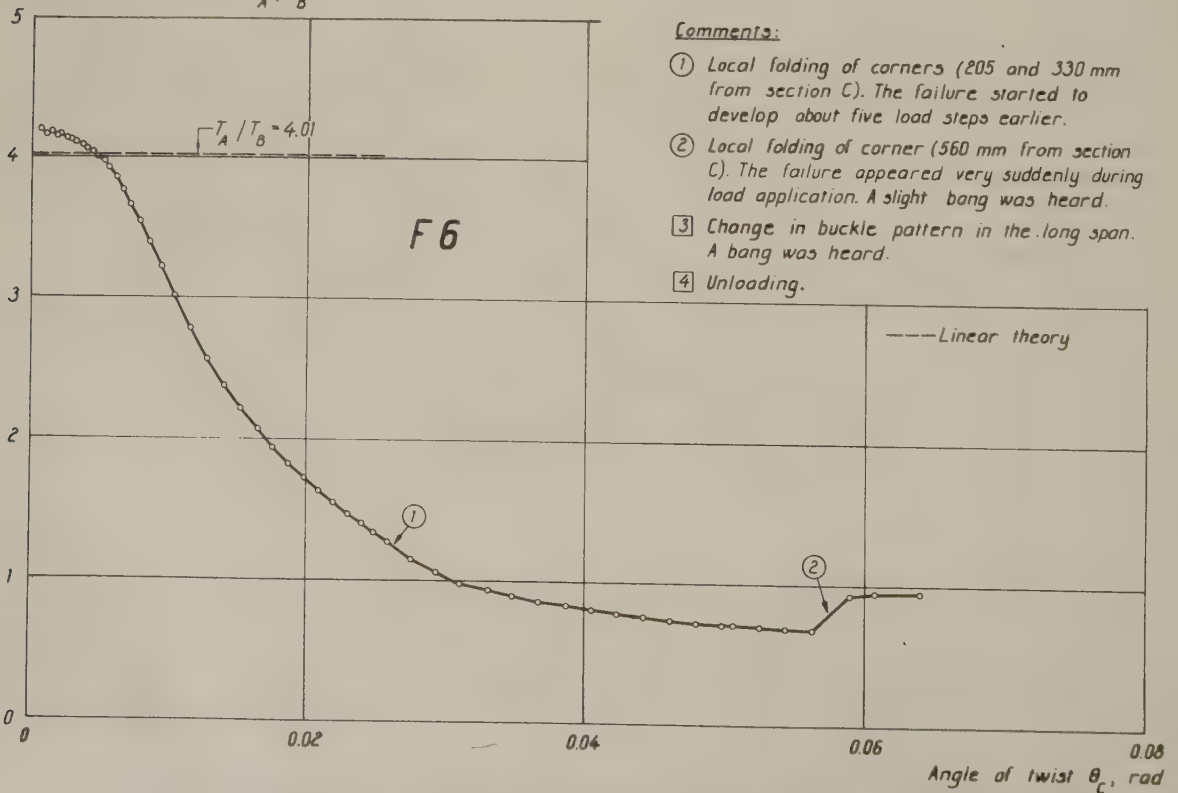
Torsional moment ratio T_A / T_B 

Fig. 4.3.11 a. Relationships between torsional moments and angle of twist for test beam F6.

b. Torsional moment ratio versus angle of twist.

test beams F2-F6, the applied torsional moment T_0 is sometimes more than 1 per cent greater than the sum of the torsional moments of the supports. For test beam F1, the error is nearly 4 per cent which means that the result obtained from this test must be interpreted with caution.

Figs. 4.3.6a-4.3.11a also show where on the test beams and at what angles of twist failure occurred.

Normal deflections.

The initial normal deflections of the test beams F1-F6 are presented in Appendix E.

The normal deflections of some points on the upper sides of test beams F4 and F6 are shown in Figs. 4.3.12 and 4.3.13. The number of the buckles often changed during the tests. For test beam F1, for example, the number of buckles on the upper side reduced from 3 to 2 in the short span, see Fig. 4.3.6.

Character of failure.

Failure occurred as yielding in one (or two) of the corners at which a local folding appeared. Cf. Section 4.2.6. The character of the failure can be described as rather soft. In the long spans, however, the failure was often more brittle and accompanied by a soft audible bang.

Ultimate torsional moments.

Measured ultimate torsional moments of the test beams F1-F6 are given in Table 4.3.4 as cross-sectional quantities on the one hand and as total applied quantities on the other.

Table 4.3.4. Measured ultimate torsional moments of test beams F1-F6.

Test beam	$T_{A,u}^{\text{exp}}$ kNm	$T_{B,u}^{\text{exp}}$ kNm	$T_{O,u}^{\text{exp}}$ kNm
F1	4.77	4.80	9.82
F2	5.10	4.72	9.26
F3	4.74	4.34	7.58
F4	5.28	4.53	7.46
F5	3.93	3.86	7.74
F6	4.18	3.87	6.85

4.3.7 Discussion of results.

Angle of twist.

The relationship between applied torsional moment and angle of twist has been studied using the theory of pure Saint Venant torsion. In these calculations, the influence of elastic supports has been included. The elasticity of the supports, obtained from the tests, are given in Table 4.3.5. On account of the stiffening effect of the screw joints the length of each span has been reduced by 70 mm. The results of the calculations are given in Figs. 4.3.6a-4.3.11a. The agreement with measured values is fairly good at low loads. For the test beams F1-F4, however, there is a tendency for the theoretical deformations to be too small. An explanation may be that local buckles appear at lower loads than for beams F5 and F6 (cf. Figs. 4.3.12 and 4.3.13).

In Figs. 4.3.6a-4.3.11a, the critical cross-sectional torsional moments are also given. For test beams F5 and F6 this is done for both the webs and the flanges.

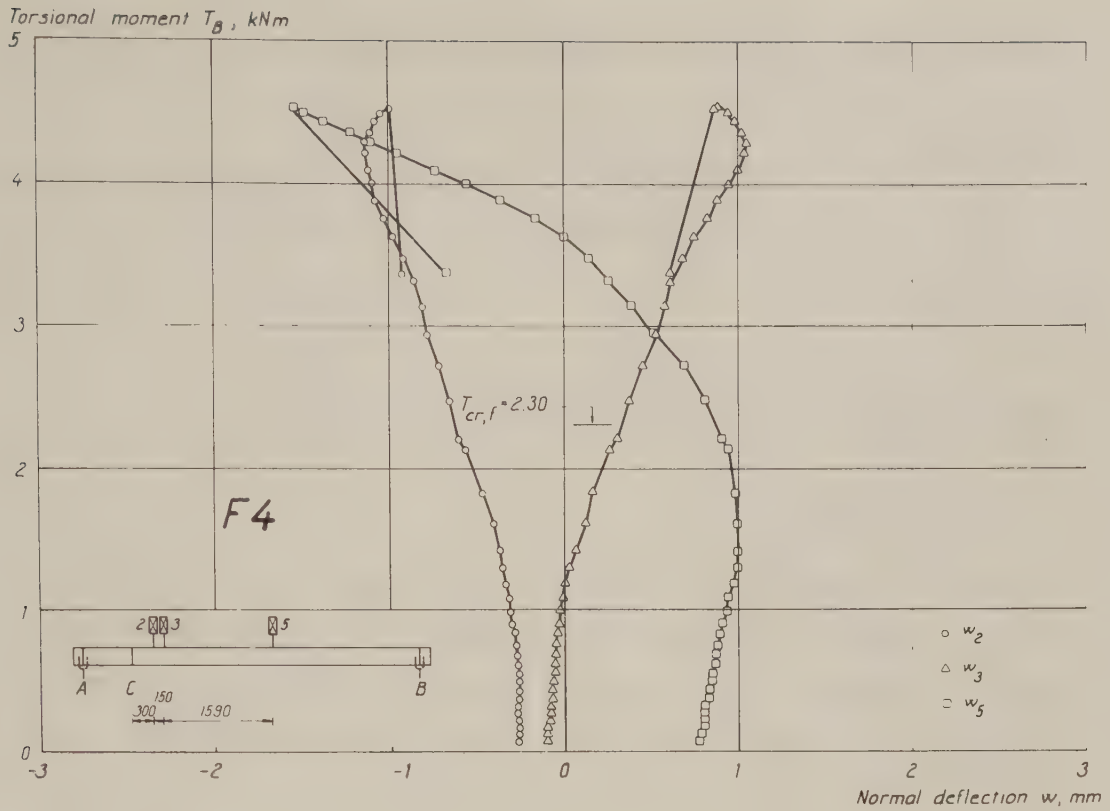


Fig. 4.3.12. Normal deflection as function of torsional moment for some points on the upper flange of test beam F4.

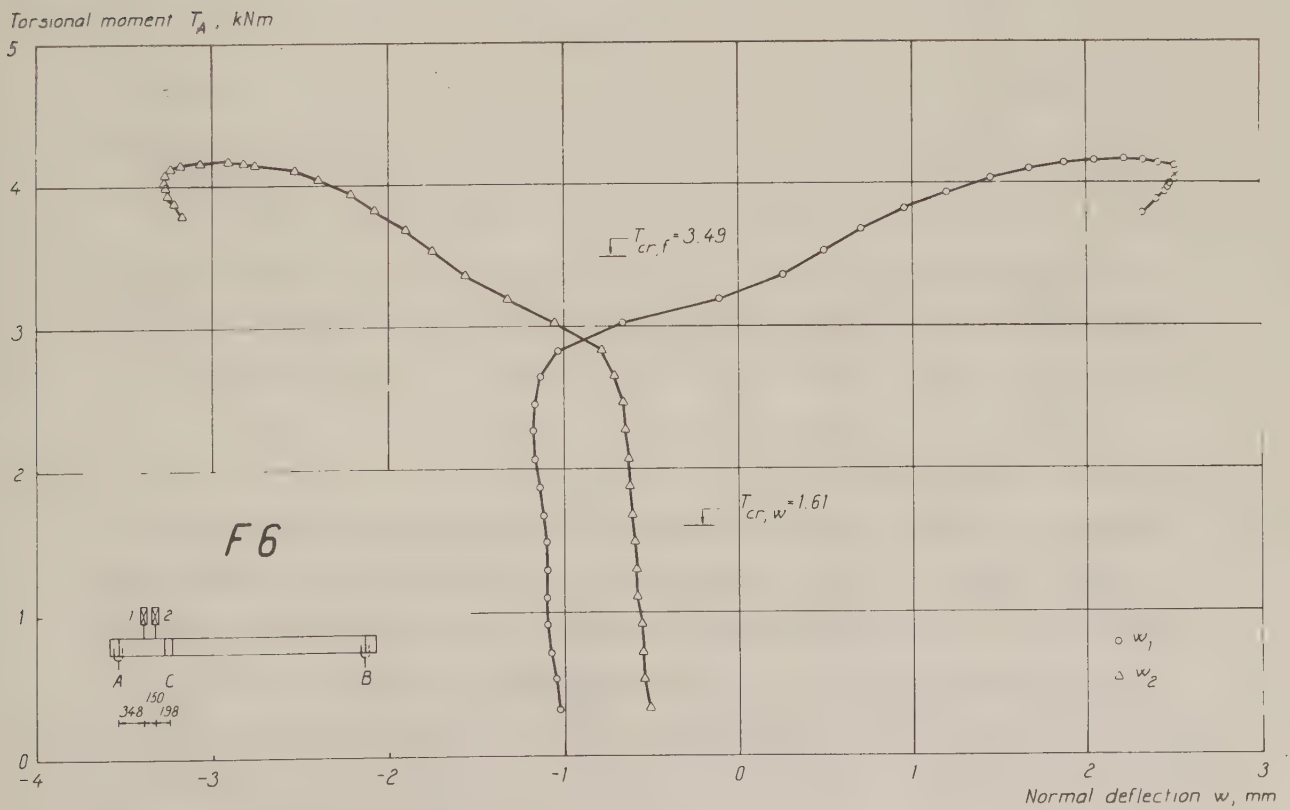


Fig. 4.3.13. Normal deflection as function of torsional moment for some points on the upper flange of test beam F6.

Table 4.3.5. The elastic constants of the supports.

Test beam	c_A kNm/rad	c_B kNm/rad
F1	3600	3600
F2-F4	6500	6500
F5-F6	5300	5300

Normal deflections.

The normal deflections of test beam F4, shown in Fig. 4.3.12, gradually increase and no typical critical load can be seen. For test beam F6, however, there is a very marked critical load, cf. Fig. 4.3.13. It is interesting to note that this critical load is higher than that of the webs and instead approaches the critical load of the flanges. This is probably an effect of the welds and the big deformations in the webs obtained on account of welding.

Torsional moment ratios.

In Figs. 4.3.6b-4.3.11b the ratios between the torsional moments of the supports T_A and T_B are given as a function of the angle of twist θ_c . Theoretical torsional moment ratios, calculated according to pure Saint Venant torsion theory, are also shown. The measured and the theoretical curves usually agree fairly well at low loads but the measured curve soon starts to drop. A drop in the curve can also be seen when failure occurs in the short span. In the same way, the measured curve rises when failure appears in the long span.

Ultimate torsional moments.

Using Eq. (4.1.30), the theoretical ultimate torsional moments of the test beams have been determined. The results of the calculations are given in Table 4.3.6 where a comparison with the measured values is also made. The measured torsional moments in this case refer to the long span. If the second term within the square brackets of Eq. (4.1.28) had been included, the theoretical ultimate torsional moment would increase. Thus for span lengths 0.15 L, 0.25 L,, 0.75 L, 0.85 L of test beams F1-F4 (L = 4800 mm) the increases will

be respectively 4.4 %, 1.5 %, 0.7 %, 0.4 %, 0.3 %, 0.2 %, 0.2 % and 0.1 %. For span lengths 0.2 L, 0.4 L, 0.6 L and 0.8 L of test beams F5 and F6 (L = 3480 mm) the corresponding increases will be respectively 3.4 %, 0.8 %, 0.3 % and 0.1 %. The influence of the span length on the ultimate torsional moment will be studied in more detail in the following.

Table 4.3.6. Theoretical ultimate torsional moments. Ratios of measured to theoretical ultimate torsional moments.

Test beam	T_u^{th} kNm	$\frac{T_u^{exp}}{T_u^{th}}$
F1	4.59	1.05
F2	4.63	1.02
F3	4.22	1.03
F4	4.33	1.05
F5	3.88	0.99
F6	3.90	0.99

Test beams F5 and F6, which were cut out from the same beam, are compared in Fig. 4.3.14 with respect to the ultimate torsional moments by means of a dimensionless ratio $T_u^{exp}(L)/T_u^{exp}(L_{max})$

where $T_u^{exp}(L_{max})$ = Ultimate torsional moment for the longest span ($L_{max} = 2784$ mm)

$T_u^{exp}(L)$ = Ultimate torsional moment of actual span.

From Fig. 4.3.14 it can be seen that the ultimate moment is hardly affected by the span length for L greater than 2 m. For the shortest span length used the ultimate moment is about 8 % higher. The influence of span length on the ultimate torsional moment if the actual dimensions of the plate elements are inserted in Eq. (4.1.28) is as shown by the dashed curve in Fig. 4.3.14. It must be emphasized that Eq. (4.1.28) has been derived on the assumption of simply supported edges. In our case the edges of the plate elements are

practically fully restrained at the ends, implying in an increase of the influence of the span length. In this connection it must be pointed out that the span lengths were too long to cause a direct transmission of tension fields from one stiffener (support or device of load application) to another.

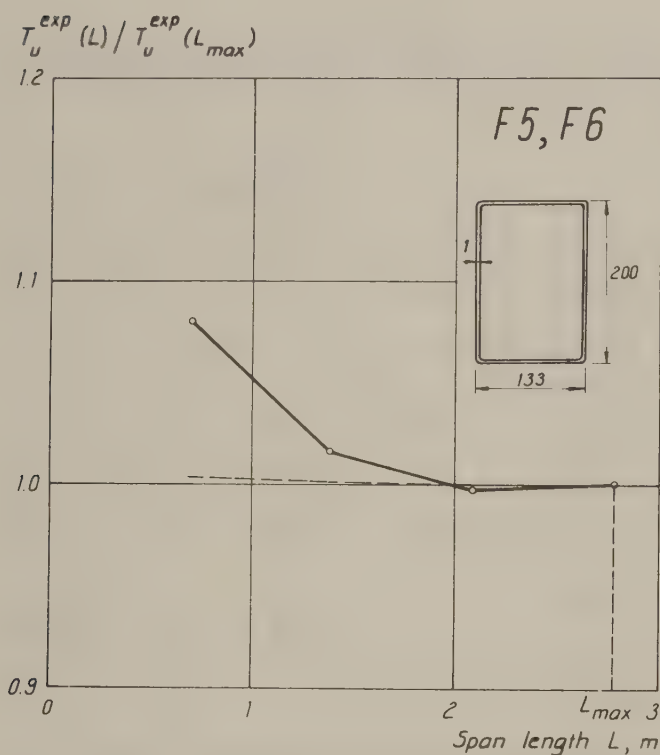


Fig. 4.3.14. Influence of span length on ultimate torsional moment for test beams F5 and F6.

For test beams F1-F4 which do not have identical cross-sectional dimensions it is more difficult to illustrate the influence of the span length upon the ultimate torsional moment. However, since the ultimate moment seems to be rather unaffected by span lengths greater than 2 m (cf. Fig. 4.3.14) it is reasonable to calculate the ratio of the ultimate moment of the short to that of the long span. The result obtained from such a calculation is given in Fig. 4.3.15. The tendency is the same as in Fig. 4.3.14 but still more emphasized for short span lengths because of the greater cross-sectional dimensions of test beams F1-F4. As for the test beams F5 and F6 Eq. (4.1.28) has been used to estimate the theoretical influence of the span length on the load-carrying capacity (dashed curve in Fig. 4.3.15).

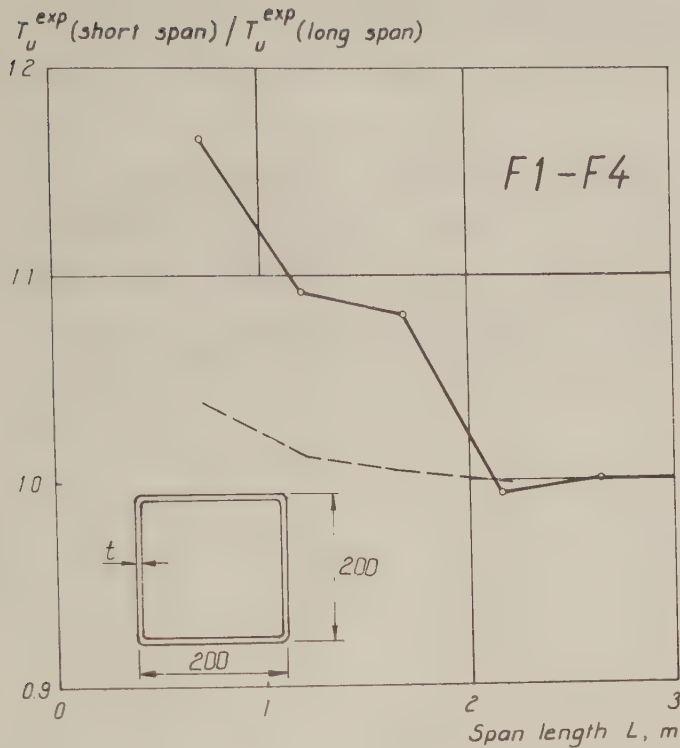


Fig. 4.3.15. Influence of span length on ultimate torsional moment for test beams F1-F4.

Ratios of the applied external ultimate torsional moment $T_{o,u}$ to the ultimate torsional moment of the long span $T_{B,u}$ are given in Table 4.3.7. A ratio equal to 2 means complete equality between the two moments of the supports provided that the ultimate moment of the short span is not greater than that of the long span.

In the third column of Table 4.3.7, results obtained from linear theory are given. The theoretical span width ratios of Table 4.3.1 have been used, assuming the supports to be rigidly fixed and the extension of the supports to be negligible.

If, instead, we consider long beams with the same span width ratios as the actual test beams, the ultimate torsional moments can be determined in the following way. In the case of a long beam subjected to a constant torsional moment, the relation between the torsional moment T and angle of twist θ can be described by a curve as shown in Fig. 4.3.16. As soon as the maximum torsional moment T_1 has been reached, local failure will occur and a sudden deloading take place. Denoting the angle of twist at maximum load for test beams F1-F6 by

Table 4.3.7. Experimental and calculated ultimate torsional moment ratios. Possible increase of applied torsional moment over that indicated by linear theory.

Test beam	$\left(\frac{T_{o,u}}{T_{B,u}}\right)^{\text{exp}}$	$\left(\frac{T_{o,u}}{T_{B,u}}\right)^{\text{th}}$	$\left(\frac{T_{o,u}}{T_{B,u}}\right)^{\text{th}}$	Possible increase of applied torsional moment over that indicated by linear theory:	
	Actual test beam	Linear theory	Long beam	Actual test beam	Long beam
F1	2.05	1.83	1.97	12 %	8 %
F2	1.96	1.54	1.82	27 %	18 %
F3	1.75	1.34	1.60	31 %	19 %
F4	1.65	1.18	1.36	40 %	15 %
F5	2.00	1.67	1.90	20 %	14 %
F6	1.77	1.25	1.41	42 %	13 %

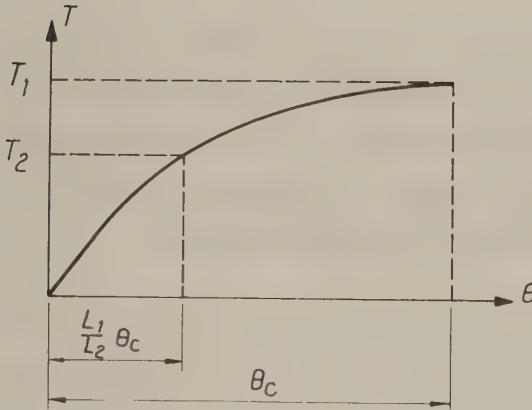


Fig. 4.3.16. Graph of torsional moment T versus angle of twist θ , for the determination of the ultimate torsional moment of long beams.

θ_c , the torsional moment T_2 of the long span can be obtained as the moment corresponding to the angle $\theta_c L_1/L_2$ in the graph of T versus θ , Fig 4.3.16. The sum of the torsional moments T_1 and T_2 is equal to the applied ultimate torsional moment $T_{o,u}$. In column four of Table 4.3.7 the ratio of $T_{o,u}$ to $T_{B,u}$ is shown (i.e. the ratio of $T_1 + T_2$ to T_1). For test beams F1-F4, the curve T_B from

Fig. 4.3.9 a has been used, since this curve is taken from the beam with the longest span. For test beams F5 and F6, the curve T_B from Fig. 4.3.11 a has been used.

The last two columns of Table 4.3.7 give the possible increase of applied torsional moment over that indicated by linear theory. As can be seen there is an obvious dependence on span length. The possibility of redistribution of torsional moments is limited but is nevertheless worth consideration.

5 COMBINED BENDING AND TORSION OF BOX BEAMS.

5.1 Beam with both ends restrained from twisting subjected to torsion and bending, where the torsion predominates.

5.1.1 Purpose.

In Section 4.3, experiments with beams having both ends restrained from twisting were described. A natural continuation of the investigation was to study the behaviour of beams with both ends restrained from twisting which were subjected not only to a torque but also to a bending moment.

5.1.2 Scope.

The experimental investigation comprised a series of four test beams, G1-G4, subjected to simultaneous torsion and bending.

The distribution of load as well as the calculated distribution of shear force, bending moment and torsional moment according to linear theory can be seen in Fig. 5.1.1. The ratio of P_1 to P_2 , defined in Fig. 5.1.1, was chosen as parameter. $P_1/P_2 = 1$ (test beam F3) represents pure torsion while $P_1/P_2 = -1$ represents pure bending. Load relations and nominal dimensions of the test beams G1-G4 are given in Table 5.1.1.

Table 5.1.1. Load relations and nominal dimensions of test beams G1-G4.

Test beam	P_1/P_2	L mm	b_o mm	h_o mm	t mm
G1	0.65	4800	200	200	1
G2	0.40	4800	200	200	1
G3	0.15	4800	200	200	1
G4	-0.15	4800	200	200	1

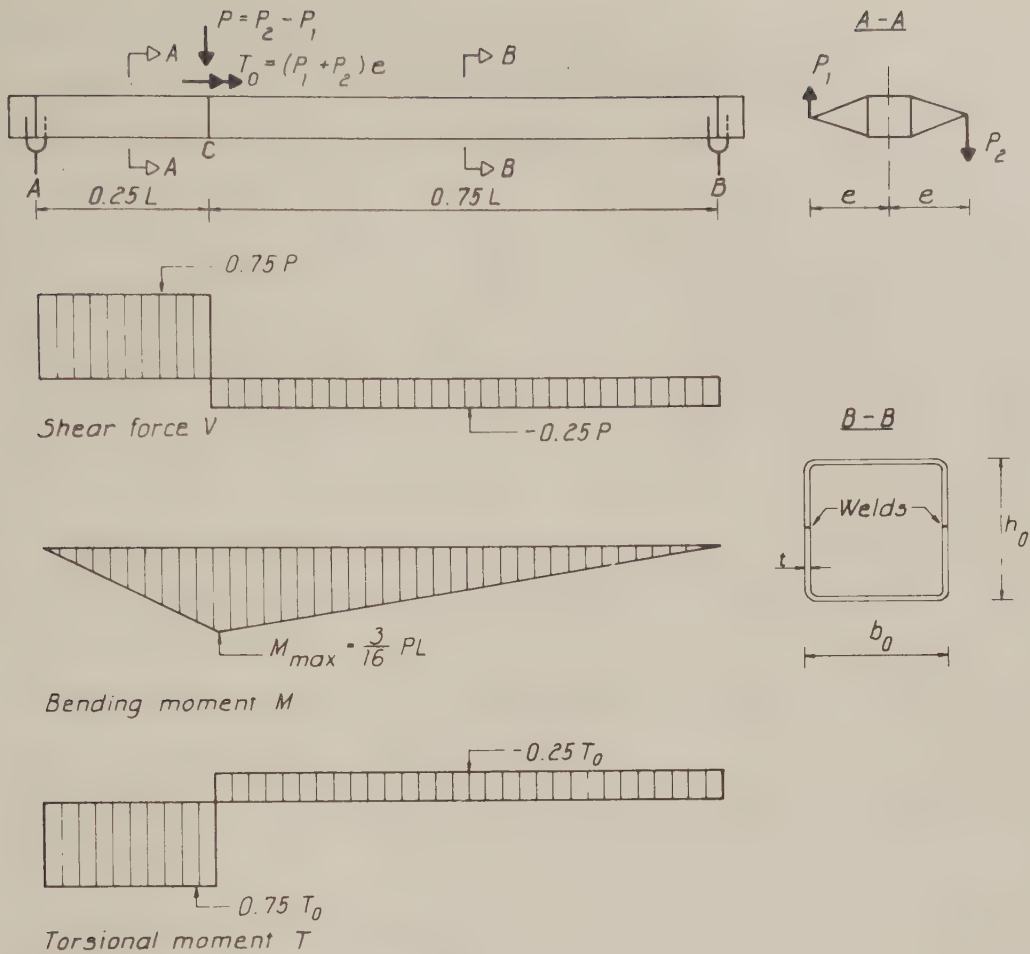


Fig. 5.1.1. Test beams G1-G4. Distribution of load. Calculated distribution of shear force, bending moment and torsional moment according to linear theory. Notations for cross section.

5.1.3 Loading and support arrangements.

The loading and support arrangements are the same as those described in Section 4.3.3 with the exception that the jack was placed eccentrically to make the two wire forces unequal, see Fig. 5.1.2.

In the testing of beam G4 the loading device was slightly modified. Thus to obtain a force directed downwards on the left side, the wire was fastened directly to the load distributing I-beam.

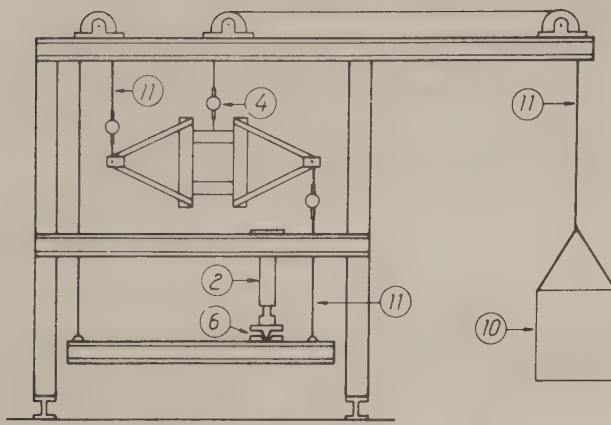


Fig. 5.1.2. Details of loading and support arrangements for test beams G1-G4. Notations, see Fig. 4.3.2.

5.1.4 Dimensions. Material properties.

The process of manufacture and the material properties of the test beams G1-G4 were described in Section 3.4.4.

Mean values of measured cross-sectional dimensions of the test beams are given in Table 5.1.2. The lengths of the beams were in very good agreement with the nominal ones given in Table 5.1.1. Cross-sectional constants and critical stresses, defined according to Appendix A, are given in Table 5.1.3.

Table 5.1.2. Measured cross-sectional dimensions of test beams G1-G4.

Test beam	b_o mm	h_o mm	t mm	R mm
G1	199.8	200.3	1.084	3
G2	200.8	199.0	1.061	3
G3	199.1	200.6	1.085	3
G4	199.1	200.9	1.079	3

A mean value of the load eccentricity used in the tests is $e = 652$ mm.

Table 5.1.3. Cross-sectional constants and critical stresses for test beams G1-G4.

Test beam	b/t	h/t	$I_o \cdot 10^{-4}$ mm ⁴	$I_T \cdot 10^{-4}$ mm ⁴	$\sigma_{cr,f}$ N/mm ²	$\sigma_{cr,w}$ N/mm ²	σ_{cr} N/mm ²	$\tau_{cr,f}$ N/mm ²	$\tau_{cr,w}$ N/mm ²
G1	183.3	183.8	570	854	22.2	131.8	29	29.6	29.4
G2	188.3	186.6	552	834	21.0	127.9	28	28.1	28.6
G3	182.5	183.9	571	852	22.4	131.6	30	29.9	29.4
G4	183.5	185.2	570	849	22.1	129.7	29	29.5	29.0

5.1.5 Measuring devices and measurement procedure.

The measuring devices and measurement procedures were almost identical to those described in Section 4.3.5.

It was more difficult to maintain constant deformation during each load step in this case because the test beams tended to deflect as well as to rotate. Thus "constant deformation" was chosen to mean constant angle of twist during each loadstep.

Load measurement.

See Section 4.3.5.

Measurement of deflection and angle of twist.

See Section 4.3.5.

Measurement of normal deflections.

See Section 4.3.5.

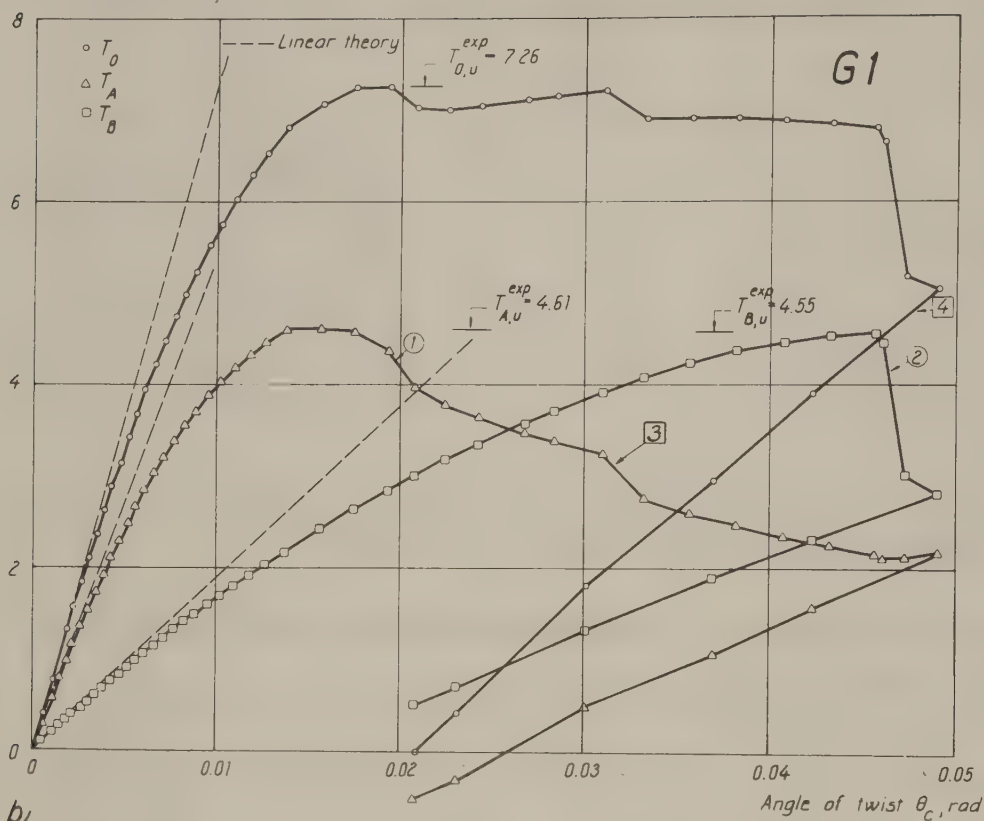
The positions of the potentiometers were somewhat changed.

5.1.6 Results.

Angles of twist.

Figs. 5.1.3a-5.1.6a show the relationship between measured torsional moment and angle of twist. No adjustment for errors in

a,

Torsional moment T , kNm

Comments:

- ① Local folding of corner (140 mm from section C). The failure developed very slowly.
- ② Local folding of corners (420 and 600 mm from section C). A sudden failure.
- ③ Local folding of another corner in the short span (430 mm from section C).
- ④ Unloading.

b,

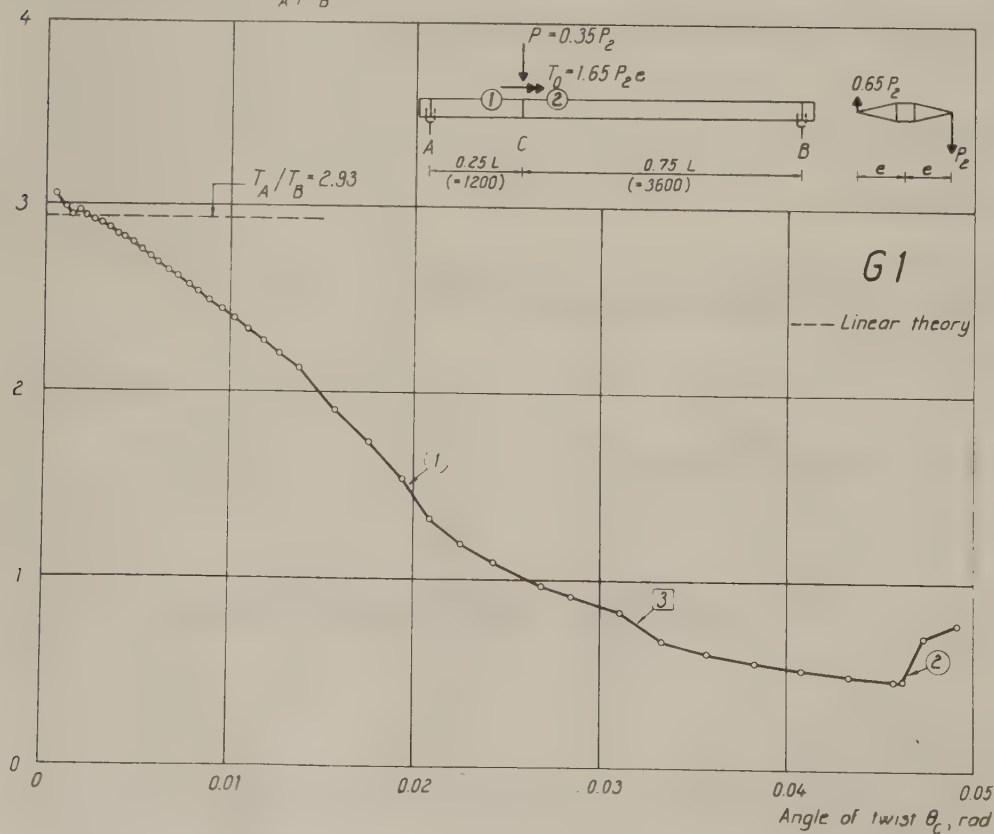
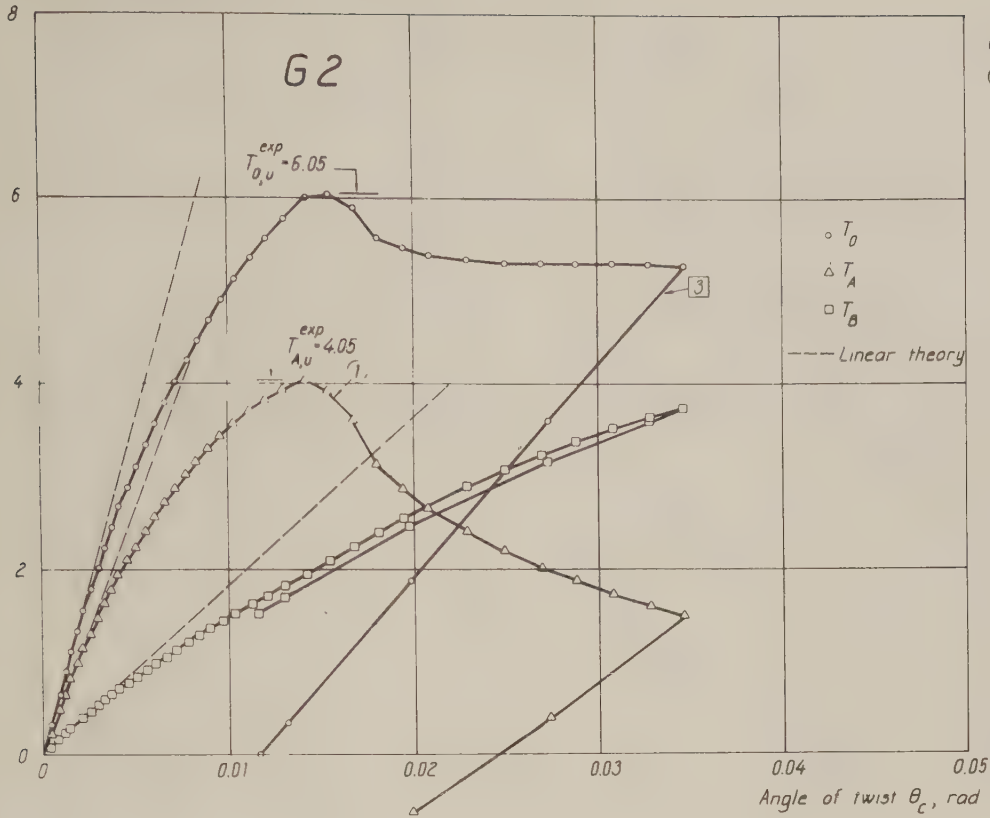
Torsional moment ratio T_A/T_B 

Fig. 5.1.3. a) Relationship between torsional moment and angle of twist for test beam G1
b) Torsional moment ratio versus angle of twist.

a,

Torsional moment T , kNm

Comments:

- ① Local folding of corners (255 and 275 mm from section C). The failure developed slowly. One of the fold appeared a few loadsteps later.
- ② Change in buckle pattern.
- ③ Unloading

b,

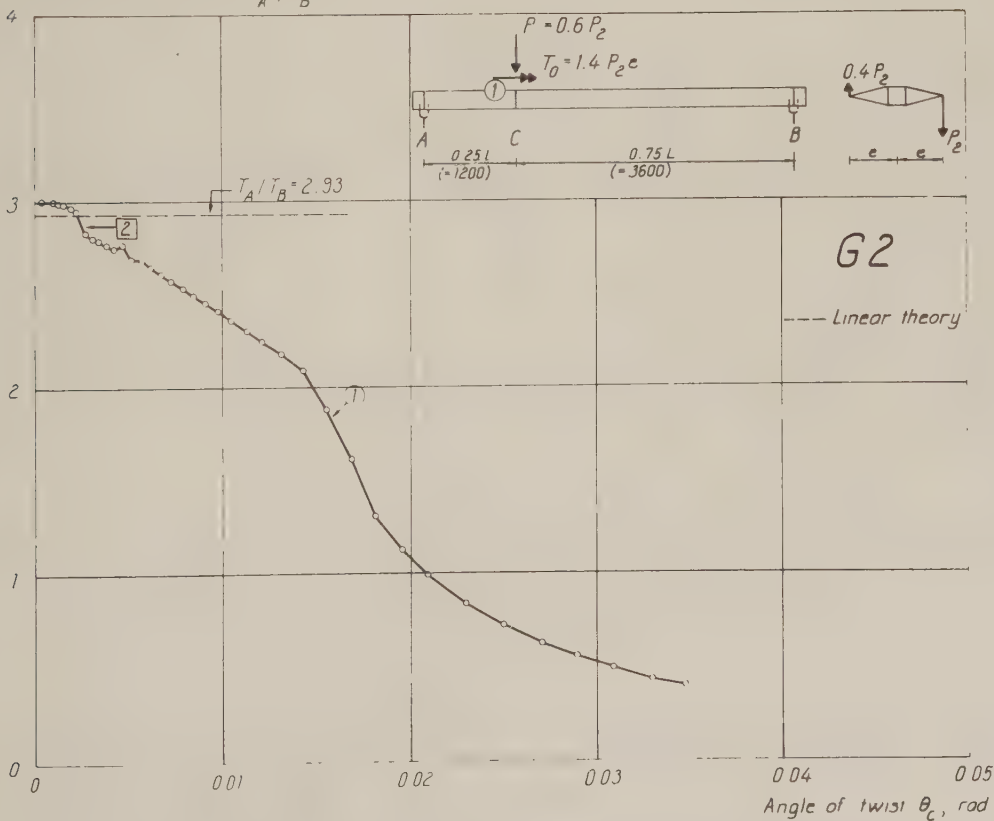
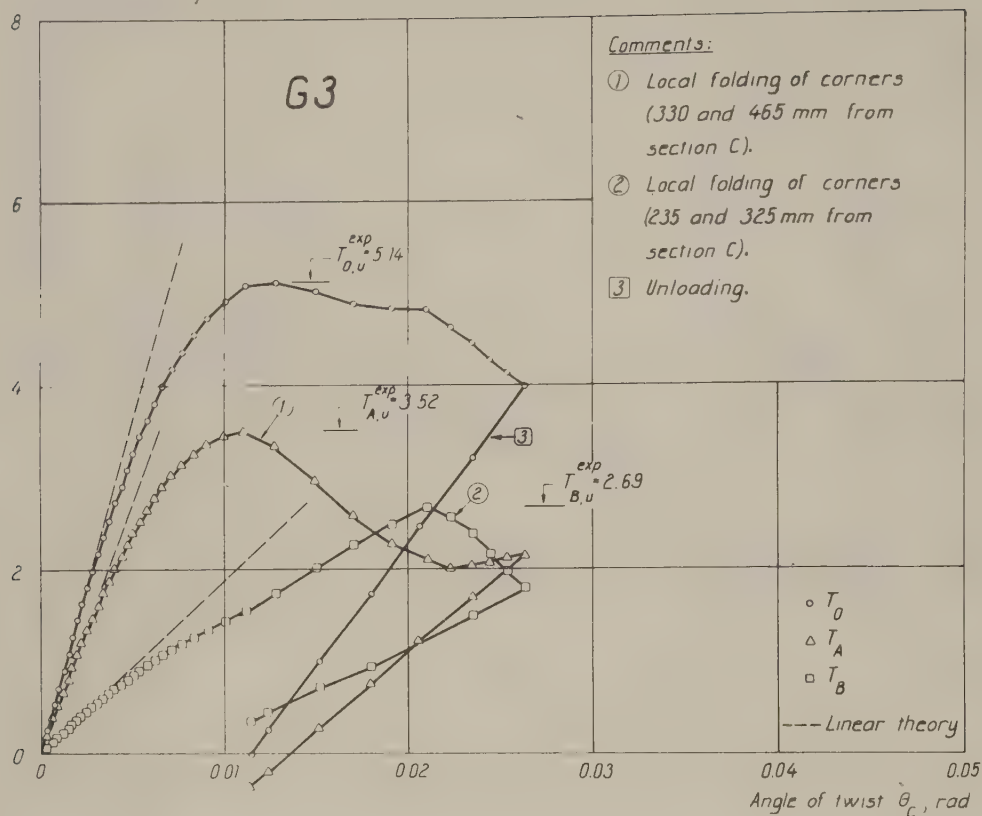
Torsional moment ratio T_A / T_B 

Fig. 5.1.4. a) Relationship between torsional moment and angle of twist for test beam G2.

b) Torsional moment ratio versus angle of twist.

a)

Torsional moment T , kNm



b)

Torsional moment ratio T_A/T_B

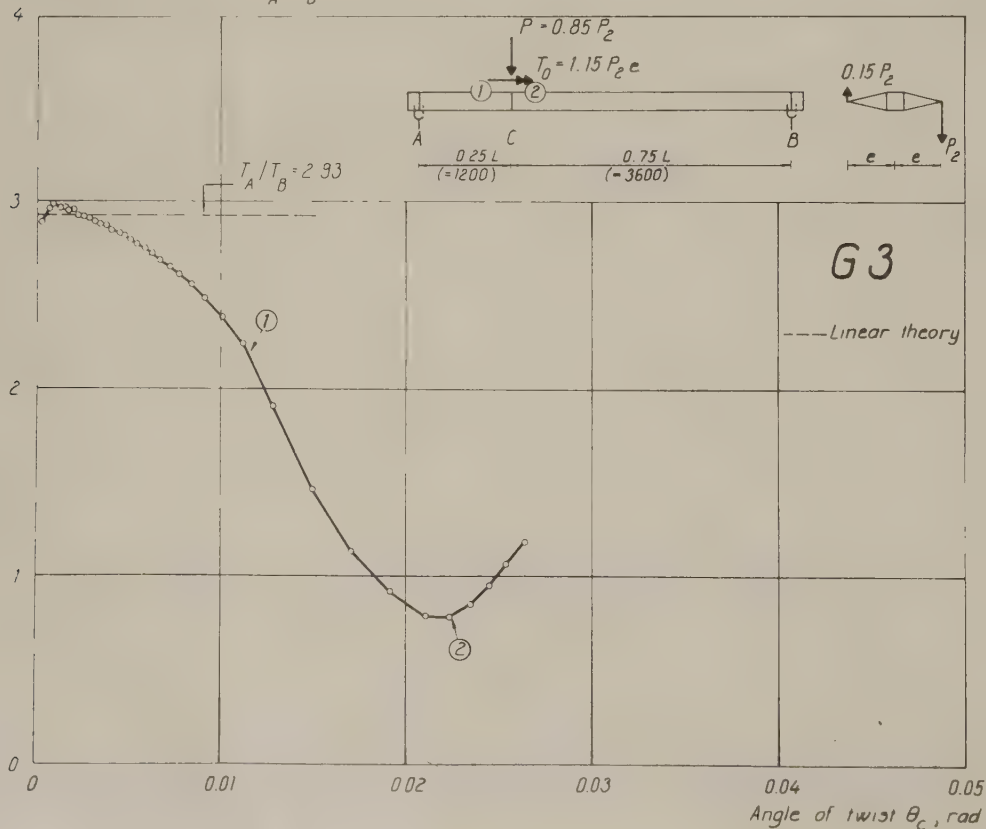
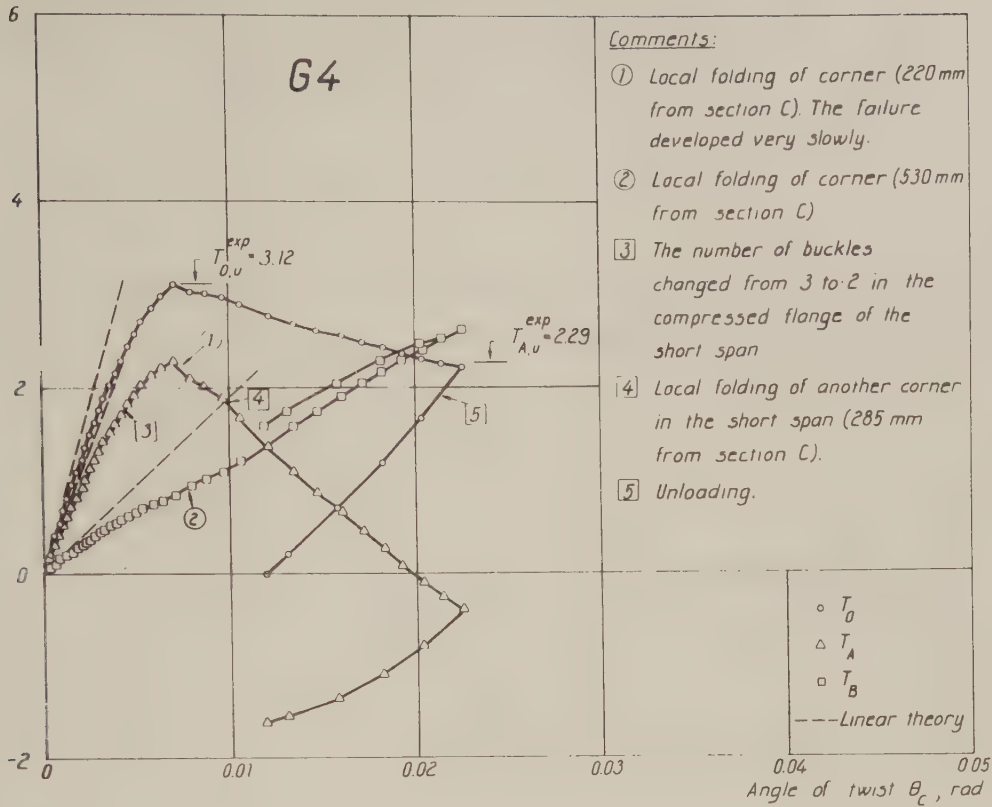


Fig. 5.1.5. a) Relationship between torsional moment and angle of twist for test beam G3.
b) Torsional moment ratio versus angle of twist.

a)

Torsional moment T , kNm

b)

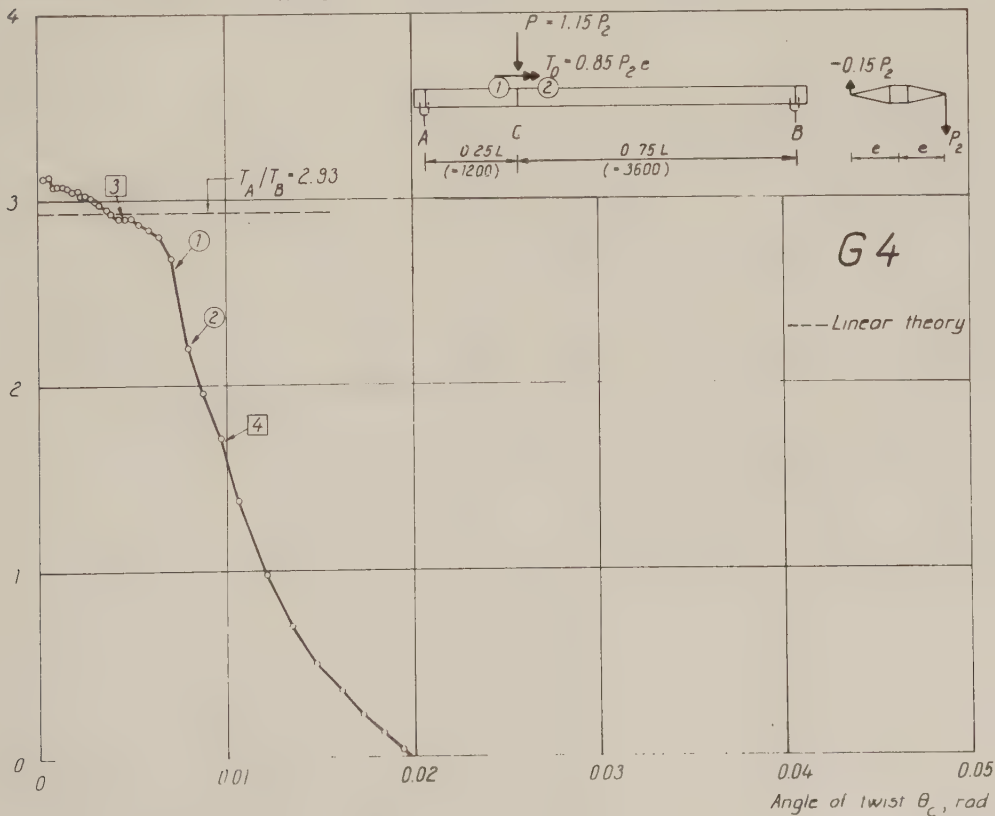
Torsional moment ratio T_A/T_B 

Fig. 5.1.6. a) Relationship between torsional moment and angle of twist for test beam G4.

b) Torsional moment ratio versus angle of twist.

measurement has been made. Thus the applied torsional moment T_0 is sometimes nearly 1 per cent greater than the sum of the moments of the supports. In the figures the position of the local failures and the angle of twist at which they occur can also be studied.

Deflections.

In Figs. 5.1.7-5.1.10 the applied load P is given as a function of the deflection v_c for test beams G1-G4.

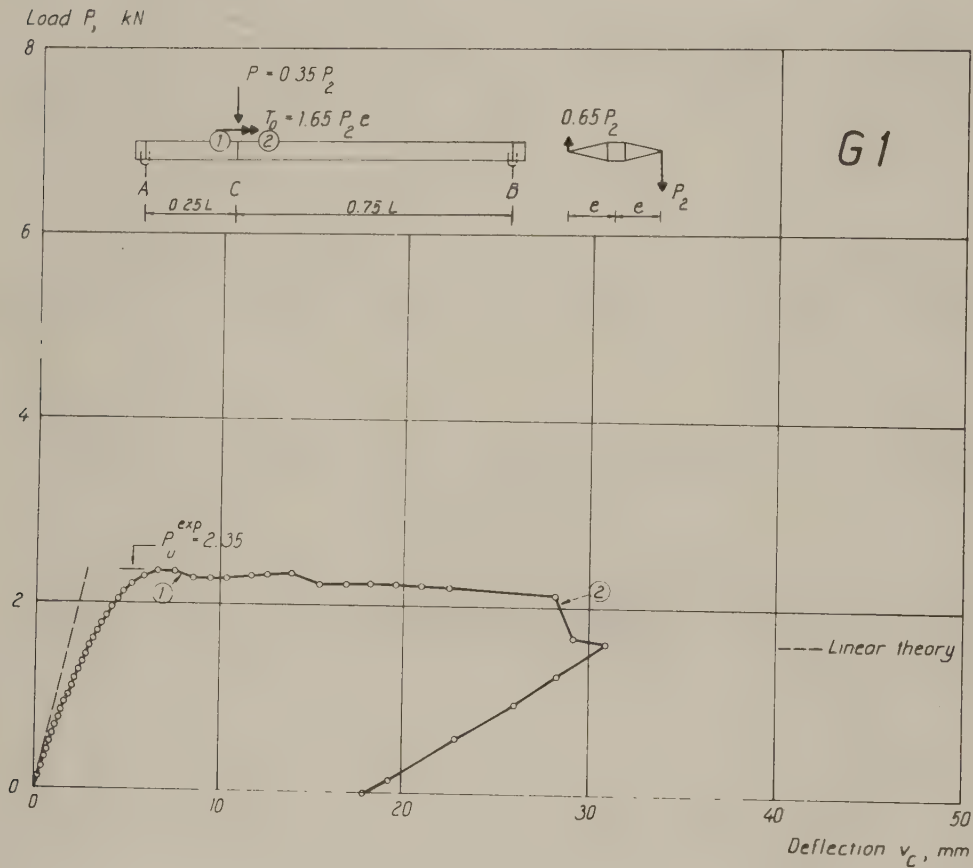


Fig. 5.1.7. Load-deflection curve for test beam G1.

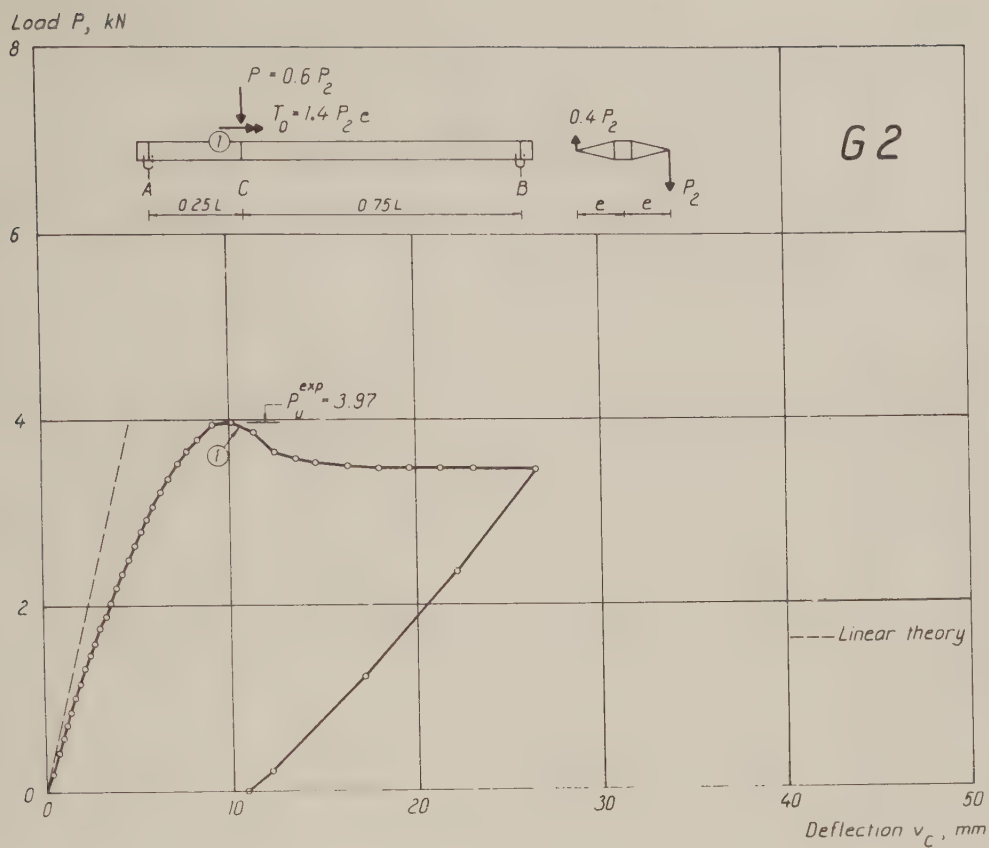


Fig. 5.1.8. Load-deflection curve for test beam G2.

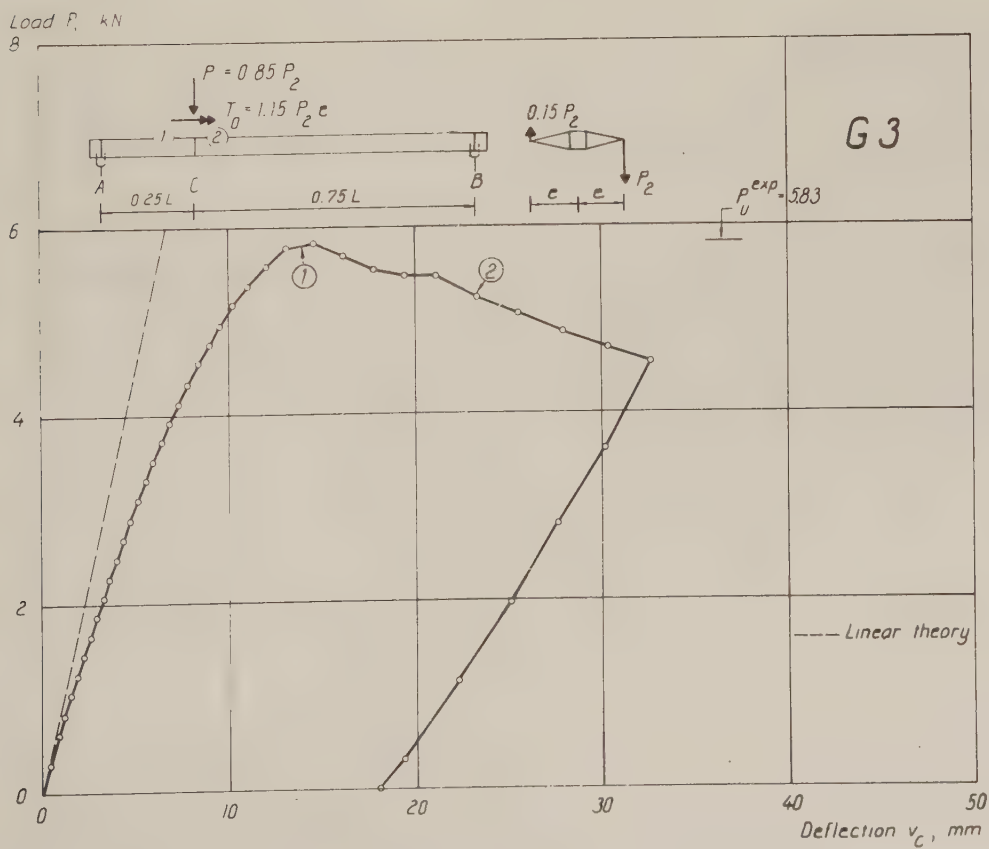


Fig. 5.1.9. Load-deflection curve for test beam G3.

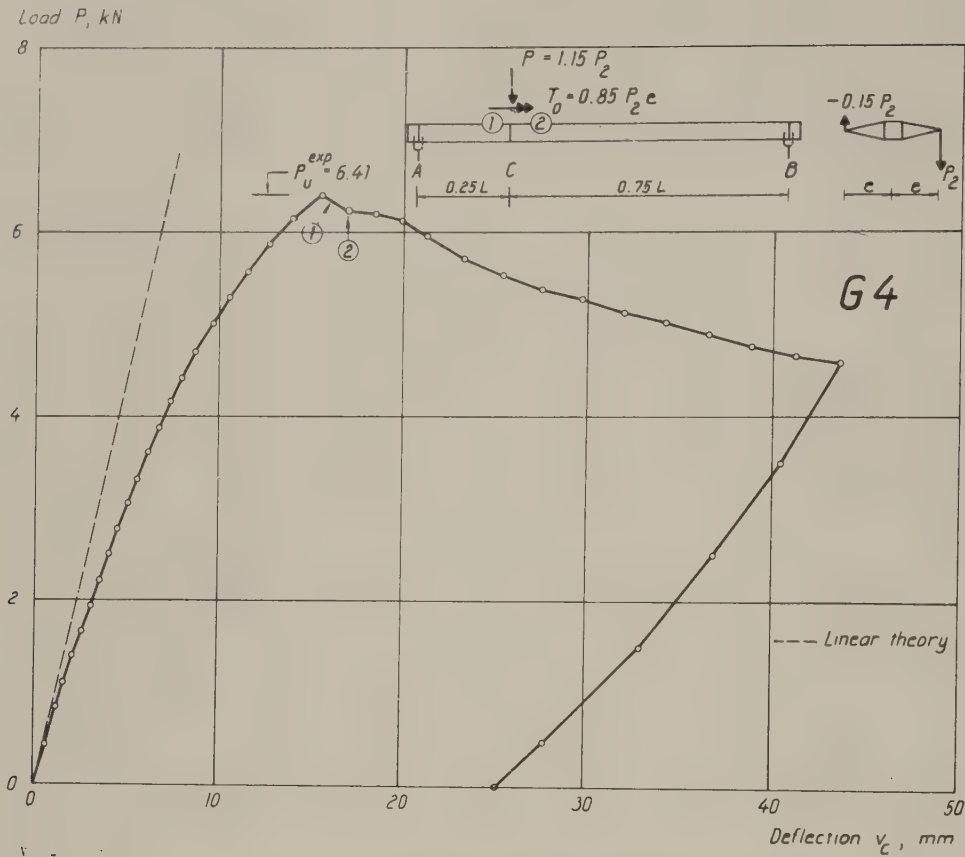


Fig. 5.1.10. Load-deflection curve for test beam G4.

Normal deflections.

The initial normal deflections of test beams G1-G4 are presented in Appendix E.

The normal deflections of some points on the compressed flange of test beam G4 are shown in Fig. 5.1.11. The jump in the curves is due to a sudden change in the number of buckles appearing in the compressed flange of the short span (cf. Fig. 5.1.6).

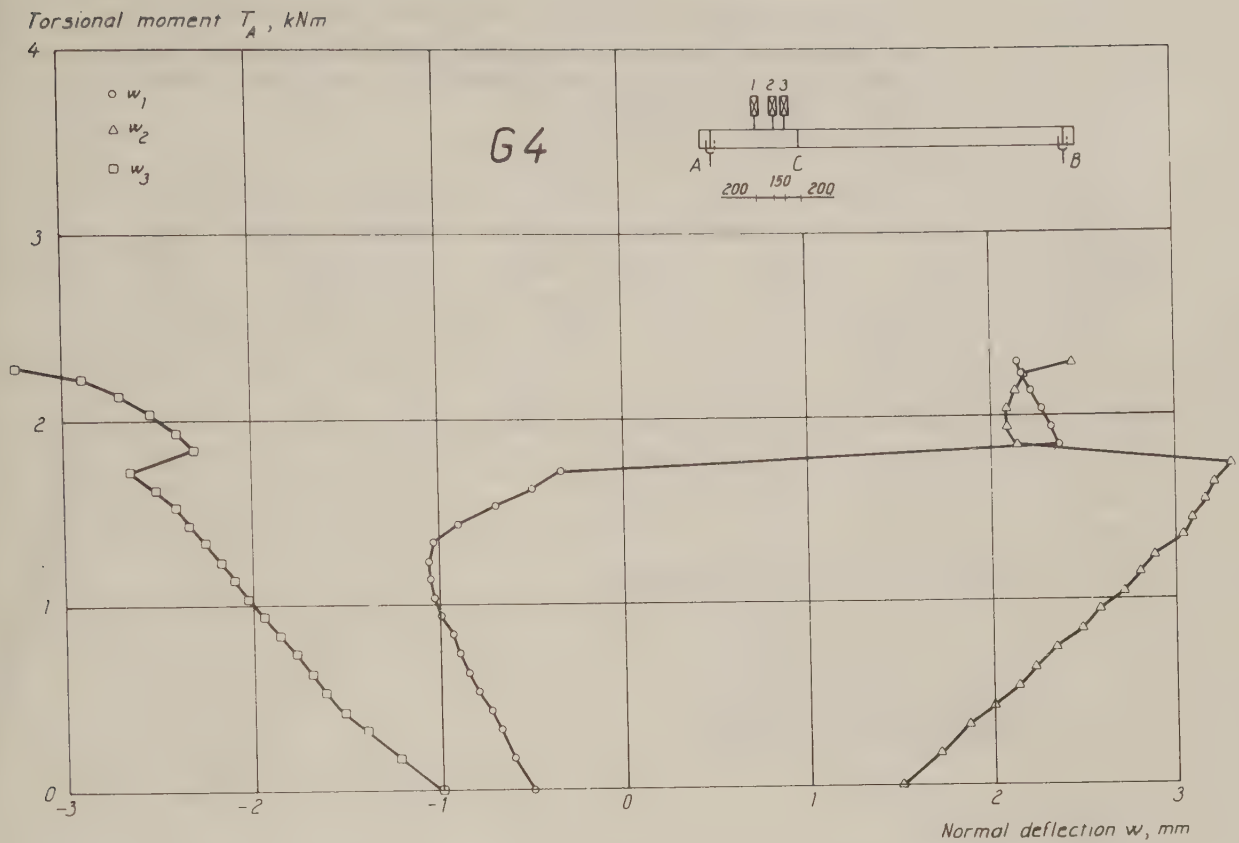


Fig. 5.1.11. Normal deflection as a function of torsional moment for some points on the compression flange of test beam G4.

Ultimate moments and ultimate forces.

The applied ultimate torsional moment $T_{O,u}^{\text{exp}}$ and the corresponding ultimate load P_u^{exp} are given in Table 5.1.4. Further, the measured internal torsional moments ($T_{A,u}^{\text{exp}}$ and $T_{B,u}^{\text{exp}}$), shear forces ($V_{A,u}^{\text{exp}}$ and $V_{B,u}^{\text{exp}}$) and bending moments ($M_{A,u}^{\text{exp}}$ and $M_{B,u}^{\text{exp}}$) are given in sections where local failure occurs. The suffixes A and B refer to the short and long span respectively. The bending moments refer to sections 150 mm to the left and to the right of section C, i.e. at a distance of half the beam height h from the outer edge of the screw joint.

Table 5.1.4. Measured applied ultimate loads and ultimate torsional moments. Measured internal torsional moments, shear forces and bending moments in sections where local failure occurs.

Test beam	$T_{0,u}^{exp}$ kNm	P_u^{exp} kN	$T_{A,u}^{exp}$ kNm	$V_{A,u}^{exp}$ kN	$M_{A,u}^{exp}$ kNm	$T_{B,u}^{exp}$ kNm	$V_{B,u}^{exp}$ kN	$M_{B,u}^{exp}$ kNm
G1	7.26	2.35	4.61	1.71	1.80	4.55	0.56	1.90
G2	6.05	3.97	4.05	2.96	3.11	-	-	-
G3	5.14	5.83	3.52	4.28	4.52	2.69	1.36	4.70
G4	3.12	6.41	2.29	4.77	5.05	0.85*	1.61*	5.53*

* The failure was not completely developed.

5.1.7 Discussion of results.

Angles of twist.

The relation between applied torsional moment and angle of twist has been determined according to the linear torsion theory according to Saint Venant (cf. Section 4.3.7). In Table 5.1.5, the elastic constants of the supports are given. The results of the calculations are shown in Figs. 5.1.3a-5.1.6a. The agreement with measured values is rather good at low loads.

Table 5.1.5. The elastic constants of the supports.

Test beam	c_A kNm/rad	c_B kNm/rad
G1-G4	6000	6000

In Fig. 5.1.12a, the measured relations between applied torsional moment and angle of twist of test beams F3 and G1-G4 are compared. Notice that there is a slight difference in cross-sectional dimensions of the beams. From the figure it can be concluded that the ultimate torsional moment and the angle of twist θ_c at ultimate load increase with decreasing proportion of bending moment.

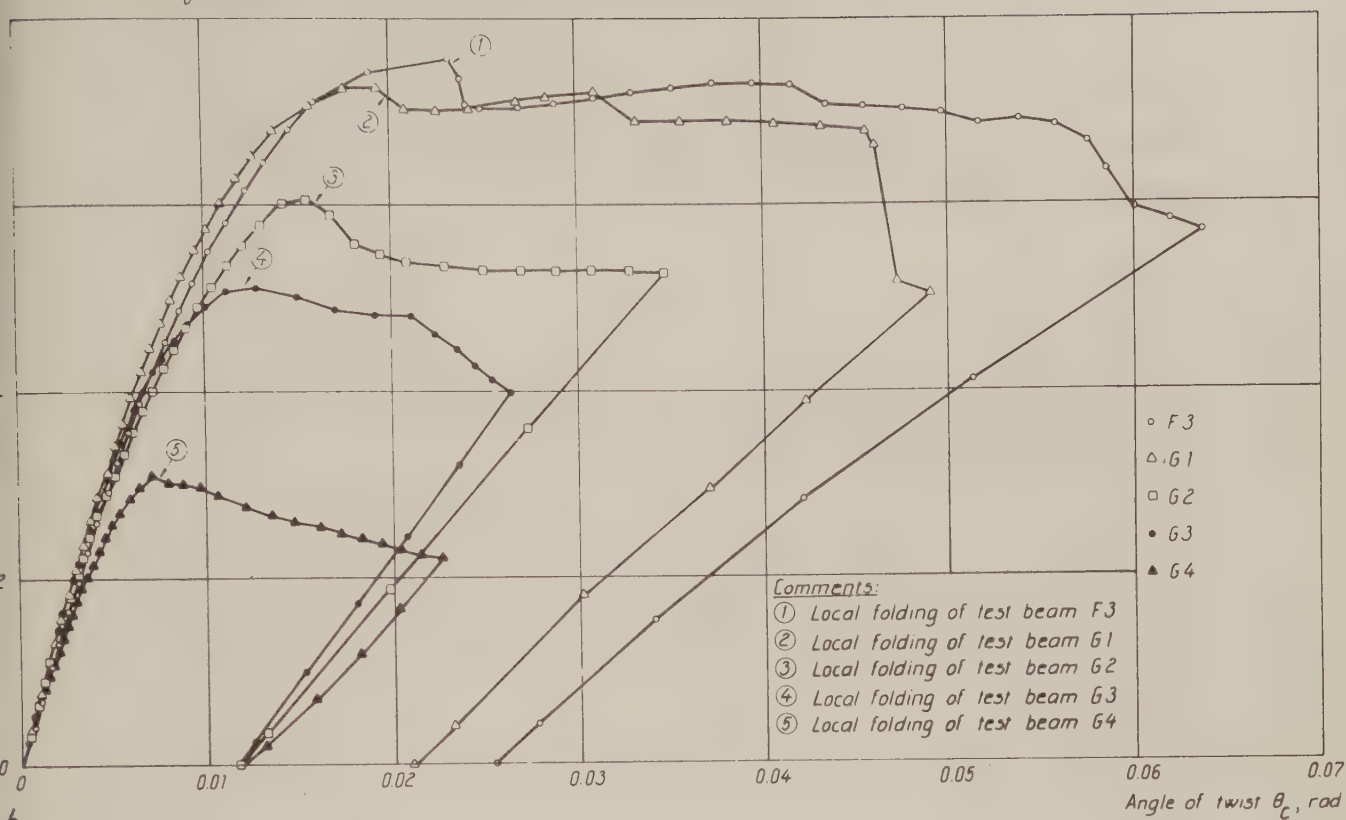
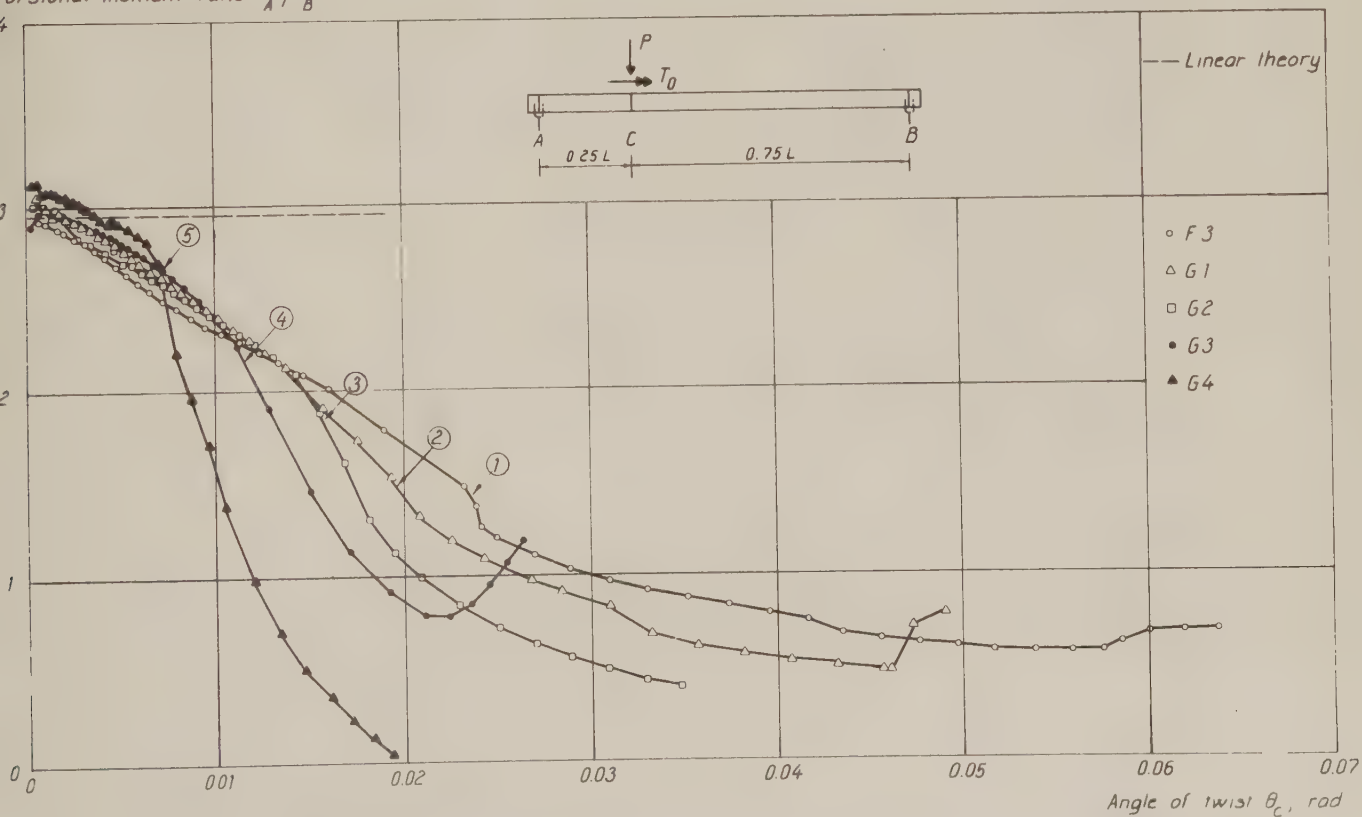
Torsional moment T_0 , kNmTorsional moment ratio T_A/T_B 

Fig. 5.1.12. a) Relationship between applied torsional moment and angle of twist for test beams F3 and G1-G4.
b) Torsional moment ratio versus angle of twist.

Further, it is evident that the curves coincide at low loads.

Deflections.

The deflections at the point of load application can be determined by simple linear theory:

$$v_C = \frac{3}{256} \frac{PL^3}{EI}$$

The influence of shear forces ($\approx 1\%$) and the extension of the screw joints have been neglected.

In Fig. 5.1.13 the load-deflection curves of the test beams G1-G4 have been collected. At low loads the curves coincide. Further, the maximum loads increase as the proportion of torsional moment is decreased.

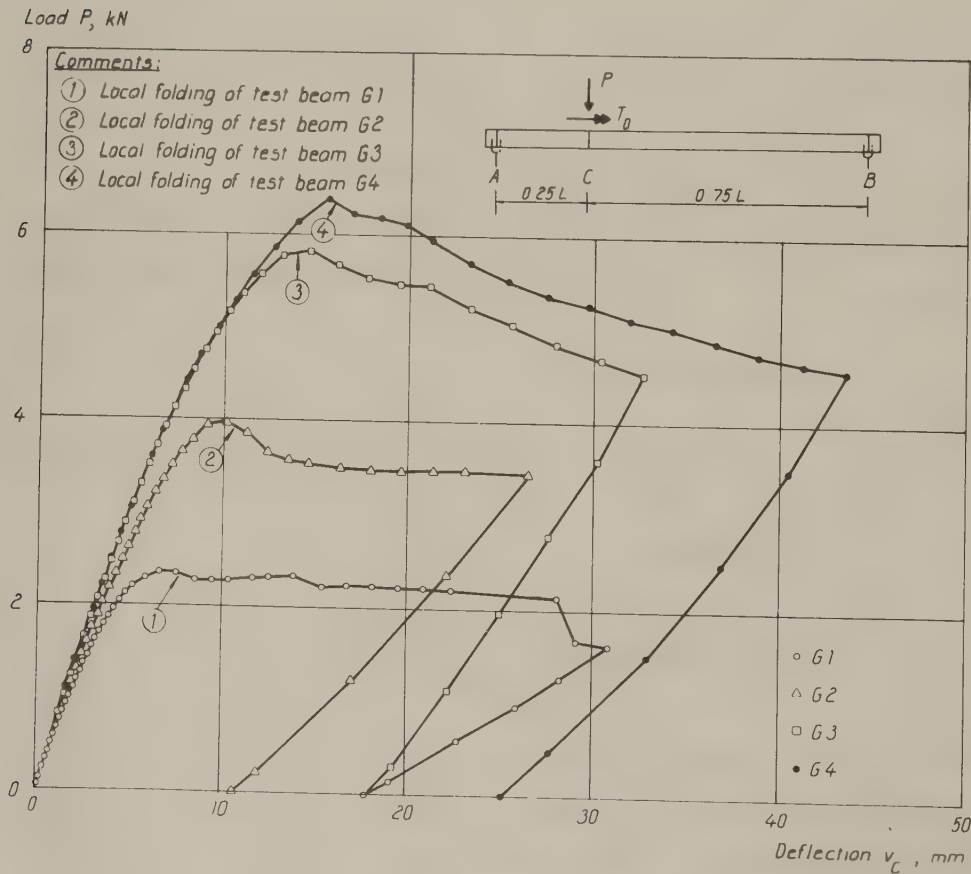


Fig. 5.1.13. Load-deflection curves for test beams G1-G4.

Torsional moment ratios.

In Figs. 5.1.3b-5.1.6b the ratio of the torsional moments of the supports is shown as a function of the angle of twist. The ratio has been theoretically determined according to the previously described linear theory. For further discussion of results see Section 4.3.7.

For test beams F3 and G1-G4, the variation of the torsional moment ratio with reference to the angle of twist is shown in Fig.

5.1.12b. Since the load-carrying capacity of a beam in practice reaches its maximum immediately before the first local failure appears, it is interesting to study how much the torsional moments have been redistributed on this occasion. Thus in Fig. 5.1.12b the point at which the first local failure of each test beam appears is marked. From this, it is clearly seen that the ability of torsional moment redistribution to occur before the first local failure appears decreases when the proportion of bending moment is increased.

Character of failure.

During the testing of beam F3, a beam identical to the G series but subjected to pure torsion, two local failures occurred, one in the short span and one in the long. If a beam from the G series had been subjected to pure bending only, one local failure would have appeared just to the right of point C in the long span. It would thus be expected that only one failure would occur in test beams loaded mainly in bending. This was confirmed by test beam G4, where the failure in the long span was never completely developed.

For test beam G2, only one failure was obtained. If the deformation had been further increased it is probable that another failure would have occurred.

In the short span, the failures in test beams G1-G4 developed rather slowly. In the long span, the failure of test beam G1 was rapid while the failure of test beam G3 developed slowly.

All failures occurred as a folding of the corners in the compressed flange. Depending on the relationship between bending and torsion, the local foldings were in the transverse direction under mainly bending and in a more longitudinal direction under mainly torsion. A typical example of local failure is shown in Fig. 5.1.14.

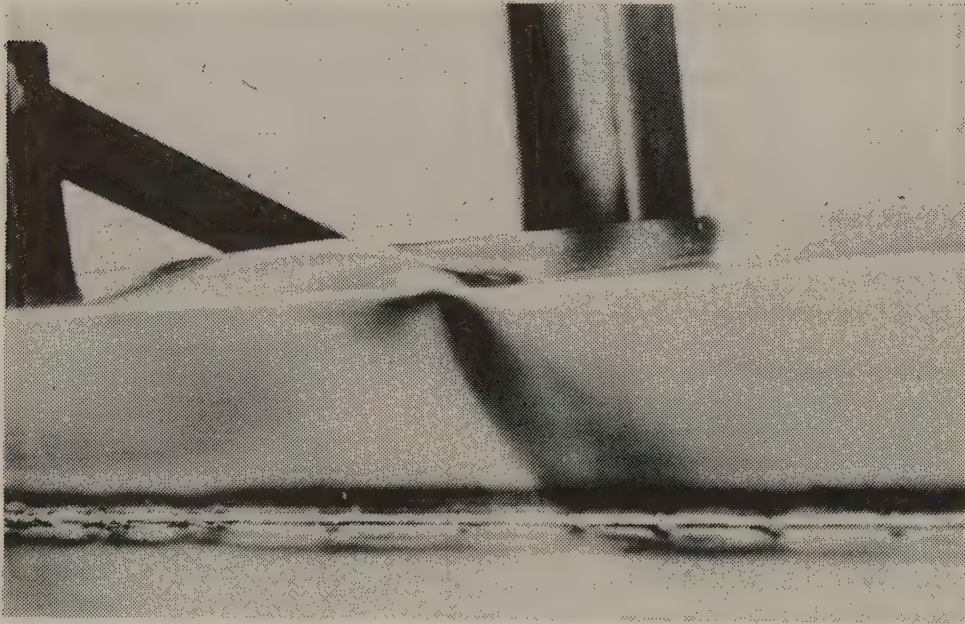


Fig. 5.1.14. Typical example of local failure.

Ultimate moments and ultimate forces.

In Table 5.1.6, calculated ultimate moments for beam sections subjected to pure bending or pure torsion are given. The theoretical determination of ultimate bending moments has been performed in accordance with the procedure given in Section 3.1.14 with effective width according to Winter and with "reduced" webs. To calculate the ultimate torsional moment for the different test beams, Eq. (4.1.30) has been used with the assumption of long spans.

The ratios of measured to calculated ultimate moments have been determined at sections of local failure. The calculated ultimate torsional moment in the short span has not been corrected with reference to the length of the span (cf. Section 4.3.7). The re-

sults are given in Table 5.1.7 and are also shown in Fig. 5.1.15. The relation can be described rather well by a circle.

Table 5.1.6. Calculated ultimate moments for beam section subjected to pure bending or pure torsion.

Test beam	M_u^{th} kNm	T_u^{th} kNm
G1	6.24	4.70
G2	5.96	4.47
G3	6.26	4.71
G4	6.20	4.65

Table 5.1.7. Ratios of measured to calculated ultimate moments at sections of local failure.

Test beam	$\frac{T_{A,u}^{exp}}{T_u^{th}}$	$\frac{M_{A,u}^{exp}}{M_u^{th}}$	$\frac{T_{A,u}^{exp} + bV_{A,u}^{exp}}{T_u^{th}}$	$\frac{T_{B,u}^{exp}}{T_u^{th}}$	$\frac{M_{B,u}^{exp}}{M_u^{th}}$	$\frac{T_{B,u}^{exp} + bV_{B,u}^{exp}}{T_u^{th}}$
G1	0.98	0.29	0.96	0.97	0.30	0.99
G2	0.91	0.52	0.95	-	-	-
G3	0.75	0.72	0.85	0.57	0.75	0.63
G4	0.49	0.81	0.64	0.18	0.89	0.25

The following discussion provides a basis for an analysis of the influence of the shear forces. Assuming the shear stresses τ_T of the torsional moment T to be uniformly distributed along the contour according to Fig. 5.1.16a, we obtain

$$\tau_T = \frac{T}{2bht}$$

Further, by assuming the shear stresses τ_V originating from the shear force V to be uniformly distributed along the webs (Fig. 5.1.16b), we obtain

$$\tau_V = \frac{V}{2ht}$$

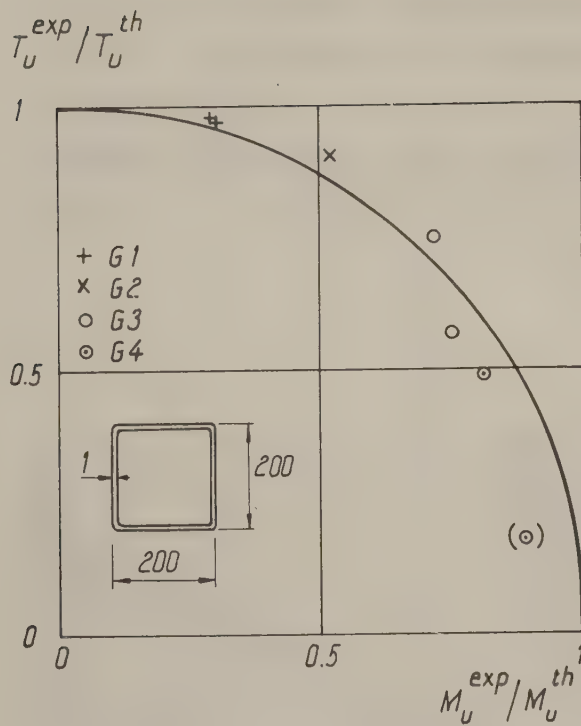


Fig. 5.1.15. Relationship between ultimate torsional moments and ultimate bending moments at sections of local failure. Test beams G1-G4.

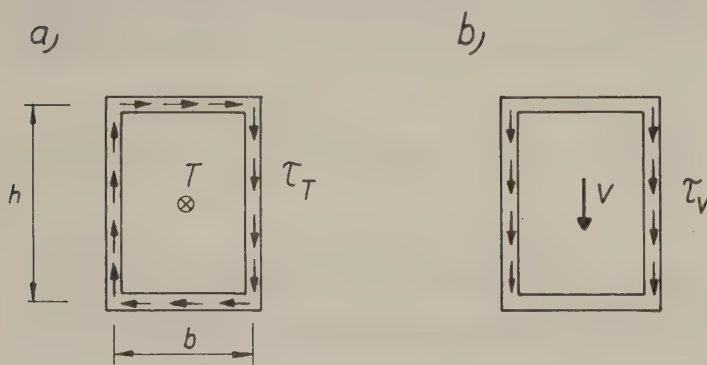


Fig. 5.1.16. Shear stresses in box beam section.

- Shear stresses originating from torsional moment T .
- Shear stresses in the webs originating from the shear force V .

Thus the shear stress in the most stressed web will be

$$\tau = \tau_T + \tau_V = (T + bV) \frac{1}{2bht}$$

We now can define a failure condition for the case of combined

torsional moment and shear force namely $\tau = \tau_{\max}$ where $\tau_{\max}$ is the maximum average stress given by Eq. (4.1.30).

Considering the influence of the shear forces as described above by forming the quotient between $T_u^{\text{exp}} + bV_u^{\text{exp}}$ and T_u^{th} we obtain the results given in Table 5.1.7 and shown in Fig. 5.1.17. In this case the theoretical ultimate torsional moment T_u^{th} of the short span has been increased by 9.2 per cent (cf. the results concerning test beam F3). From Fig. 5.1.17 we conclude that a circle may be used to describe the relation between the ultimate forces. If this circle is used for design purposes it will probably mean that the obtained dimensions are on the safe side with respect to required ultimate strength of the box beams.

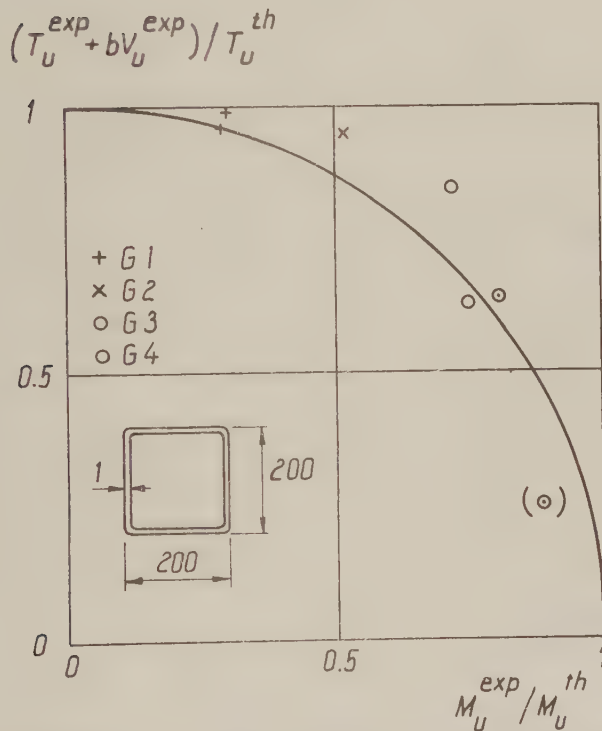


Fig. 5.1.17. Relationship between ultimate torsional moments (modified) and ultimate bending moments at sections of local failure. Test beams G1-G4.

To obtain some idea of the influence of the vertical point load on the external ultimate torsional moment, these two quantities have been plotted against each other in Fig. 5.1.18. In this case they have been divided by the ultimate load under pure bending

P_u^{th} and the ultimate torsional moment under pure torsion $T_{o,u}^{th}$ respectively. The ultimate load under pure bending P_u^{th} has been calculated according to the procedure described in Section 4.1.14 assuming the local failure to occur 150 mm to the right of section C. The ultimate torsional moments under pure torsion have been obtained by multiplying the ultimate torsional moments in Table 5.1.6 by the quotient of $T_{o,u}^{exp}$ to $T_{B,u}^{exp}$ for test beam F3. The actual values are given in Table 5.1.8. From Fig. 5.1.18 it can be seen that the relationship between the external ultimate torsional moment and the applied ultimate point load can be described rather well by a circle.

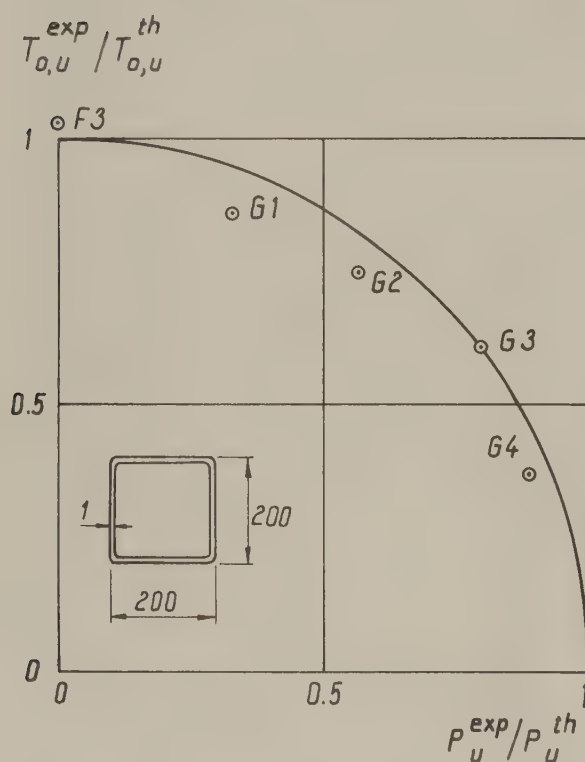


Fig. 5.1.18. Relationship between applied ultimate torsional moment and ultimate point load for test beams F3 and G1-G4.

Table 5.1.8. Calculated external ultimate torsional moments and ultimate loads for a box beam under pure torsion or pure bending. Actual torsional moment ratios and load ratios.

Test beam	$T_{o,u}^{th}$	P_u^{th}	$\frac{T_{o,u}^{exp}}{T_{o,u}^{th}}$	$\frac{P_u^{exp}}{P_u^{th}}$
	kNm	kN		
G1	8.44	7.23	0.86	0.33
G2	8.03	6.91	0.75	0.57
G3	8.46	7.26	0.61	0.80
G4	8.35	7.19	0.37	0.89

5.2 Two-span beam subjected to torsion and bending, where the bending predominates.

5.2.1 Purpose.

Experiments with two-span beams subjected to pure bending were described in Section 3.4. A natural extension of the investigation was to study the behaviour of two-span beams subjected to both bending and torsion where the bending action was dominant.

5.2.2 Scope.

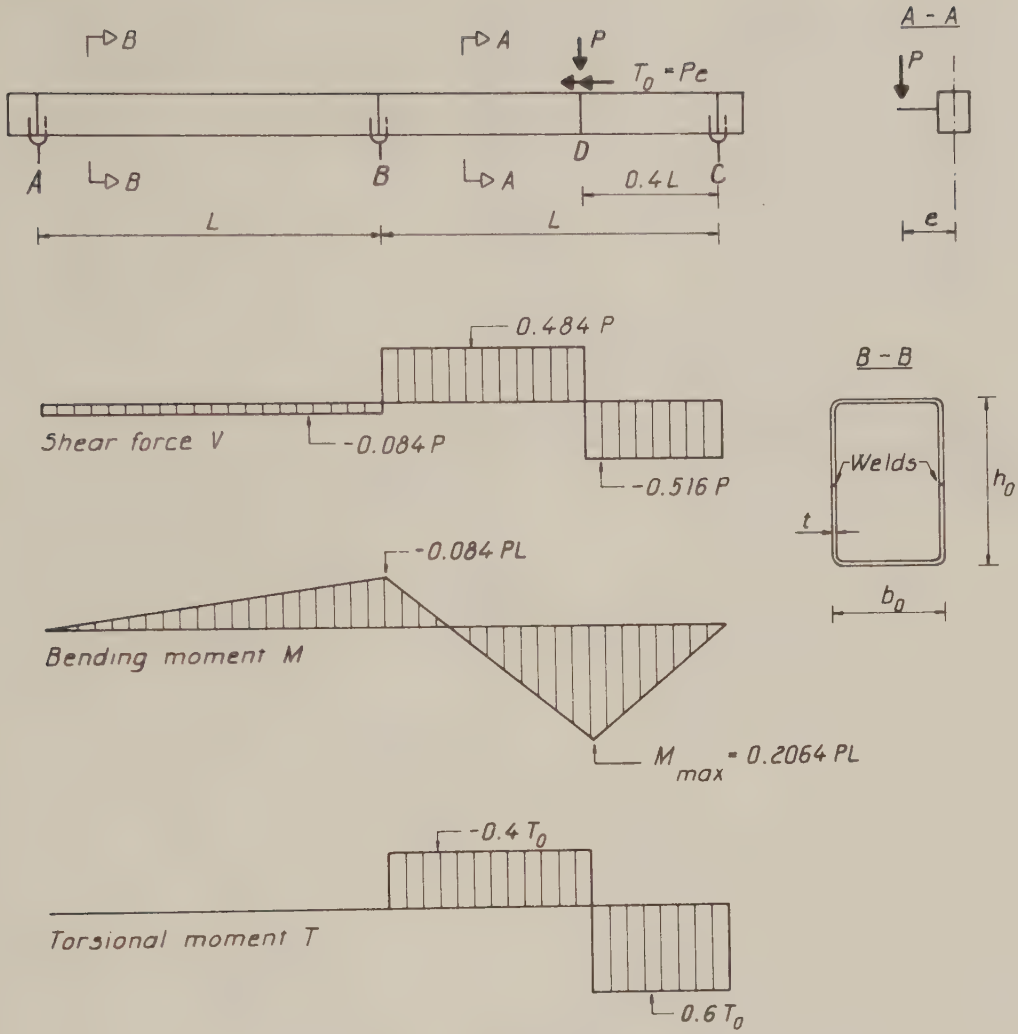
The experiments comprised a series of five test beams H1-H5.

In Fig. 5.2.1a, the distribution of load for test beams H1-H3 is shown. In the same figure, the calculated distribution of shear force, bending moment and torsional moment according to linear theory also can be seen. The corresponding quantities for test beams H4 and H5 are presented in Fig. 5.2.1b. The nominal dimensions of test beams H1-H5 are given in Table 5.2.1.

Table 5.2.1. Nominal dimensions of test beams H1-H5.

Test beam	L mm	e mm	b _o mm	h _o mm	t mm
H1	3480	200	133	200	1
H2	3480	400	133	200	1
H3	3480	750	133	200	1
H4	3480	400	133	200	1
H5	3480	750	133	200	1

a)



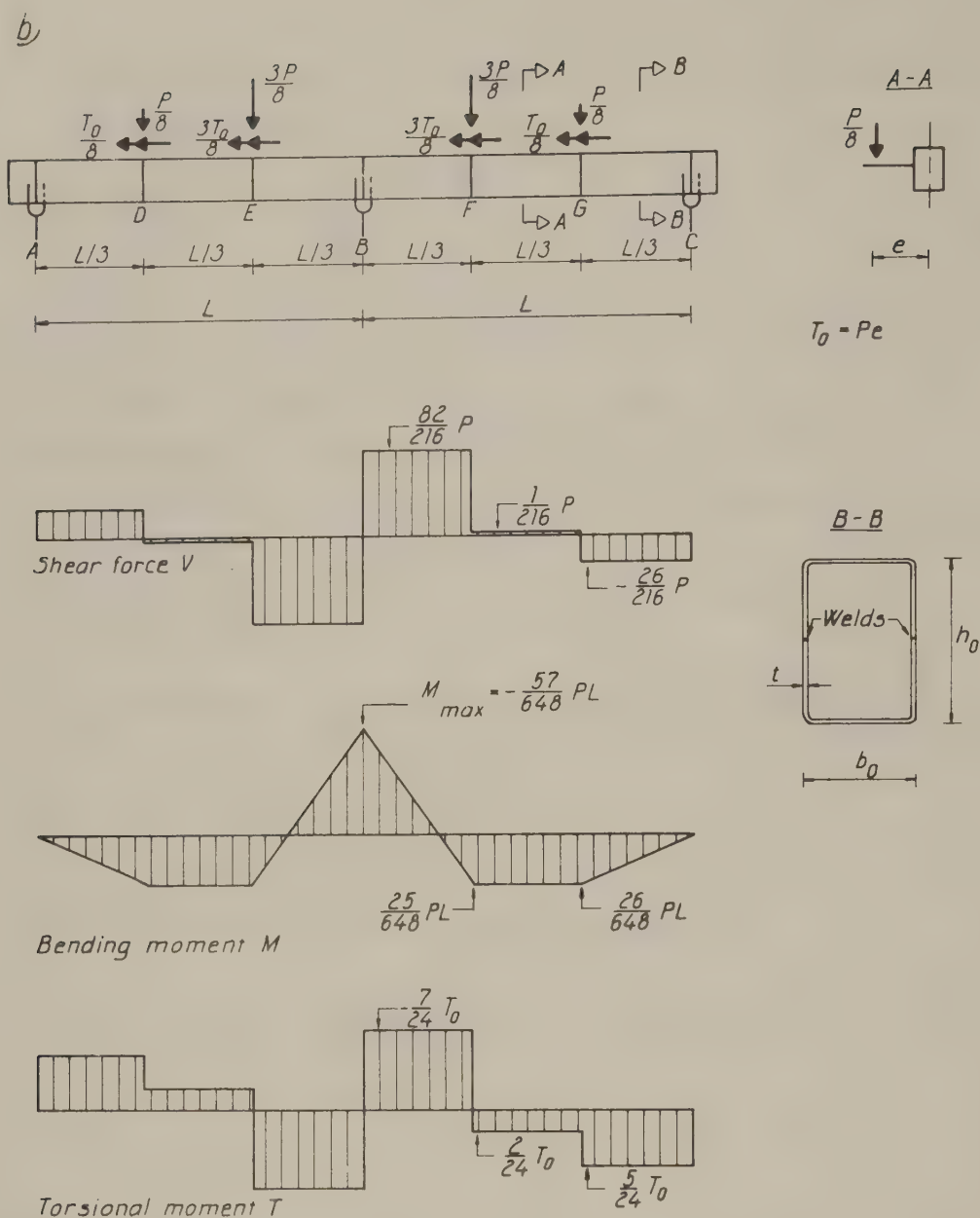


Fig. 5.2.1. Distribution of load. Calculated distribution of shear force, bending moment and torsional moment according to linear theory. Notations for cross section.

a) Test beams H1-H3.

b) Test beams H4-H5.

5.2.3 Loading and support arrangements.

The principal load and support arrangements for test beam H1 are shown in Fig. 5.2.2. The test rig was designed to satisfy, as closely as possible, the following boundary conditions for the box beam, namely that

- 1) all supports are restrained from twisting,
- 2) the box beams are simply supported with regard to vertical bending.

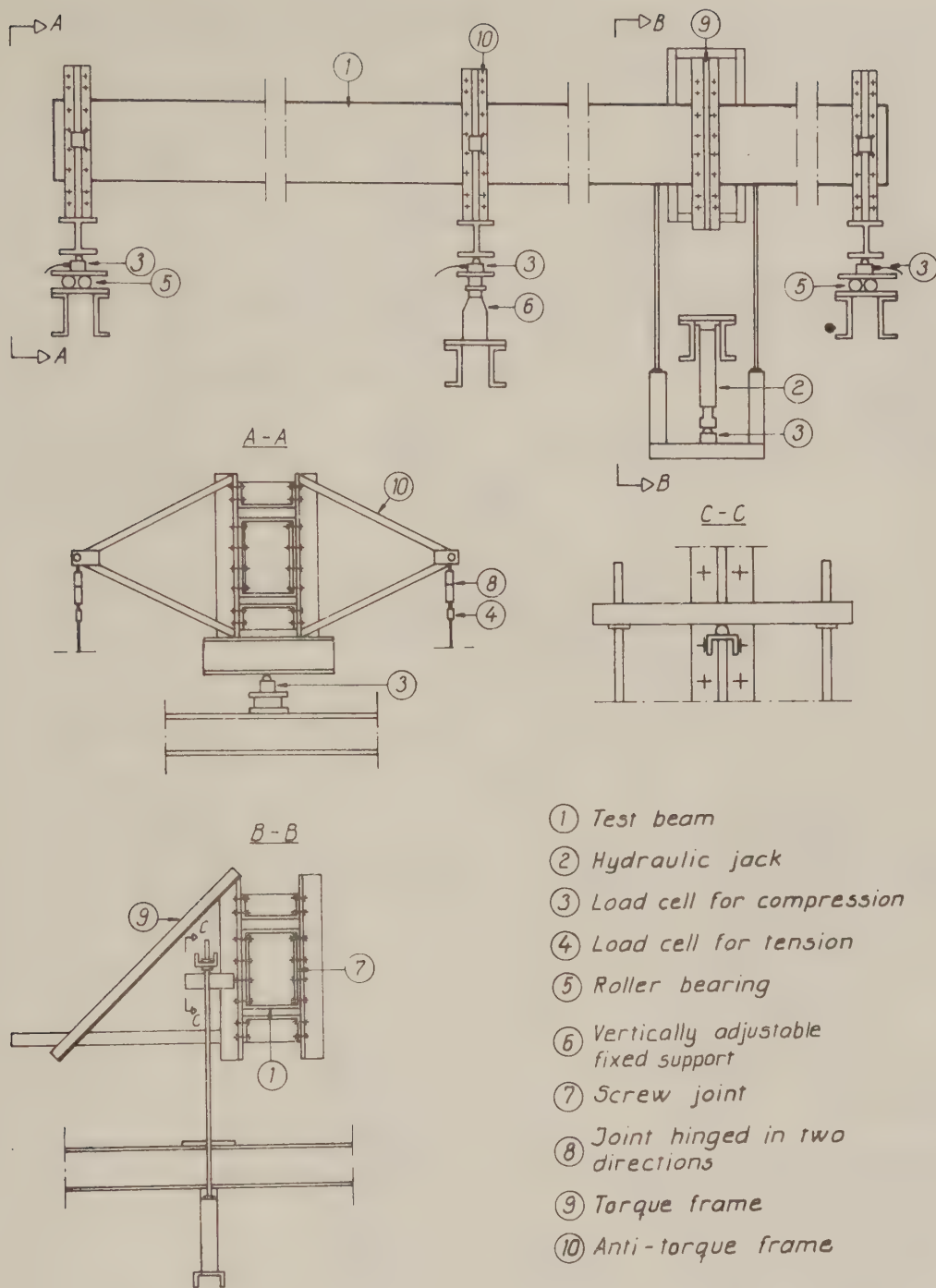


Fig. 5.2.2. Principal loading and support arrangements for test beam H1.

The eccentric force on test beam H1 was generated by a hydraulic jack and transferred to the test beam by a torque frame. At the end supports, anti-torque frames rested on roller supports. An anti-torque frame with a vertically adjustable fixed support was used at the middle support.

The load arrangement was somewhat changed for test beams H2 and H3 but the principal was the same.

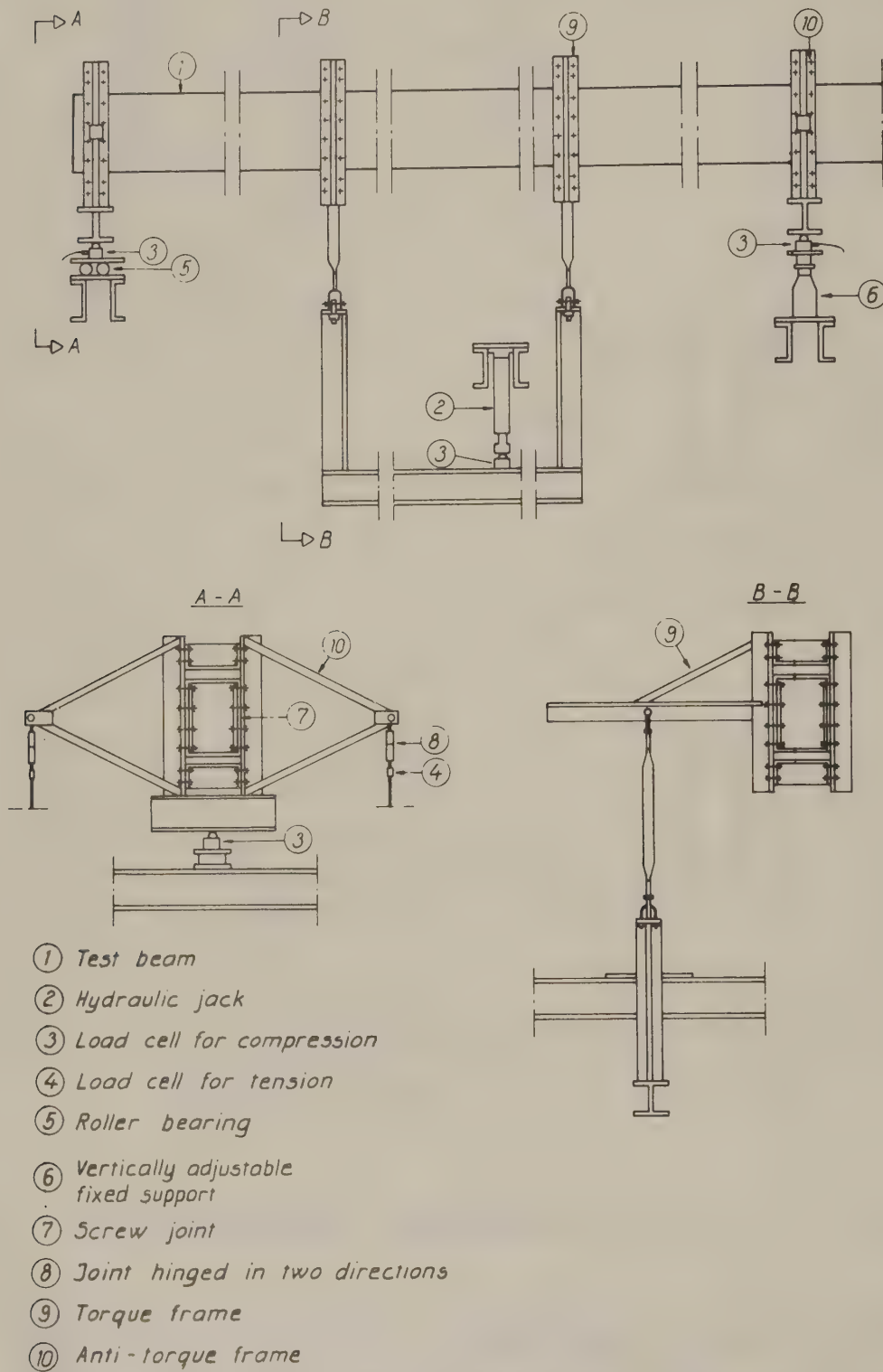


Fig. 5.2.3. Principal loading and support arrangements for test beams H4 and H5.

The load and support arrangements for test beams H4 and H5 are shown in Fig. 5.2.3. The eccentric forces were generated by two jacks connected to the same hydraulic pump. With regard to the support arrangement the test rig was identical to that used for test beams H1-H3.

For all the test beams H1-H5 the flanges were free.

5.2.4 Dimensions. Material properties.

The manufacture and material properties of test beams H1-H5 were described in Section 3.4.4.

Mean values of the measured cross-sectional dimensions of the test beams are given in Table 5.2.2. The lengths of the beams were in very good agreement with the nominal lengths given in Table 5.2.1. Cross-sectional constants and critical stresses, defined according to Appendix A, are given in Table 5.2.3.

Table 5.2.2. Measured cross-sectional dimensions of test beams H1-H5.

Test beam	b_o mm	h_o mm	t mm	R mm
H1	134.4	197.6	1.037	3
H2	134.0	197.2	1.027	3
H3	134.3	197.4	1.043	3
H4	136.0	197.1	1.052	3
H5	136.0	197.1	1.040	3

Table 5.2.3. Cross-sectional constants and critical stresses for test beams H1-H5.

Test beam	b/t	h/t	$I_O \cdot 10^{-4}$ mm ⁴	$I_T \cdot 10^{-4}$ mm ⁴	$\sigma_{cr,f}$ N/mm ²	$\sigma_{cr,w}$ N/mm ²	σ_{cr} N/mm ²	$\tau_{cr,f}$ N/mm ²	$\tau_{cr,w}$ N/mm ²
H1	128.6	189.6	398	432	45.0	123.9	59	60.1	27.7
H2	129.5	191.0	392	425	44.4	122.0	58	59.3	27.2
H3	127.8	188.3	400	433	45.6	125.6	59	60.9	28.1
H4	128.3	186.4	405	445	45.2	128.1	59	60.4	28.6
H5	129.8	188.5	400	440	44.2	125.2	57	59.0	28.0

5.2.5 Measuring devices and measurement procedure.

In these measurements the computer-controlled data acquisition system described in Appendix D was used. The measurements comprised the determination of applied loads and support reactions, the measurement of deflection and angle of twist of the test beams and the recording of normal deflections at separate points where the flanges were in compression.

In view of "creep" effects under high loads, attempts were made to ensure that the deformation was maintained constant during every load step. Since the test beams were subjected to vertical displacements as well as rotations, "constant deformation" was defined as constant deflection of the test beam.

Measurements were made three times during every load step, namely immediately after load application, ten seconds after load application and just before the next load step.

Load measurement.

See Section 4.3.5.

Positions and numbering of load cells for test beam H5 are shown in Fig. 5.2.4. The applied loads acting on the load cells numbered 1 and 2 were recorded as continuous functions of time.

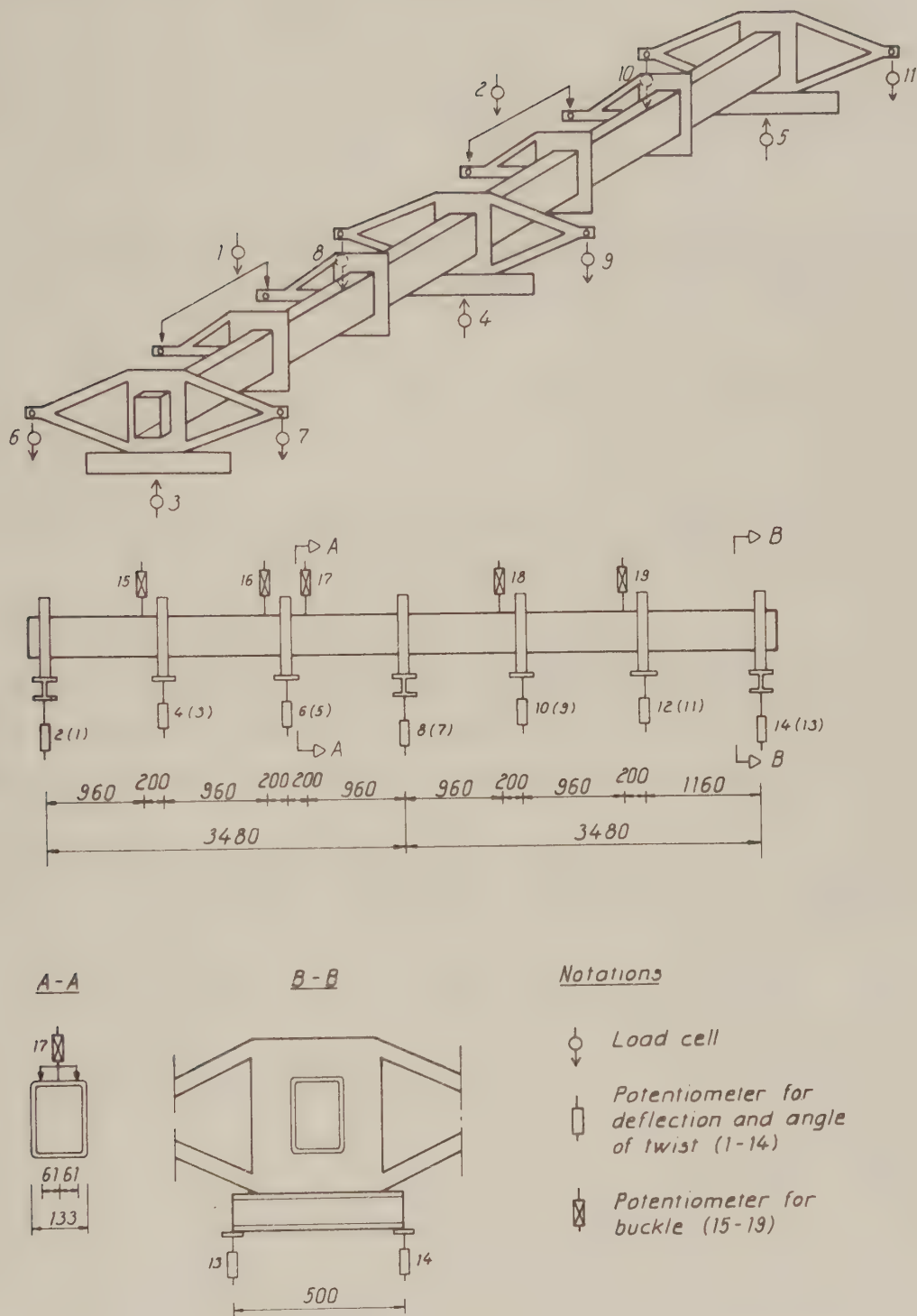


Fig. 5.2.4. Positions and numbering of load cells and potentiometers for test beam H5.

Measurement of deflection and angle of twist.

See Section 4.3.5.

Positions and numbering of potentiometers for test beam H5 are shown in Fig. 5.2.4.

Measurement of normal deflections.

See Section 4.3.5.

Positions and numbering of potentiometers for determination of normal deflections for test beam H5 are shown in Fig. 5.2.4.

5.2.6 Results.

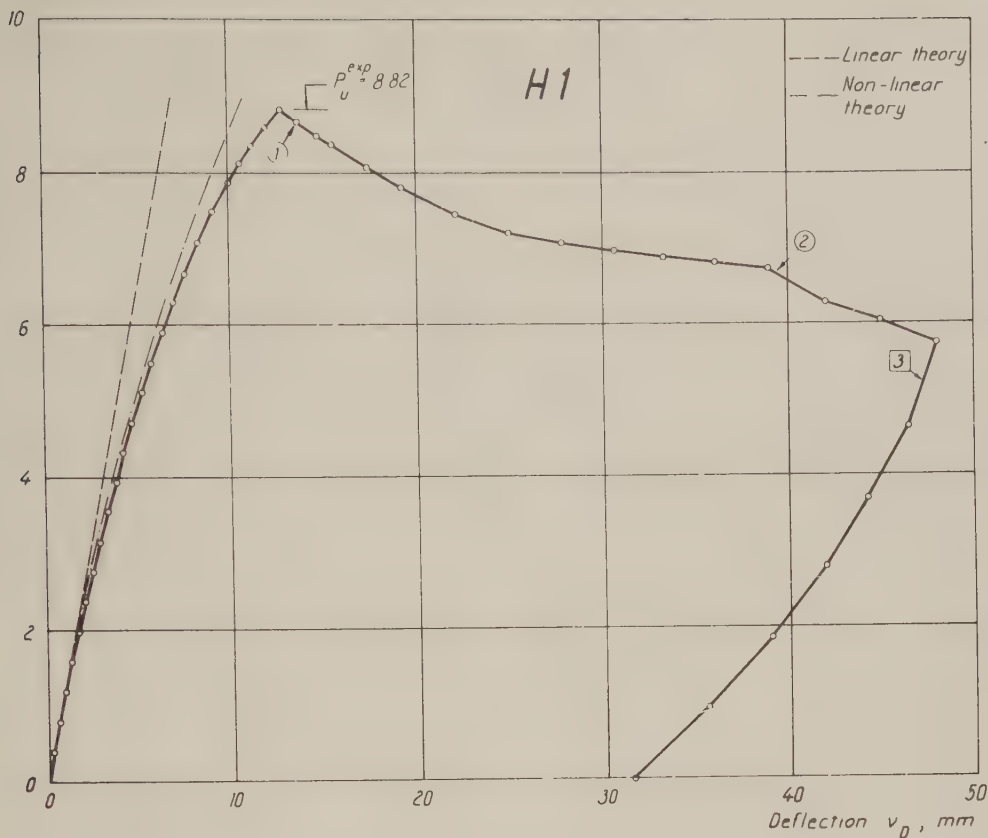
Deflections.

For test beams H1-H3 the relationship between the vertical load P and the deflection v_D is shown in Figs. 5.2.5a-5.2.7a.

In Figs. 5.2.8 a, b and 5.2.9 a, b, the deflection at the point loads is given as a function of the total applied loads for test beams H4 and H5. In order to obtain a third local failure the jack in the left span was locked after the second failure had occurred.

In Figs. 5.2.5-5.2.9, position and character of failure are also described.

a,

Load P kN

Comments:

- ① Local folding of corners (160 and 175 mm from section D). The failure developed slowly.
- ② Local folding of corners (190 and 195 mm from support B). The failure developed slowly.
- ③ Unloading

b,

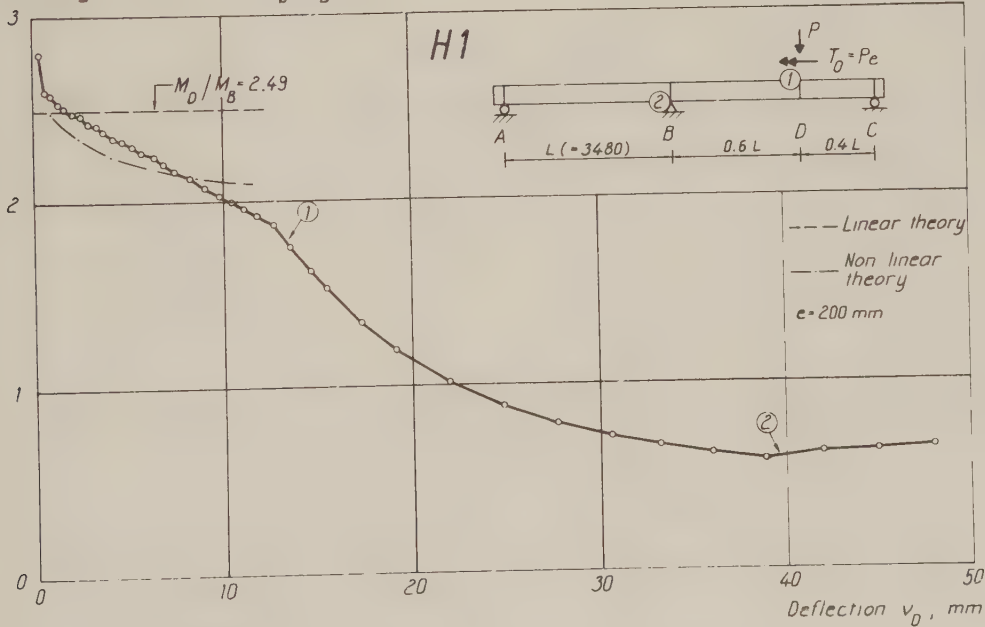
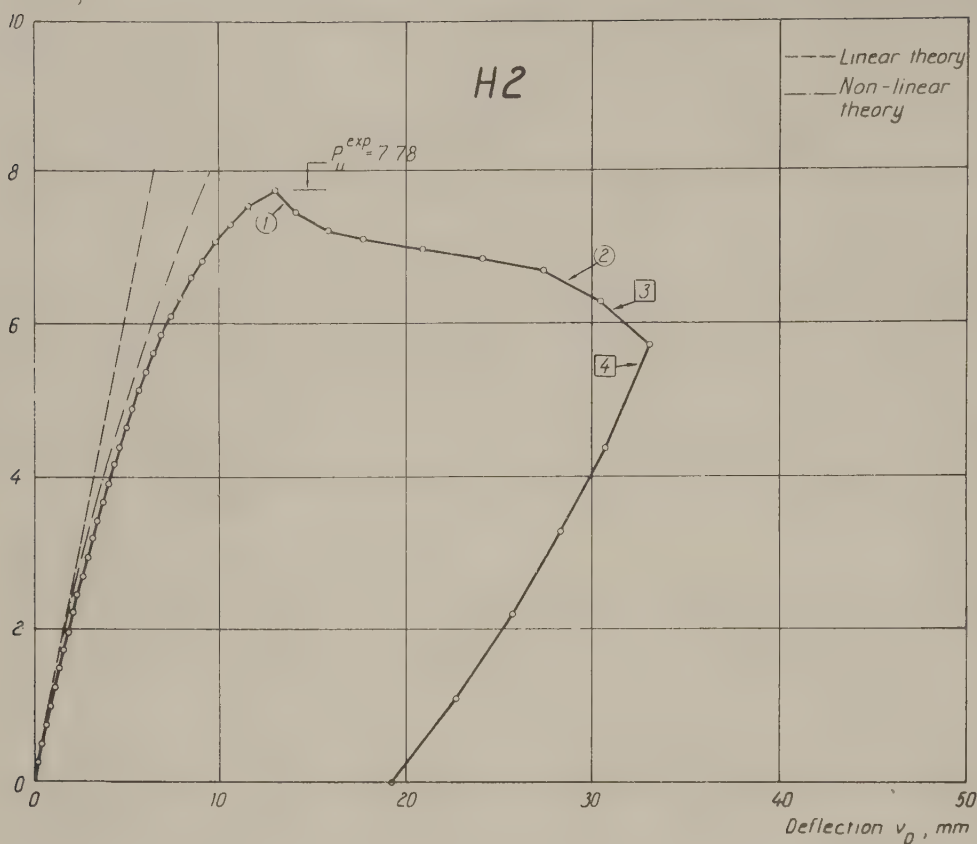
Bending moment ratio M_D/M_B 

Fig. 5.2.5. a) Load-deflection curve for test beam H1.
b) Bending moment ratio versus deflection.

a,

Load P , kN

Comments:

- ① Local folding of the beam (360 and 400 mm from section D). The failure developed slowly.
- ② Local folding of the beam (215 mm from support B). The failure developed slowly.
- ③ Local folding of another corner (125 mm from support B).
- ④ Unloading

b,

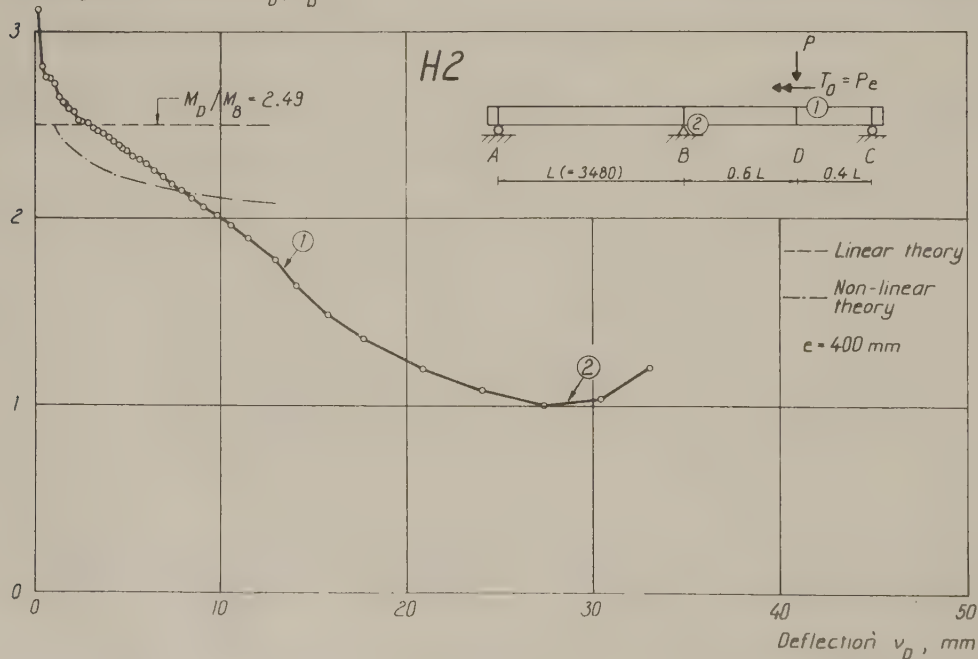
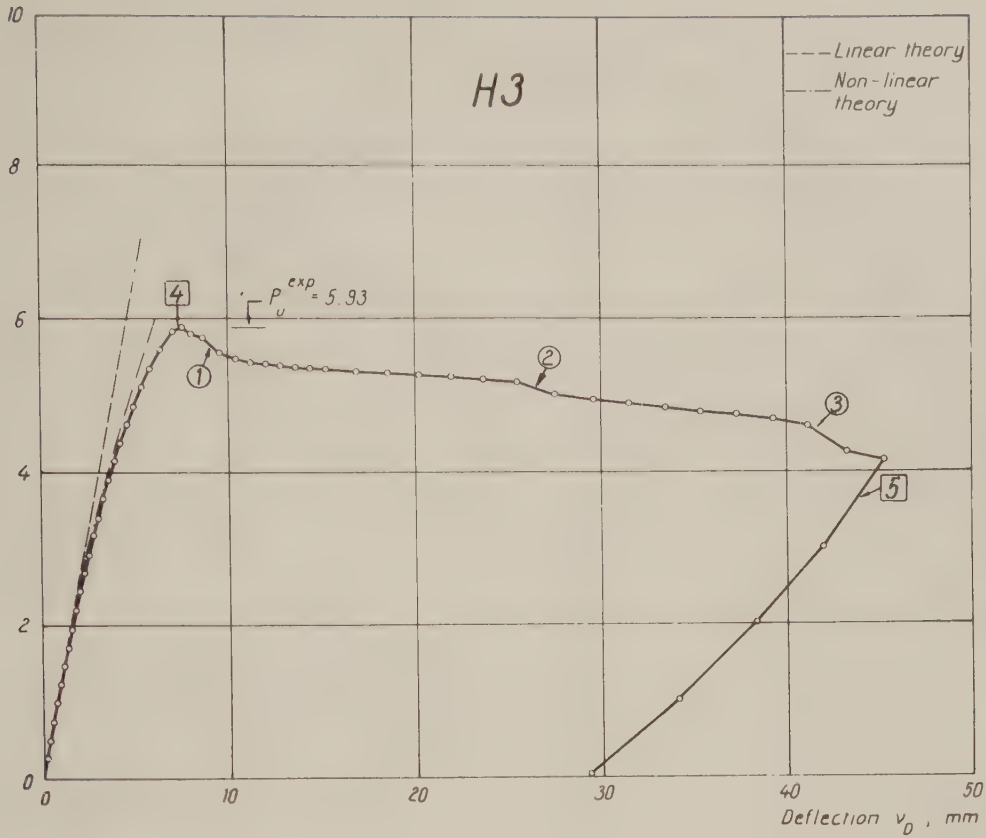
Bending moment ratio M_D/M_B 

Fig. 5.2.6. a) Load-deflection curve for test beam H2.

b) Bending moment ratio versus deflection.

a)

Load P , kN**Comments:**

- ① Local folding of corner (185 mm from section D).
- ② Local folding of corner (155 mm from section D).
- ③ Local folding of corners (150-200 mm from support B).
- ④ Local folding of one side of the compressed free flange at the centre of section D.
- ⑤ Unloading.

b)

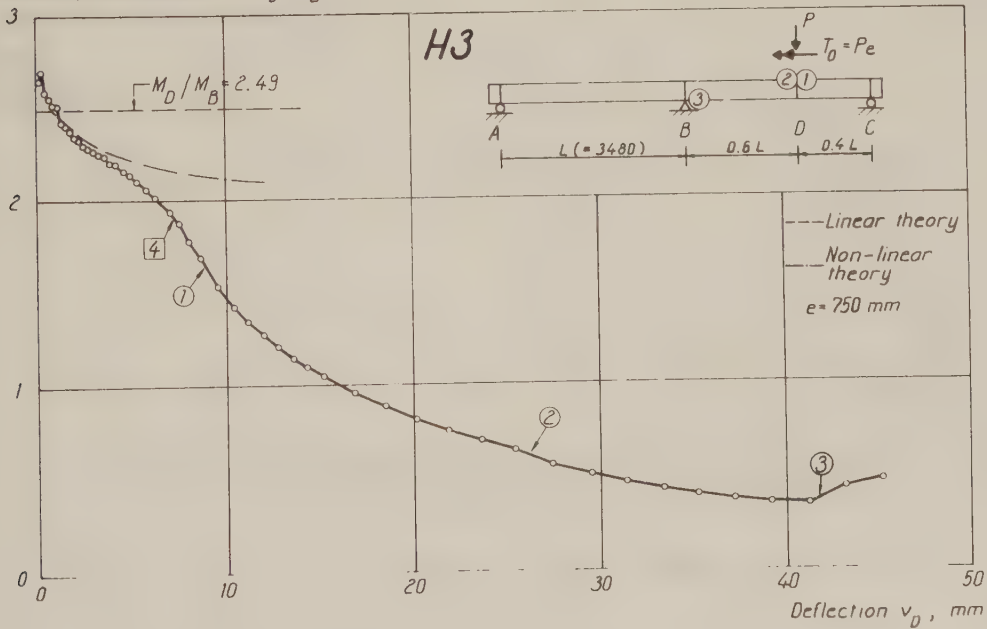
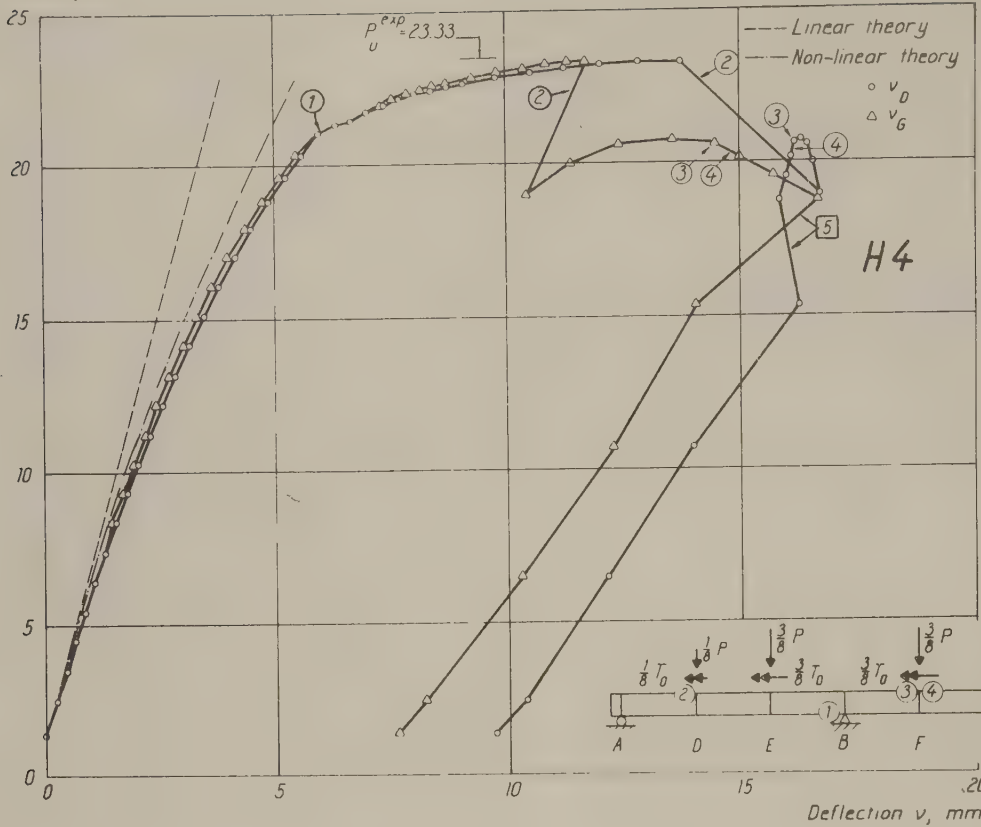
Bending moment ratio M_D/M_B 

Fig. 5.2.7. a) Load-deflection curve for test beam H3.
b) Bending moment ratio versus deflection.

a,

Load P , kN



Comments:

- ① Local folding of corners (100 and 195 mm from support B). One of the folds appeared some load steps later.
- ② Local folding of corners (135 and 220 mm from section D). The failure developed slowly over a few seconds. The hydraulic jack creating the forces at sections D and E was locked.

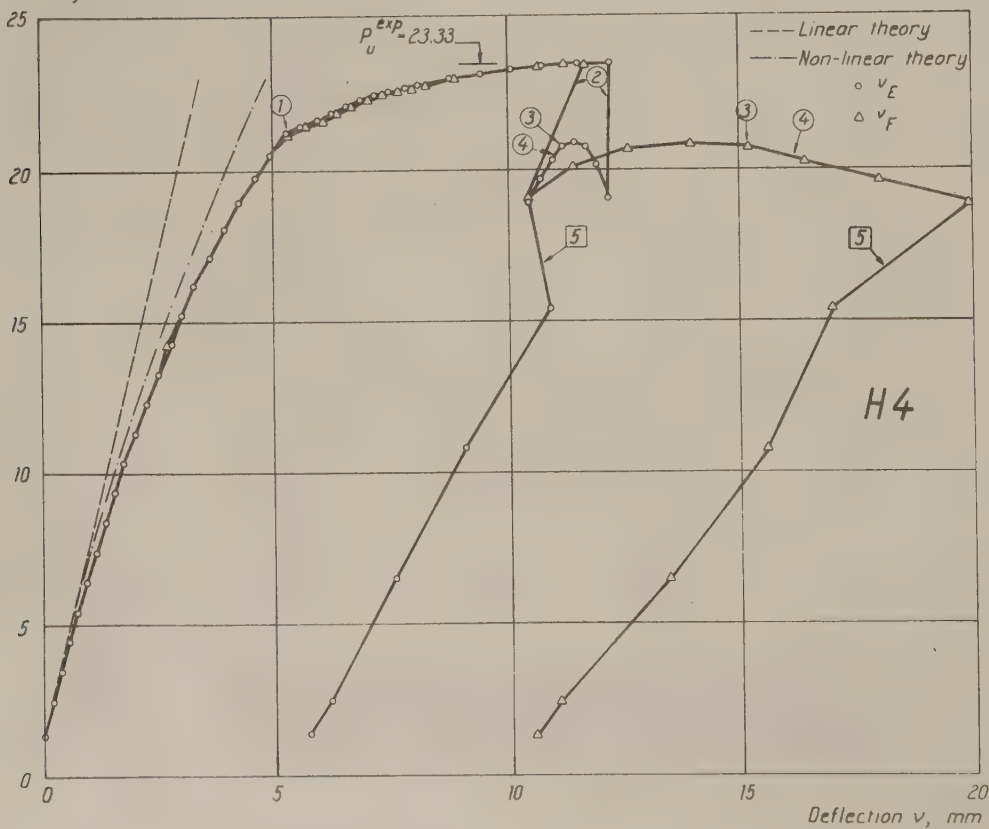
- ③ Local folding of corner (200 mm from section F).

- ④ Local folding of corners (180 and 190 mm from section F).

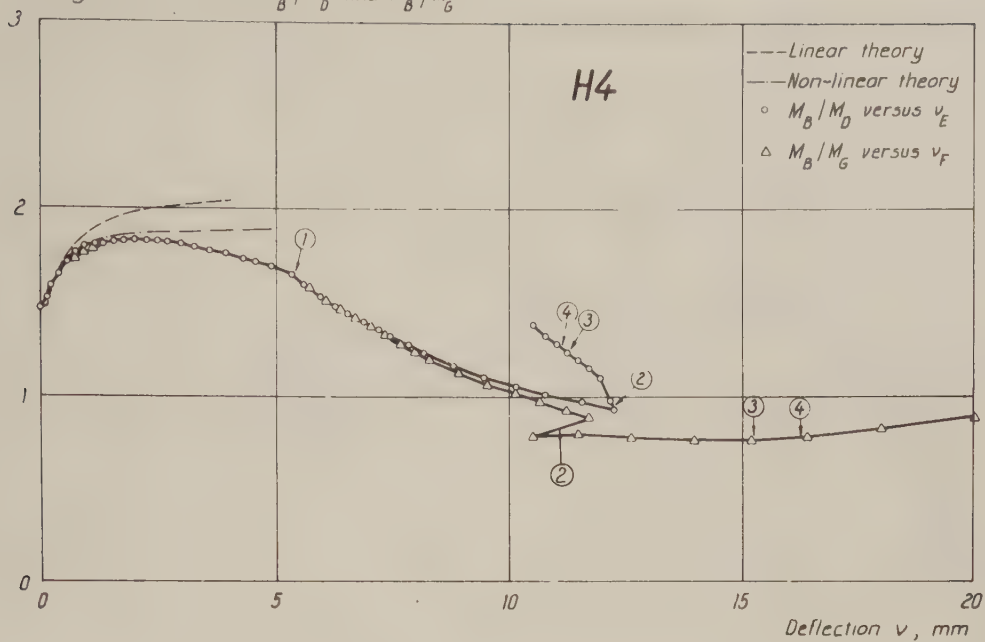
- ⑤ Unloading. The hydraulic jack was unlocked.

b,

Load P , kN



c)

Bending moment ratios M_B/M_D and M_B/M_G 

d)

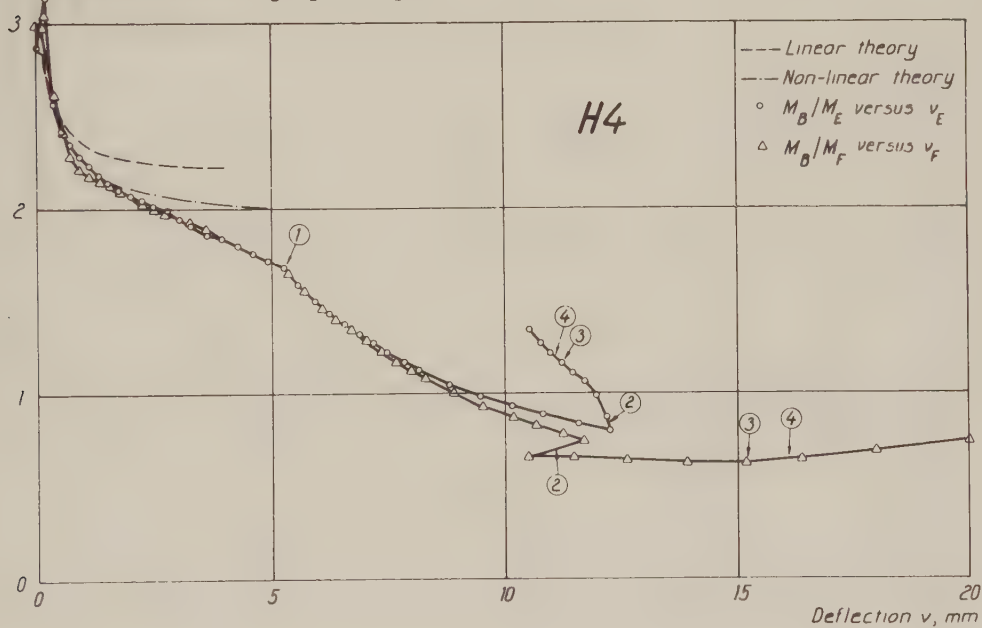
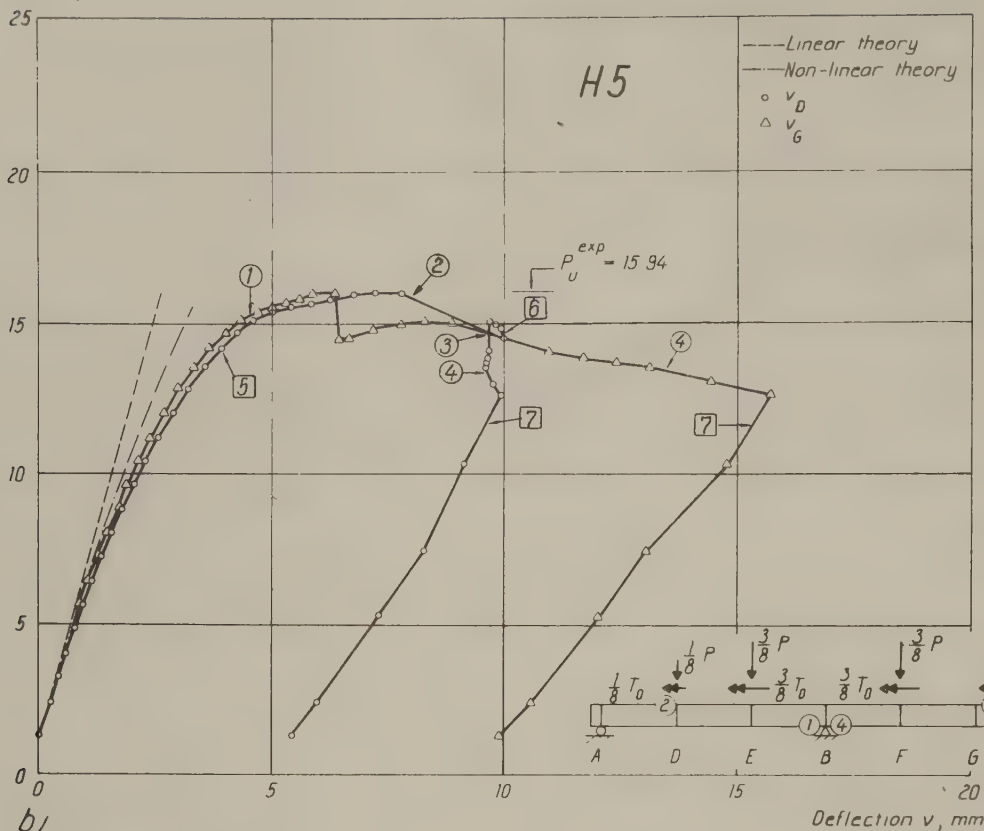
Bending moment ratios M_B/M_E and M_B/M_F 

Fig. 5.2.8. a-b) Load-deflection curves for test beam H4.
c-d) Bending moment ratio versus deflection.

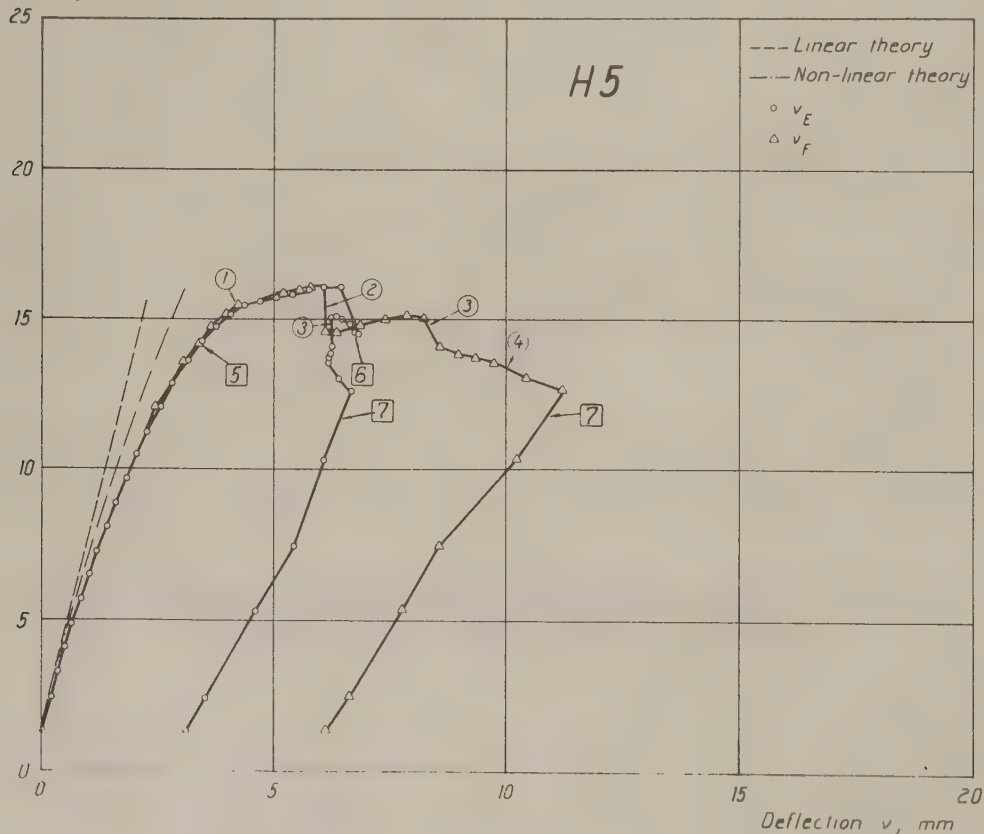
a,

Load P , kN

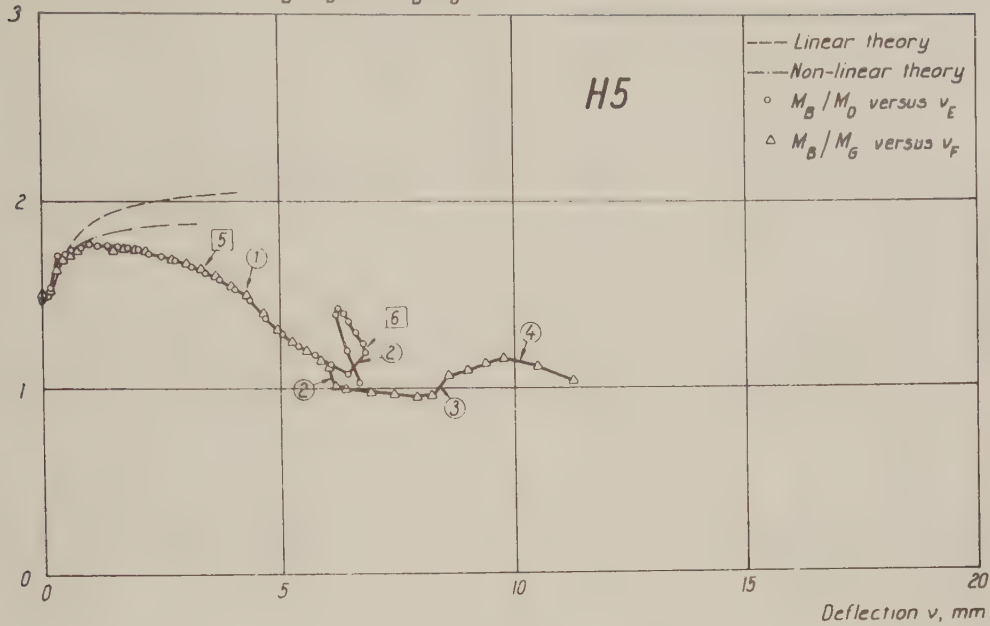


b,

Load P , kN



c)

Bending moment ratios M_B/M_D and M_B/M_G 

d)

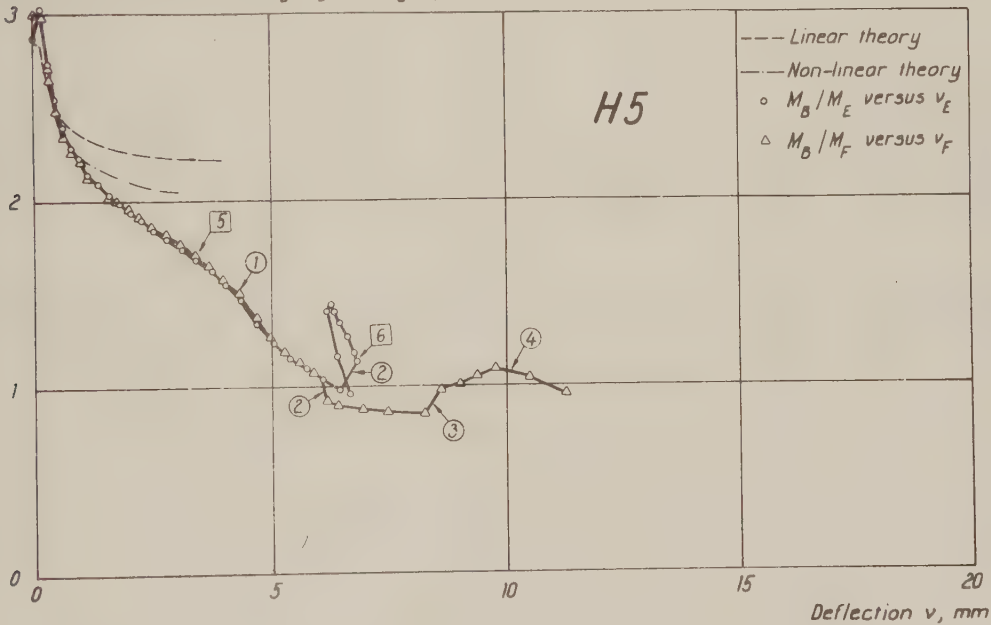
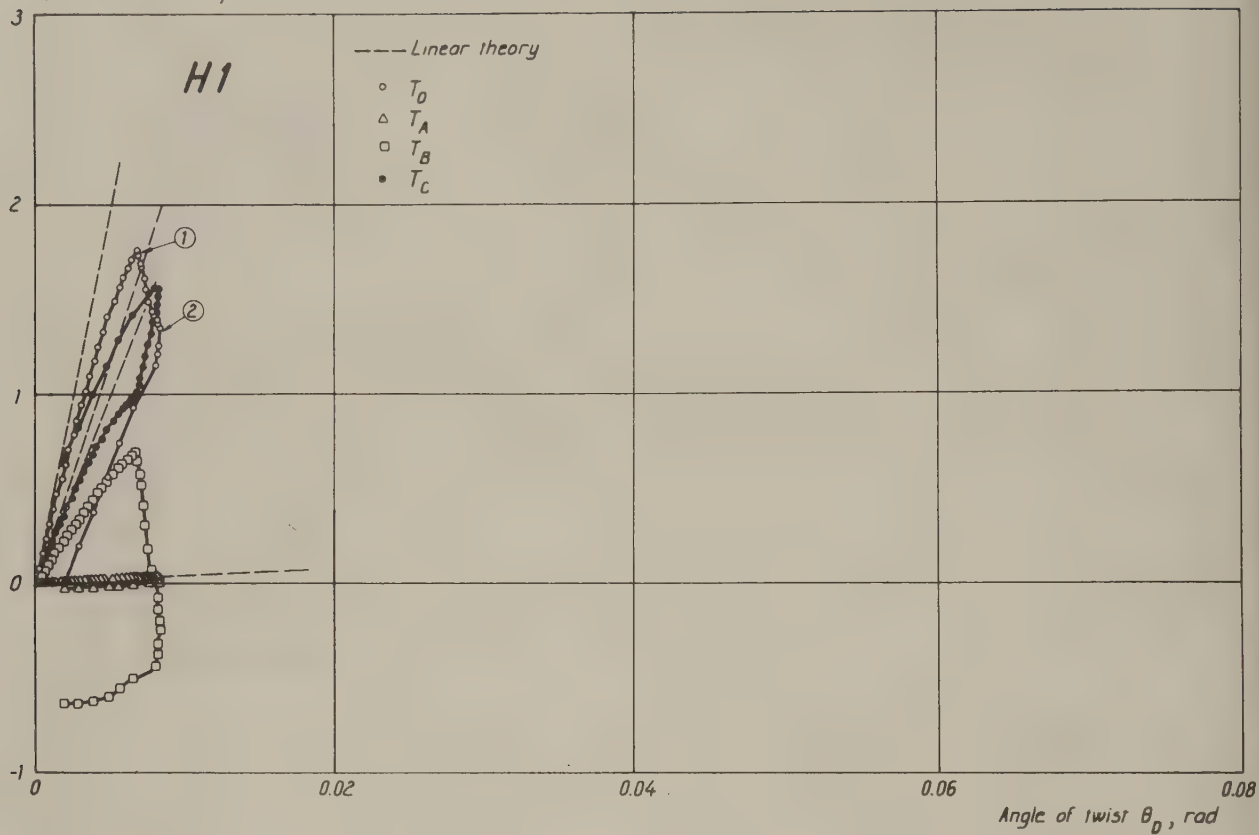
Bending moment ratios M_B/M_E and M_B/M_F 

Fig. 5.2.9. a-b) Load-deflection curves for test beam H5.
c-d) Bending moment ratio versus deflection.

Angle of twist.

In Figs. 5.2.10a-5.2.12a, the relationship between the torsional moment and the angle of twist is shown for test beams H1-H3. In Fig. 5.2.13a and 5.2.14a, the same relationship is shown for the left span of test beams H4 and H5. The suffix L, used in connection with the torsional moments $T_{0,L}$ and $T_{B,L}$, indicates that the moments refer to the left span.

a)

Torsional moment T , kNm

b)

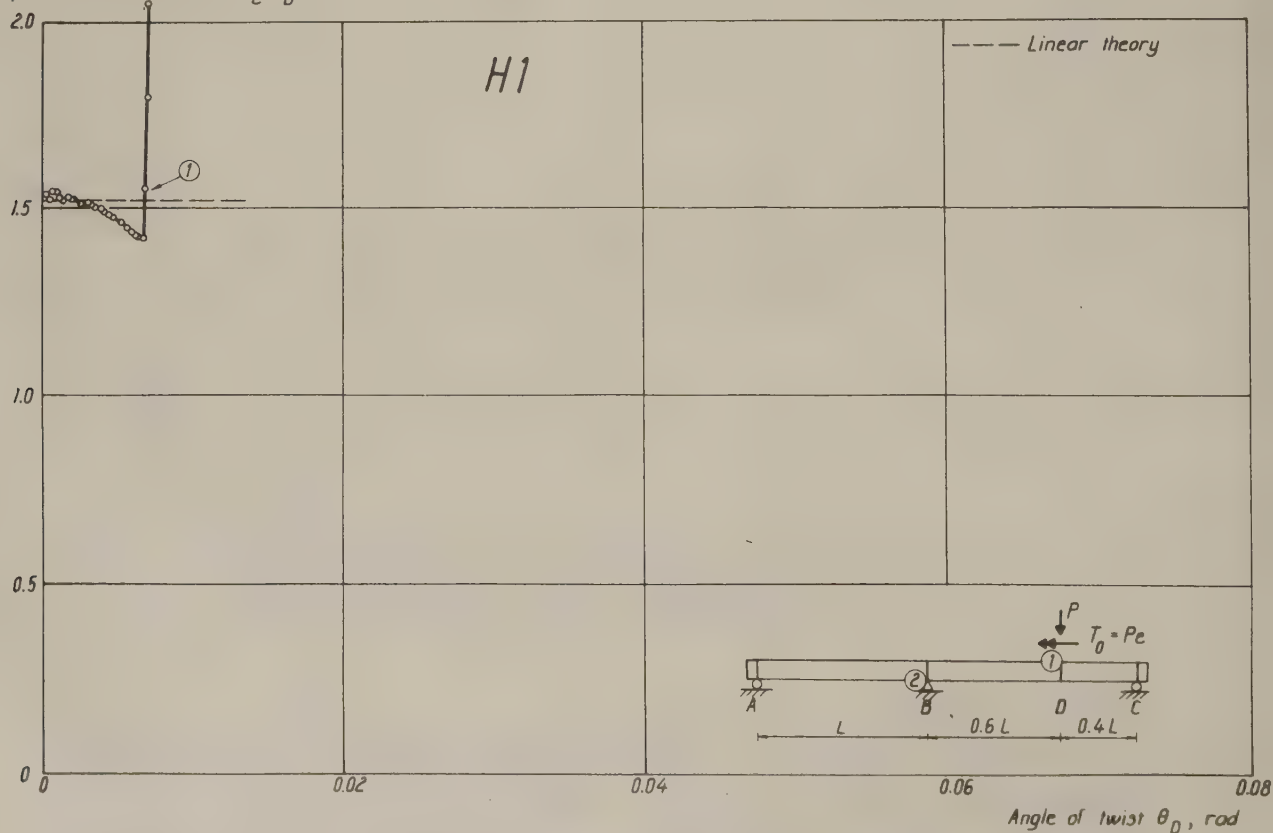
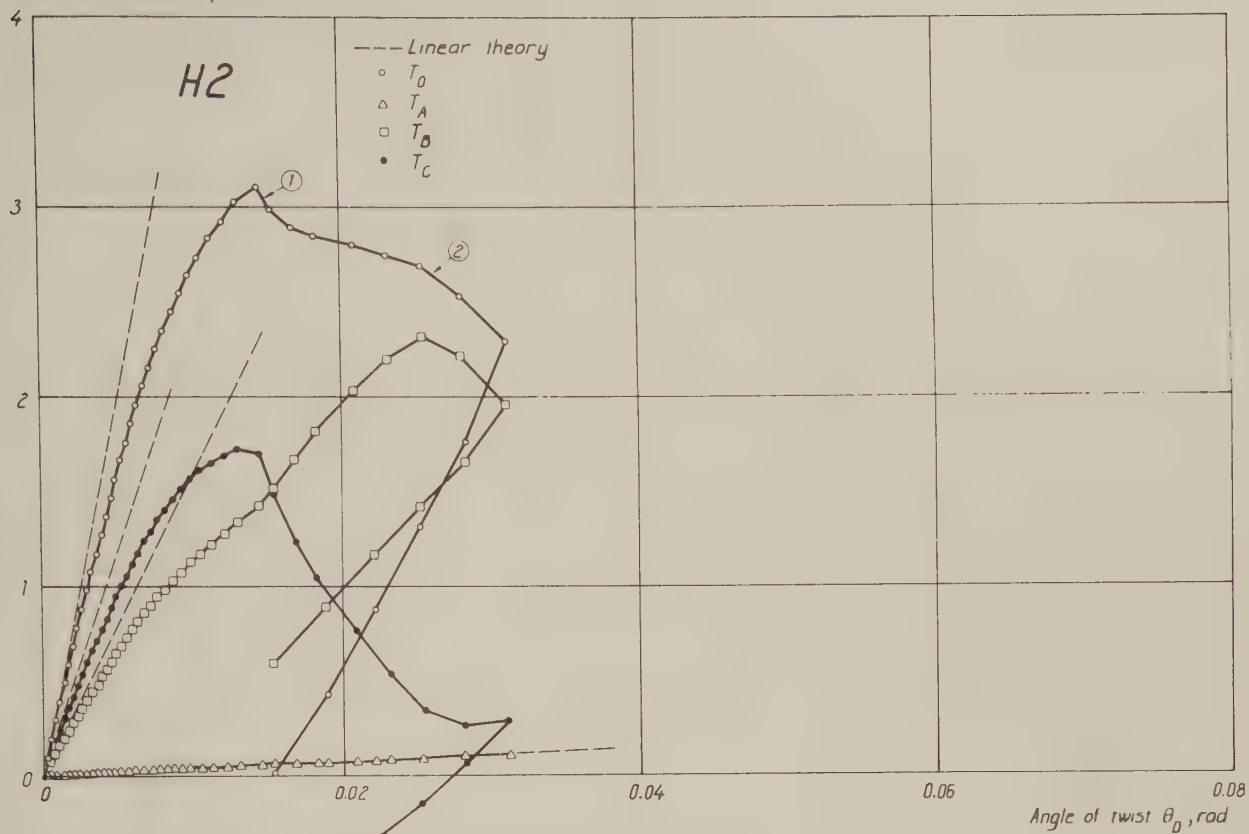
Torsional moment ratio T_C/T_B 

Fig. 5.2.10. a) Relationship between torsional moment and angle of twist for test beam H1.
b) Torsional moment ratio versus angle of twist.

a)

Torsional moment T , kNm

b)

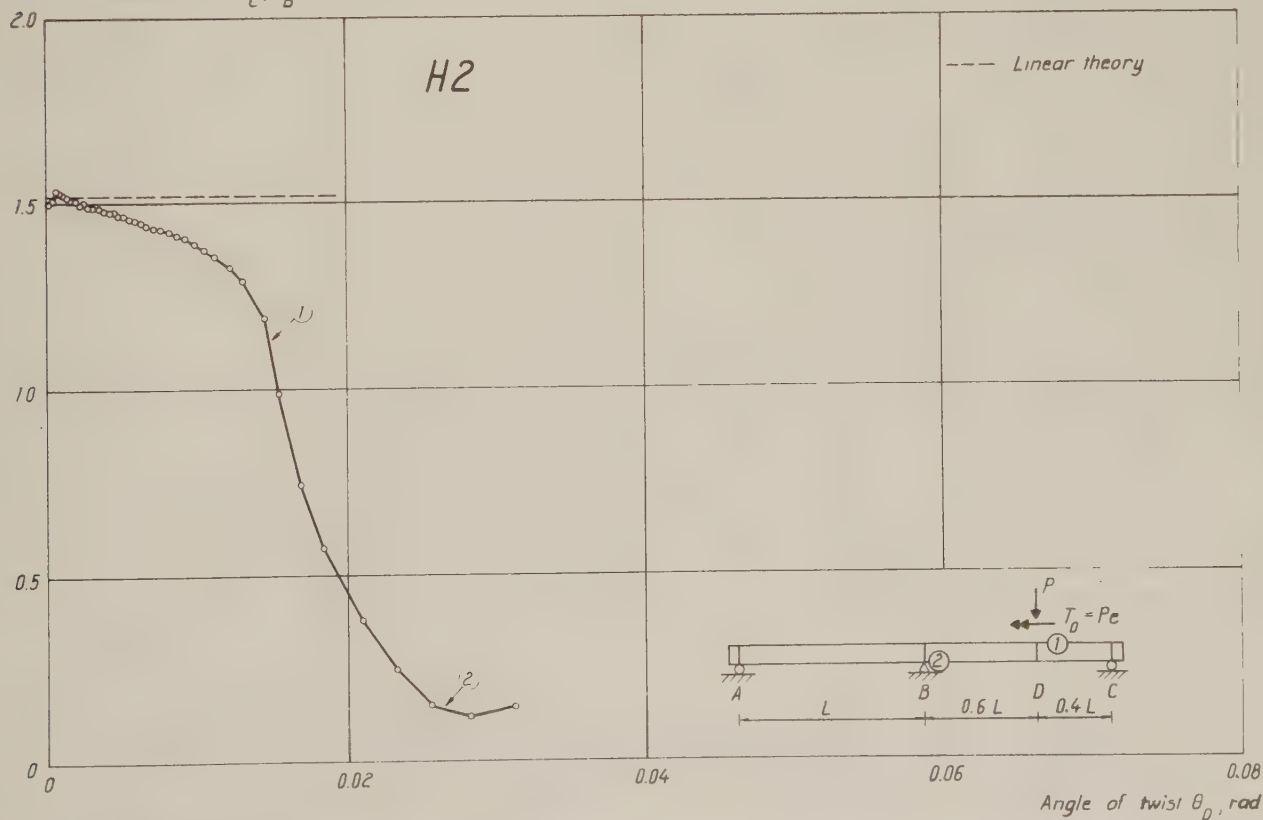
Torsional moment ratio T_C/T_B 

Fig. 5.2.11. a) Relationship between torsional moment and angle of twist for test beam H2.

b) Torsional moment ratio versus angle of twist.

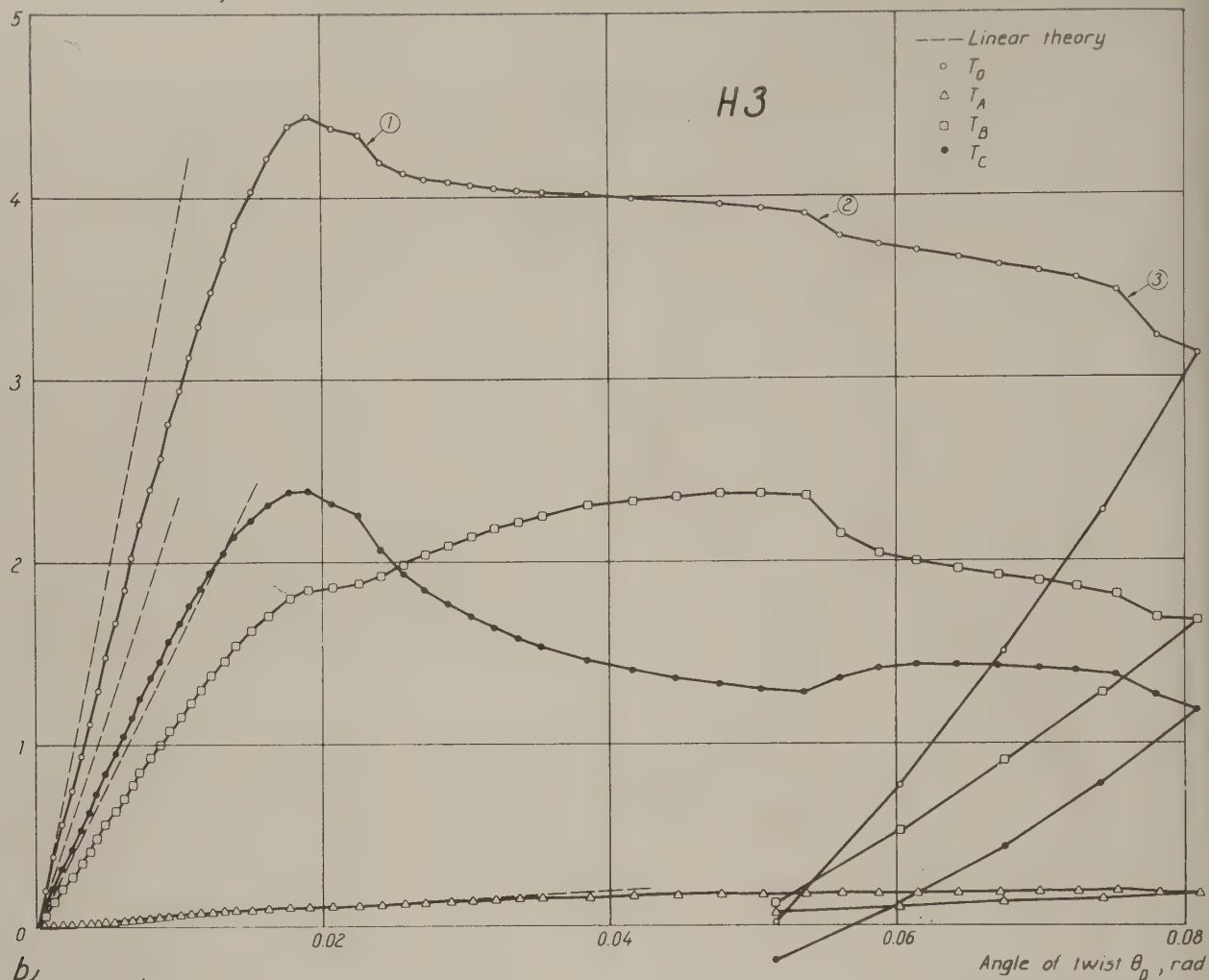
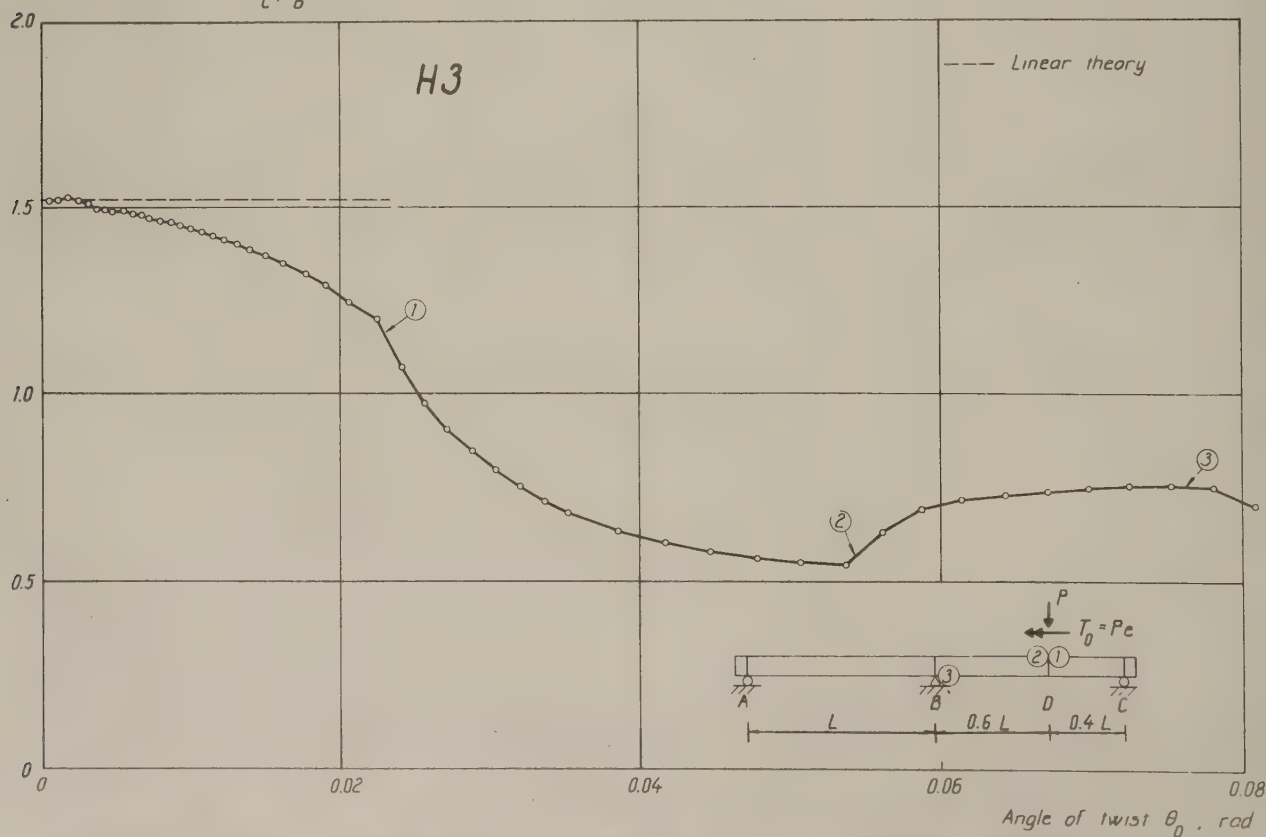
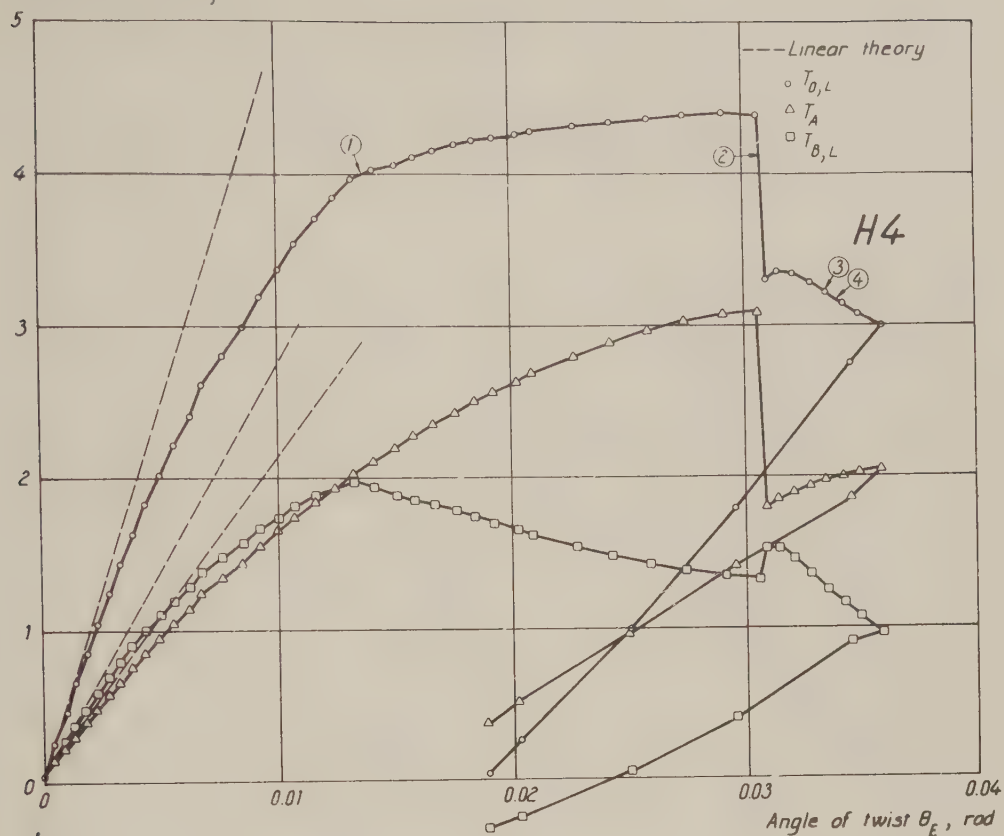
Torsional moment T , kNmTorsional moment ratio T_C/T_B 

Fig. 5.2.12. a) Relationship between torsional moment and angle of twist for test beam H3.

b) Torsional moment ratio versus angle of twist.

a,

Torsional moment T , kNm

b,

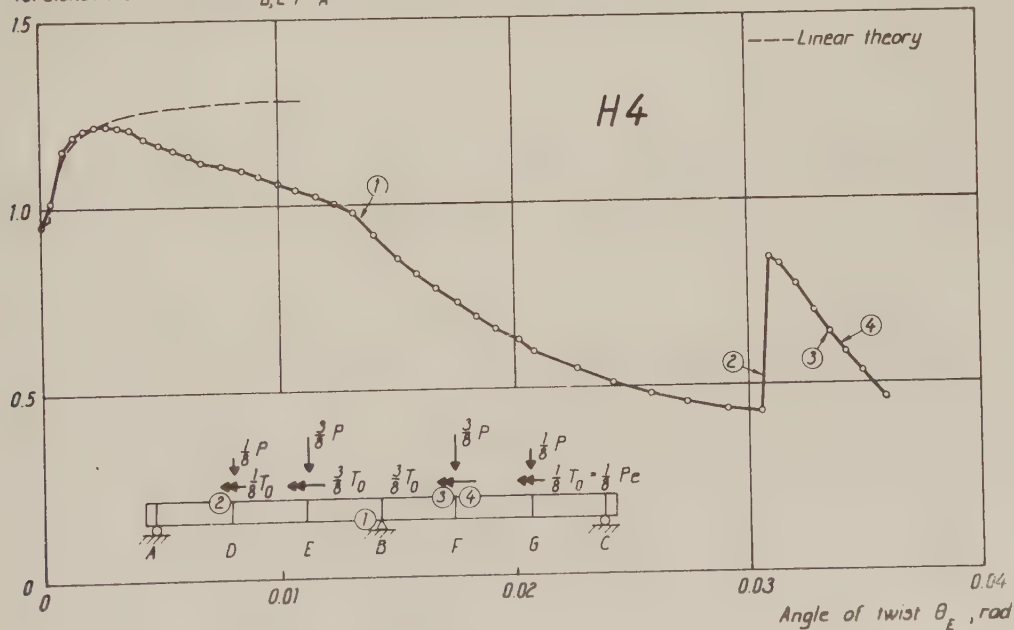
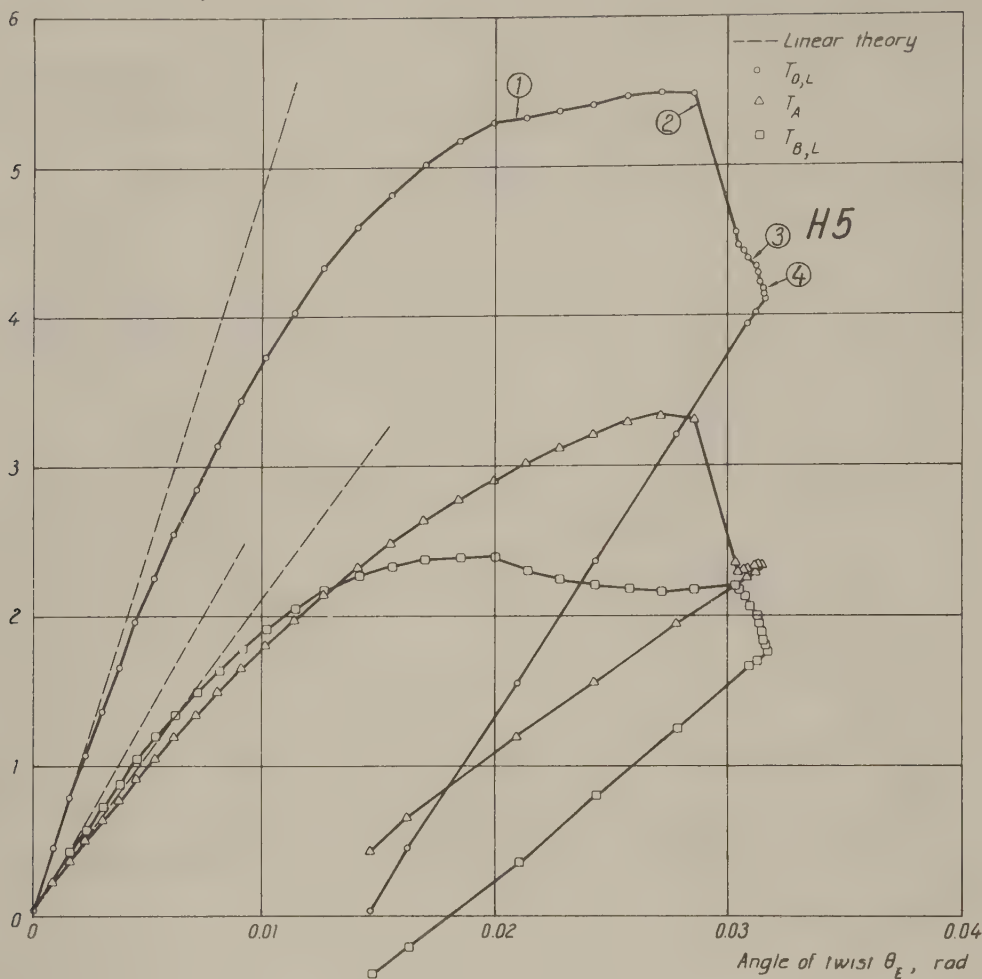
Torsional moment ratio $T_{B,L} / T_A$ 

Fig. 5.2.13. a) Relationship between torsional moment and angle of twist for test beam H4.
b) Torsional moment ratio versus angle of twist.

a,

Torsional moment T , kNm

b,

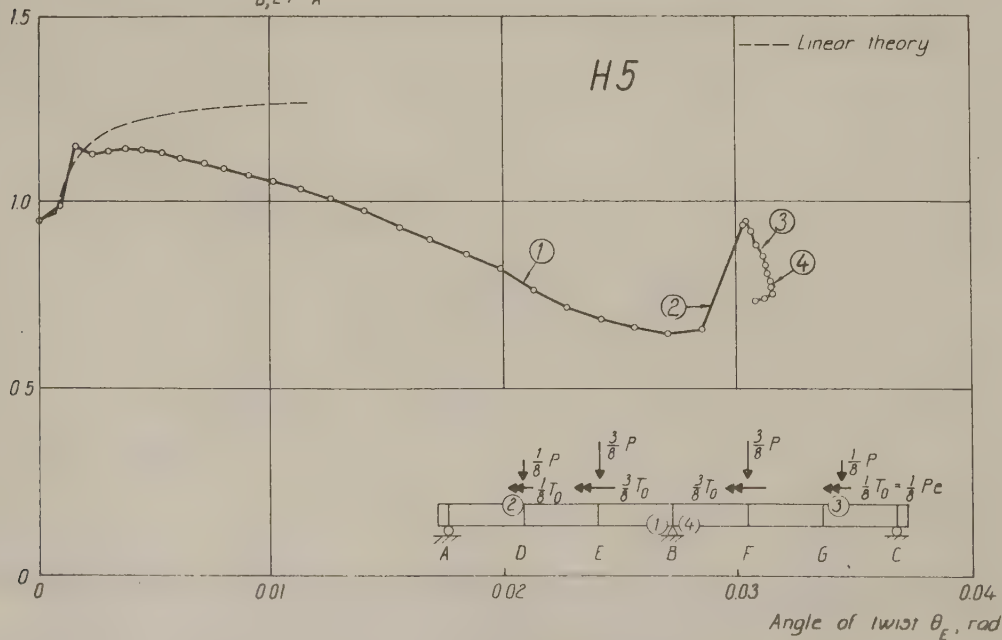
Torsional moment ratio $T_{B,L}/T_A$ 

Fig. 5.2.14. a) Relationship between torsional moment and angle of twist for test beam H5.
 b) Torsional moment ratio versus angle of twist.

Normal deflections.

The magnitudes of the initial normal deflections of test beams H1-H5 are presented in Appendix E. The normal deflections of some points on the upper flange of test beam H3 are shown in Fig. 5.2.15.

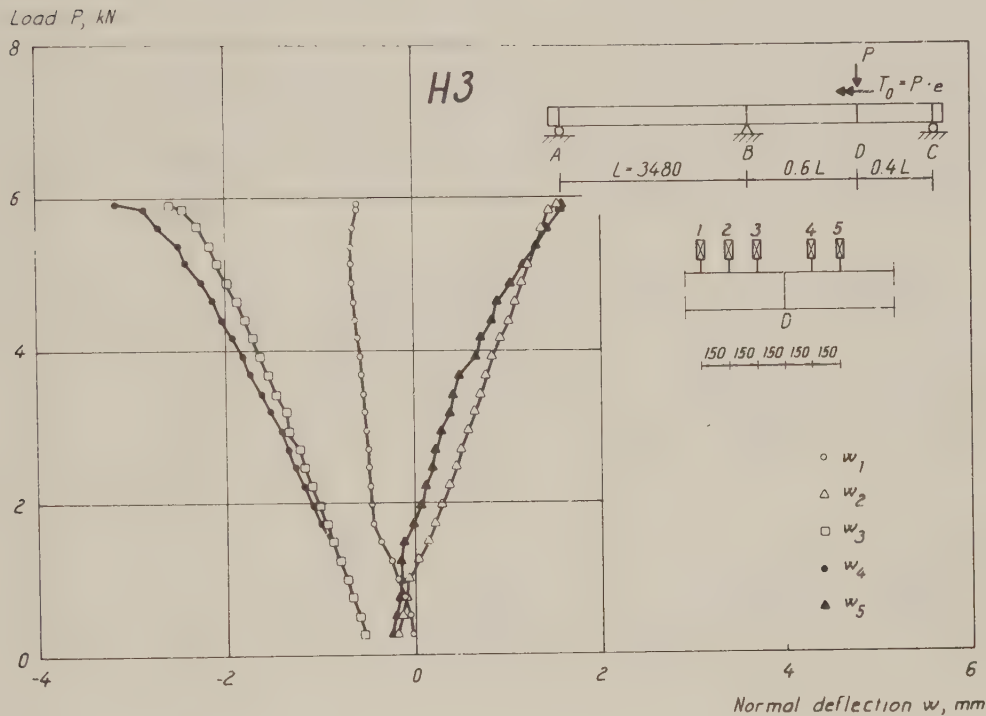


Fig. 5.2.15. Normal deflection as function of load for some points on the upper flange of test beam H3.

Ultimate moments and ultimate forces.

Measured applied ultimate loads and ultimate torsional moments of test beams H1-H5 are given in Table 5.2.4, together with the internal bending moments, torsional moments and shear forces at sections of local failure. In the determination of the internal bending moments it has been assumed that failure occurs at a distance of 150 mm from the centre of the screw joints.

Table 5.2.4. Measured applied ultimate loads and ultimate torsional moments. Measured internal bending moments, torsional moments and shear forces at sections of local failure.

Test beam	P_u kN	$T_{o,u}$ kNm	Failure number	M_u^{exp} kNm	T_u^{exp} kNm	V_u^{exp} kN
H1	8.82	1.76	1	5.32	0.74	4.38
			2	5.45	0.00	1.64
H2	7.78	3.11	1	4.77	1.70	3.84
			2	3.51	2.42	3.92
H3	5.93	4.45	1	3.55	2.40	2.86
			2 *	2.24	2.55	3.14
			3	4.65	2.05	3.28
H4	23.33	8.89	1	4.39	1.97	7.64
			2	3.79	3.08	3.75
			3	4.44	2.34	7.92
			4	5.52	1.04	0.78
H5	15.94	11.04	1	2.98	2.42	5.34
			2	2.48	3.31	2.46
			3	2.84	3.20	2.81
			4	2.58	3.13	5.11

* The failure was influenced by previous folding.

5.2.7 Discussion of results.

Deflections.

Using the elastic constants of the supports from Table 5.2.5, the theoretical deflections were calculated according to the procedure given in Section 3.4.7. The influence of the torsional moment was not taken into consideration. The results of the calculations are given in Figs. 5.2.5-5.2.9 where the results based on simple linear theory are also indicated.

Table 5.2.5. The elastic constants of the supports.

Test beam	k_A	k_B	k_C	c_A	c_B	c_C
	kN/mm	kN/mm	kN/mm	kNm/rad	kNm/rad	kNm/rad
H1-H5	23	23	23	4000	4000	4000

A comparison between theoretical and measured deflections shows that the measured deflections exceed the theoretical ones determined according to the non-linear theory even at rather low loads.

In Fig. 5.2.16a, the load-deflection curves of test beams D1 and H1-H3 are compared. With increasing load eccentricity e , the ultimate load P_u decreases. Notice that the point load on test beam D1 was acting 62 mm to the right of the corresponding point loads on the test beams H1-H3.

The measured load-deflection curves of test beams D2, H4 and H5 are given in Fig. 5.2.17a. As in Fig. 5.2.16a, the ultimate load P_u decreases with increasing load eccentricity e .

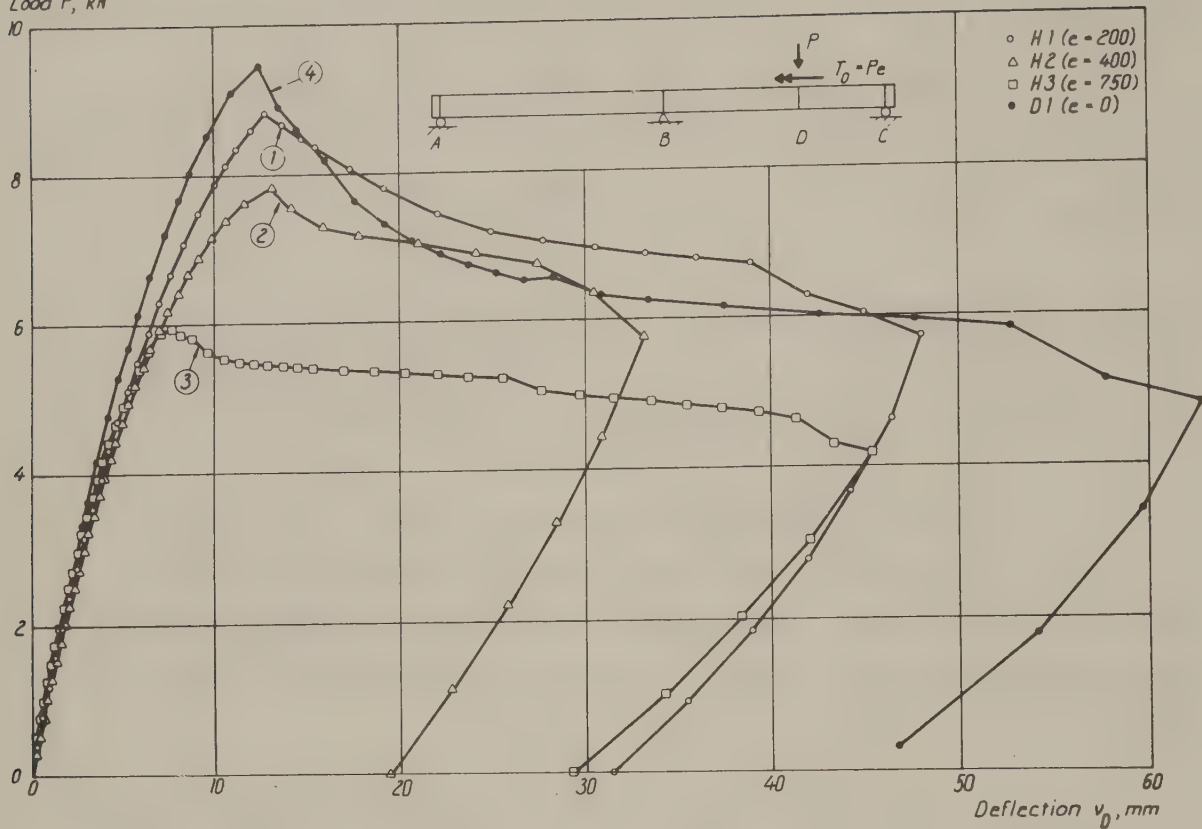
Angles of twist.

The relationship between torsional moment and angle of twist has been determined by simple linear theory according to Saint Venant. In this calculation the elasticity of the supports, see Table 5.2.5, has been taken into consideration. Further, on account of the stiffening effect of the screw joints, 40 mm has been subtracted from the length of each span. The reduction is less than for the previously tested beams because the flanges in this case are free to move. The results from the calculations are shown in Figs. 5.2.10a-5.2.14a.

Normal deflections.

The normal deflections developed gradually and no marked critical load could be seen (cf. Fig. 5.2.15). This seems natural because the critical load varies along the test beams. The number of buckles was fairly constant during the tests.

a,

Load P , kN

b,

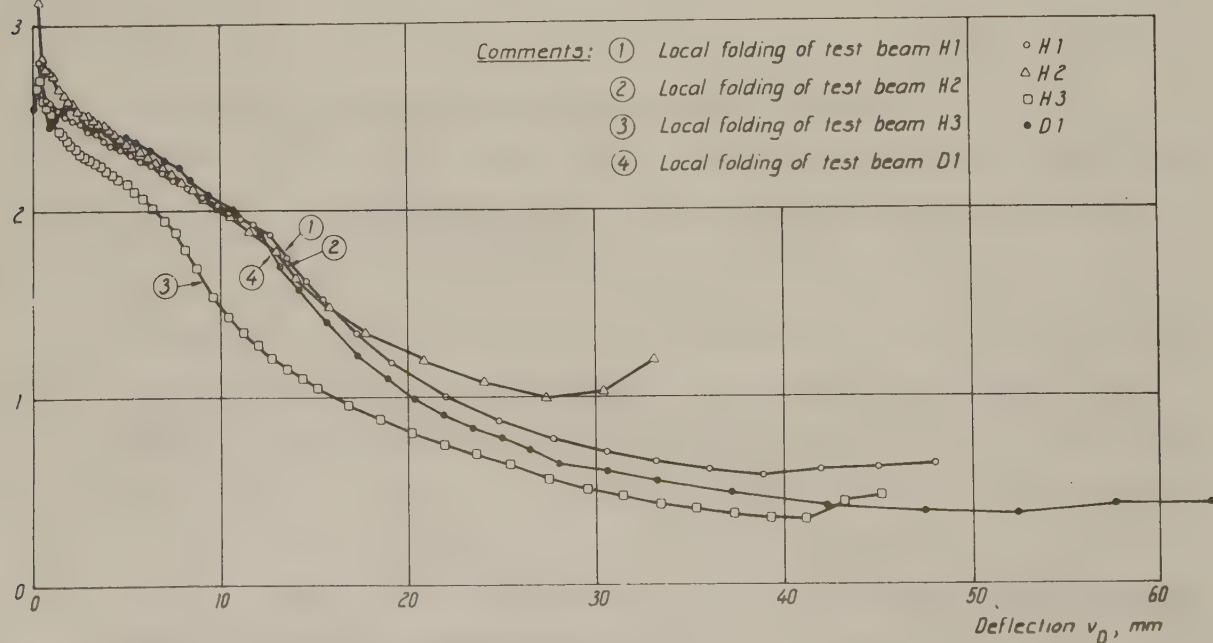
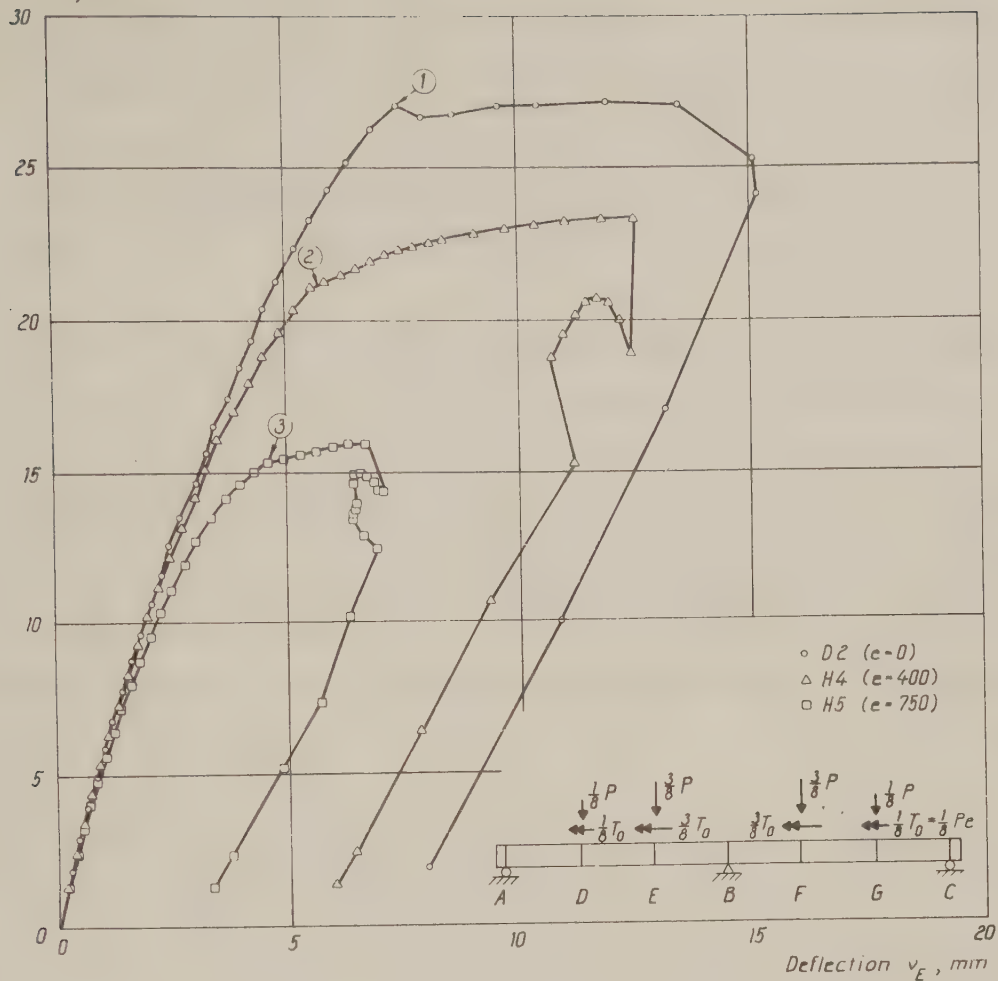
Bending moment ratio M_D / M_B 

Fig. 5.2.16. a) Load-deflection curves for test beams H1-H3 and D1.
b) Bending moment ratio versus deflection.

a,

Load P , kN

b,

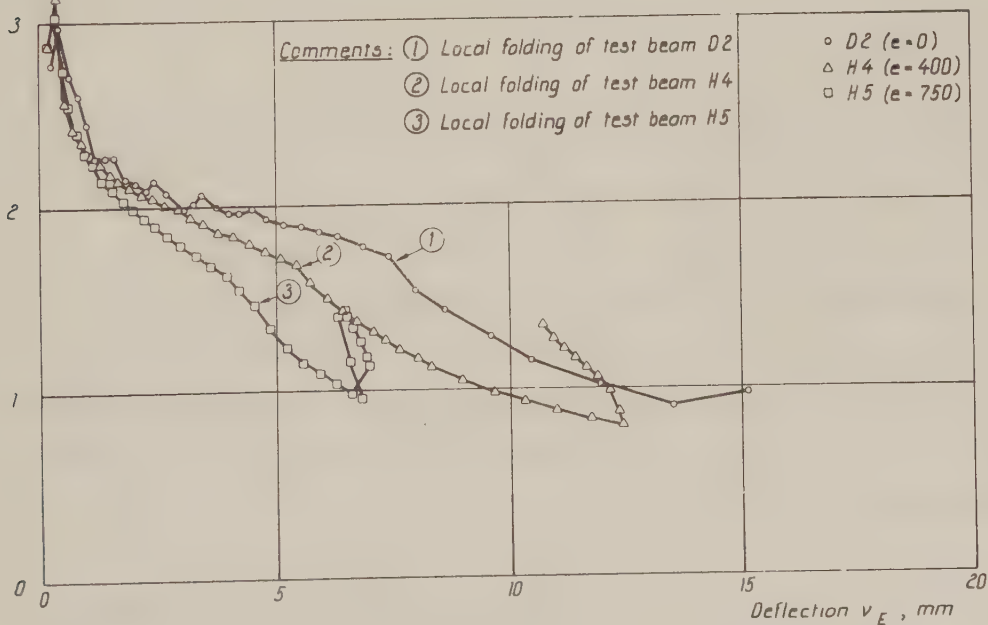
Bending moment ratio M_B/M_E 

Fig. 5.2.17. a) Load-deflection curves for test beams D2, H4 and H5.
b) Bending moment ratio versus deflection.

Bending moment ratios.

The measured bending moment ratio is shown as a function of vertical deflection in Figs. 5.2.5b-5.2.7b, 5.2.8c, d and 5.2.9c, d. The figures also include curves obtained from the previously described non-linear and linear theories. The discussion in Section 3.4.7 is also relevant in this section.

To study the effect of torsional moment on the occurrence of bending moment redistribution, the bending moment ratios of the test beams D1 and H1-H3 have been plotted against the deflection v_D in Fig. 5.2.16b. The curve for test beam H3 is the only one that distinctly separates from the others. This is due to the large applied torsional moment. However, the first failure of test beam H3 occurs at about the same bending moment ratio as that of D1, H1 and H2, which means that the possibility for bending moment redistribution to occur has not been reduced.

The bending moment ratio M_B/M_E is shown as a function of vertical deflection v_E in Fig. 5.2.17b for test beams D2, H4 and H5. In this case also the possibility for bending moment redistribution to occur has not been reduced by increasing load eccentricity e at the time of the first local failure.

Torsional moment ratios.

The relationship between measured torsional moment ratio and angle of twist is shown in Figs. 5.2.10b-5.2.14b. In the same figures the results based on simple linear theory are also given. The calculations are based on the same assumptions as were described under the heading "Angles of twist" in this section. The non-linear course of the theoretically determined curves for test beams H4 and H5 depends on the fact that the torsional moments from the loading devices are not distributed in the same way as the torsional moments applied later.

If the torsional moment ratios of test beams H1-H3 are compared, it can be seen (Fig. 5.2.18) that the possibility for torsional moment redistribution to occur at first local failure increases with increasing proportion of torsional moment. The somewhat strange curve belonging to test beam H1 depends on the very small torsional moments acting on that beam.

The torsional moment ratios of test beams H4 and H5 are compared in Fig. 5.2.19 as functions of angle of twist. At the first local folding of test beam H5 the torsional moment ratio $T_{B,L}/T_A$ is considerably less than unity depending on the large bending moments at the vicinity of support B. The external torsional moment in the left span is transmitted to support A.

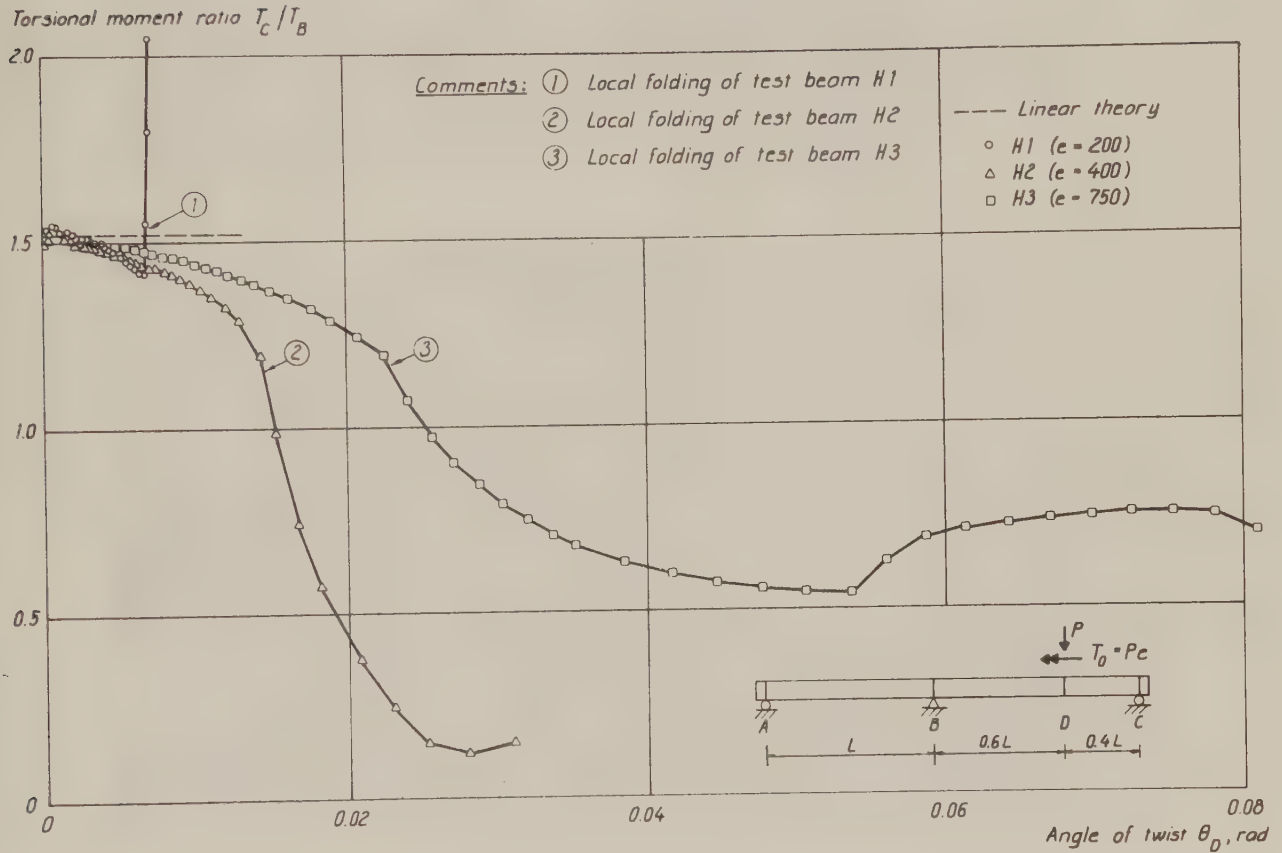


Fig. 5.2.18. Torsional moment ratio versus angle of twist for test beams H1-H3.

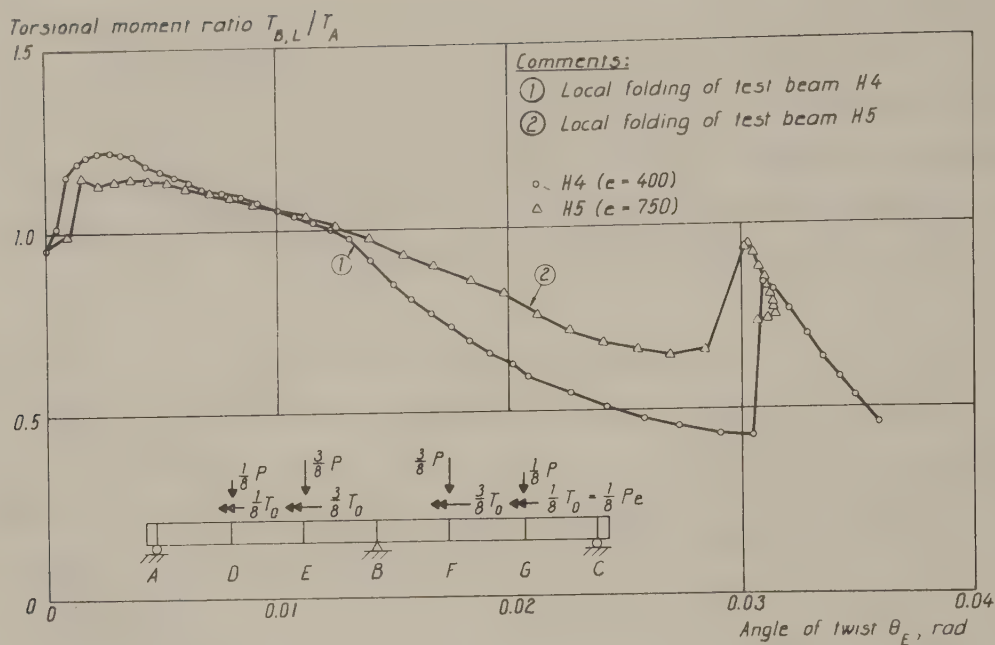


Fig. 5.2.19. Torsional moment ratio versus angle of twist for test beams H4 and H5.

Character of failure.

As the influence of the bending moments in sections of failure was in no case negligible, no typical shear failures occurred but only combined failures where the influence of bending moment was often dominating. Because of the free flanges of the test beams the failures in some cases started just at the screw joints, see Fig. 5.2.20. However, the failures were always accompanied by a folding of one or two corners at a distance from the screw joints. All the failures developed fairly slowly during the tests. No "brittle" failures occurred.

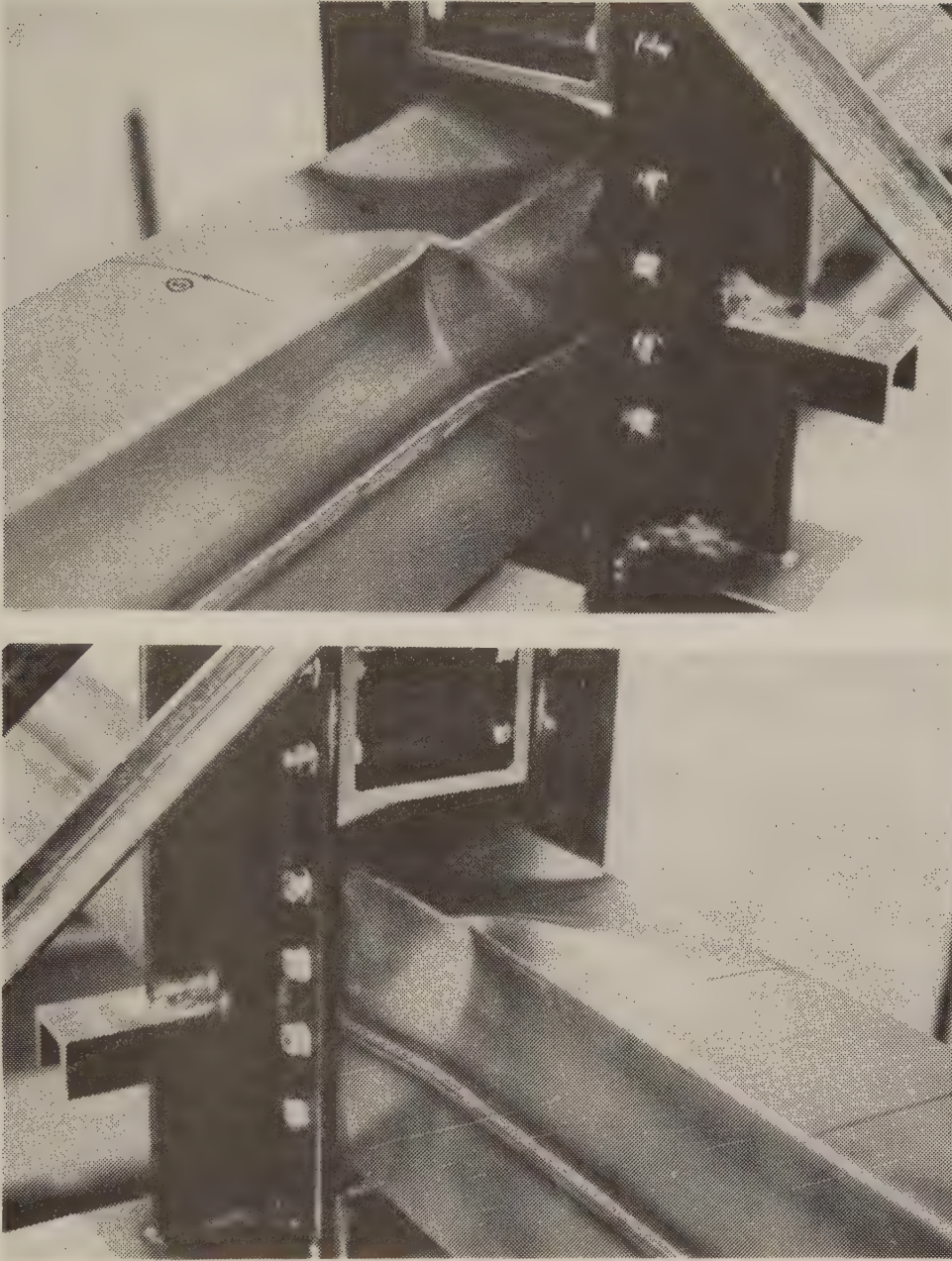


Fig. 5.2.20. Example of local failure which started to develop at the screw joint.

Ultimate moments and ultimate forces.

Table 5.2.6 gives the calculated ultimate moments for beam sections subjected to pure bending or to pure torsion. The ultimate bending moments have been obtained using the procedure given in Section 3.1.14 with "reduced" webs and with effective width according to Winter. The ultimate torsional moments have been obtained using Eq. (4.1.30). In this connection it must be pointed out that beams F5 and F6 were tested with screw joints in the flanges as well as in the webs. The test beams H1-H5 on the other hand had no screw joints in the flanges.

Table 5.2.6. Calculated ultimate moments for beam sections subjected to pure bending or pure torsion.

Test beam	M_u^{th} kNm	T_u^{th} kNm
H1	5.46	3.69
H2	5.36	3.60
H3	5.52	3.74
H4	5.60	3.83
H5	5.48	3.73

From Tables 5.2.4 and 5.2.6 the ratios of measured to calculated ultimate moments at sections of local failure can be determined. The results are given numerically in Table 5.2.7 on the one hand and graphically in Fig. 5.2.21 on the other. From Fig. 5.2.21 it is clear that the actual relationship can be described rather well by a circle.

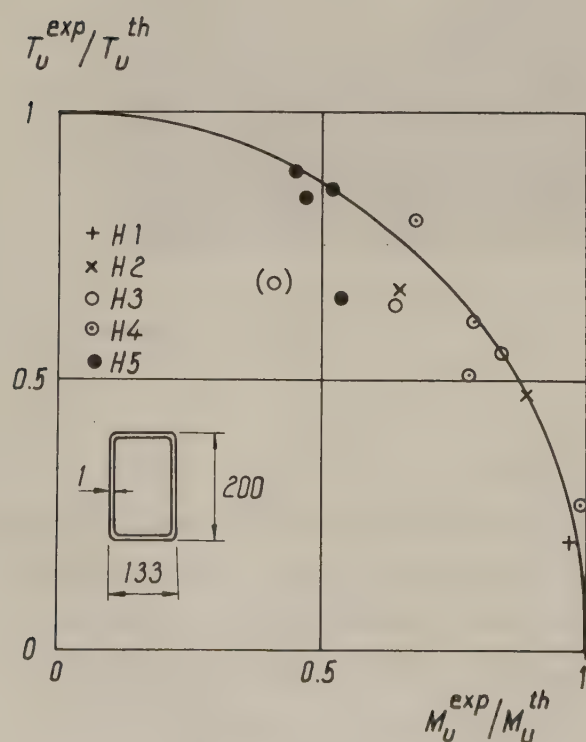


Fig. 5.2.21. Relationship between ultimate torsional moment and ultimate bending moment at sections of local failure. Test beams H1-H5.

If we include the influence of the shear forces according to the principles given in Section 5.1.7 we obtain the results given in Table 5.2.7 (final column). As in Section 5.1.7 we have taken into consideration in this case the increase of the ultimate torsional moment T_u^{th} on account of short span length. For the short spans ($0.4 L$) of test beams H2 and H3 we increase the ultimate torsional moments by 2 per cent. For test beams H4 and H5 we increase by 3 per cent. The results are also shown in Fig. 5.2.22 from which it can be concluded that the relation between the ultimate forces can be described by a circle. If this circle is used for design purposes, it will probably mean that the dimensions obtained are on the safe side.

Table 5.2.7. Ratios of measured to calculated ultimate moments in sections where local failure occurs.

Test beam	Failure number	M_u^{exp}/M_u^{th}	T_u^{exp}/T_u^{th}	$\frac{T_u^{exp} + b V_u^{exp}}{T_u^{th}}$
H1	1	0.97	0.20	0.36
	2	1.00	0.00	0.06
H2	1	0.89	0.47	0.59
	2	0.65	0.67	0.82
H3	1	0.64	0.64	0.73
	2*	0.41	0.68	0.79
	3	0.84	0.55	0.67
H4	1	0.78	0.51	0.76
	2	0.68	0.80	0.92
	3	0.79	0.61	0.61
	4	0.99	0.27	0.29
H5	1	0.54	0.65	0.82
	2	0.45	0.89	0.95
	3	0.52	0.86	0.93
	4	0.47	0.84	0.99

* The failure was influenced by earlier folding

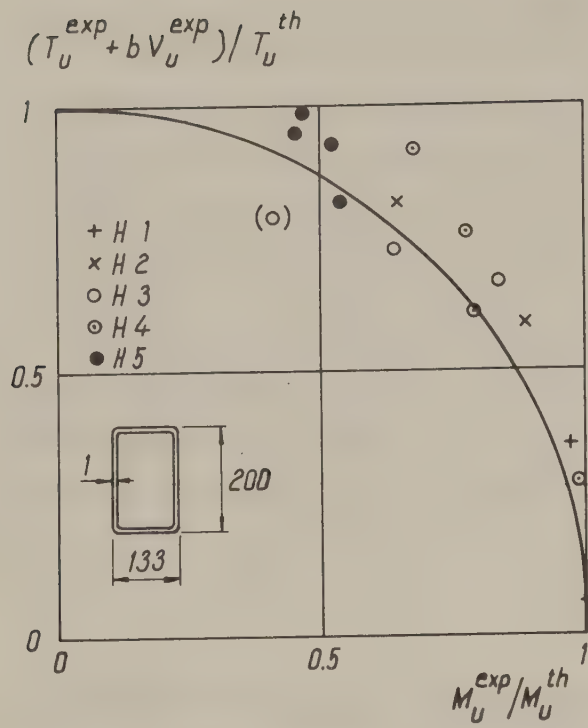


Fig. 5.2.22. Relationship between ultimate torsional moment (modified) and ultimate bending moment at sections of local failure. Test beams H1-H5.

6 PURE BENDING OF HAT-SECTION BEAMS.

6.1 Theory.

6.1.1 Definition of cross-sectional dimensions and coordinate axes.

The cross-sectional dimensions of a hat beam section are defined in Fig. 6.1.1a. All dimensions refer to the centre of the plates. The index o is used to denote outer dimensions (cf. Fig. 6.2.1). The expressions used to characterize the different plates of the beam section are given in Fig. 6.1.1b. The coordinate axes for a hat-section beam are defined in Fig. 6.1.2.

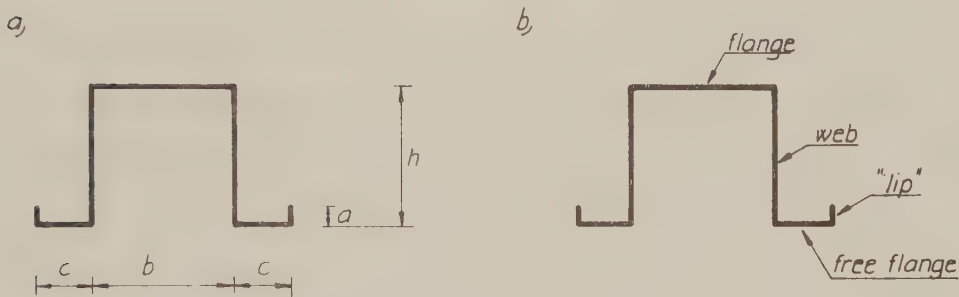


Fig. 6.1.1. Definition of
a) cross-sectional dimensions
b) notations for different plates.

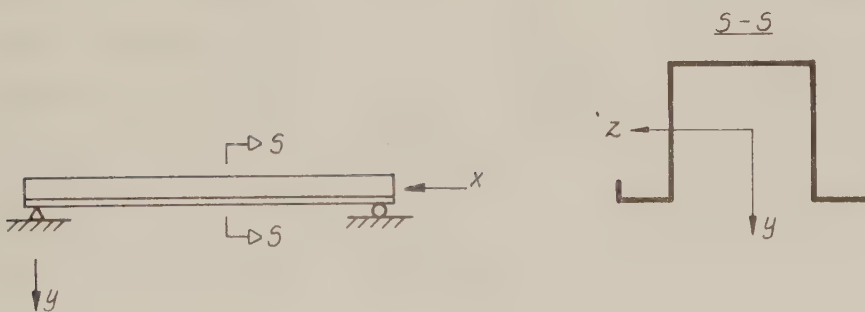


Fig. 6.1.2. Orientation of coordinate axes for a hat-section beam.

6.1.2 Determination of stresses and deflections of single-span beam.

The distributions of stresses and deflections have been calculated in two different ways:

- 1) Simplified theory, i.e. the area of each lip has been replaced by an area concentrated to a point at the end of the associated free flange. The method of solution is summarized in Appendix F.
- 2) Accurate theory, i.e. a more advanced method is used which takes into consideration the elastic constants of the "lips" as well as those of the free flanges. The method of solution is summarized in Appendix G.

6.1.3 Determination of ultimate bending moment.

Two different methods for the determination of the ultimate moment have been used:

- 1) The "lips" of the real beam section have been represented by concentrated point areas according to the simplified theory of Appendix F. (See Fig. 6.1.3a and b.) The equivalent width c_{eq} of the free flange given in Fig. 6.1.3c has been chosen to give the same bending stresses at points 2 and 3 as those calculated according to Appendix F. In the determination of c_{eq} , it has been assumed that the critical stresses of the compressed flange and of the webs have not yet been attained. Further, it has also been assumed that the elastic constant of the free flange is equal to zero, since it is unlikely that there will be much transverse flexural rigidity left in the webs or in the compression flange at instant of failure. The ultimate moments have then been calculated by introducing effective widths for the compression flange and the compressed parts of the webs (cf. Section 3.1.14) using Winter's formula. Failure is supposed to occur when the most stressed point of the cross-section has reached the yield stress of the material (in the actual tests always the upper corners).
- 2) The hat-section beam has been represented by a closed beam section according to Fig. 6.1.4. Observe that the dimensions of the free flange and of the lip comply with the relation:

$$b = 2(a + c)$$

The ultimate moment of this box beam section is calculated in

accordance with the method given in Section 3.1.14.

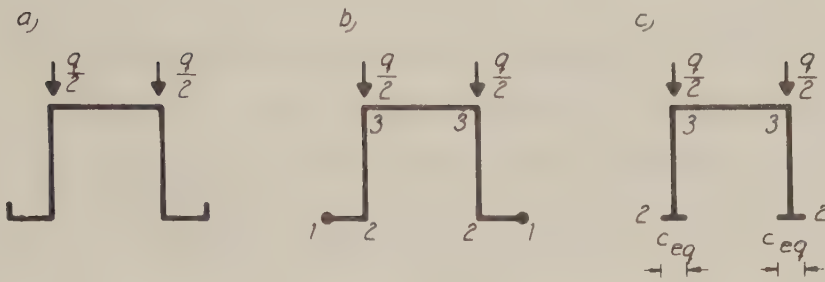


Fig. 6.1.3. a) Real beam section.
 b) Simplified beam section according to Appendix G.
 c) Equivalent beam section giving the same bending stresses in points 2 and 3 as the simplified beam section.

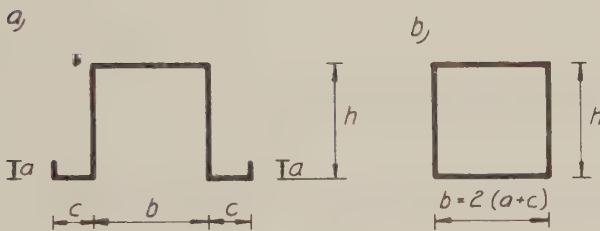


Fig. 6.1.4. a) Real beam section.
 b) Equivalent closed beam section.

6.2 Single-span beam loaded with two point loads.

6.2.1 Purpose.

In Sections 3.2 and 3.3, the behaviour of thin-walled box beams subjected to pure bending was studied. If one of the flanges of a box beam is cut and unfolded, a hat section beam is obtained. The behaviour of this beam in bending is rather different from that of a box beam. Thus the cross-sectional shape will not be retained.

The aim of the experiments was to study the influence of buckling phenomena, to investigate different relations between load and deformation, to clarify the nature of failure and to compare the results with those from experiments with box beams in pure bending.

6.2.2 Scope.

The experiments comprised two test beams J1 and J2. Distribution of load, shear force and bending moment are shown in Fig. 6.2.1. Nominal dimensions of test beams J1 and J2 are given in Table 6.2.1.

Table 6.2.1. Nominal dimensions of test beams J1 and J2.

Test beam	L_1 mm	L_2 mm	L mm	b_0 mm	h_0 mm	c_0 mm	a_0 mm	t mm
J1	607	1040	2254	173	260	65	22	1.5
J2	404	692	1500	173	173	65	22	1.5

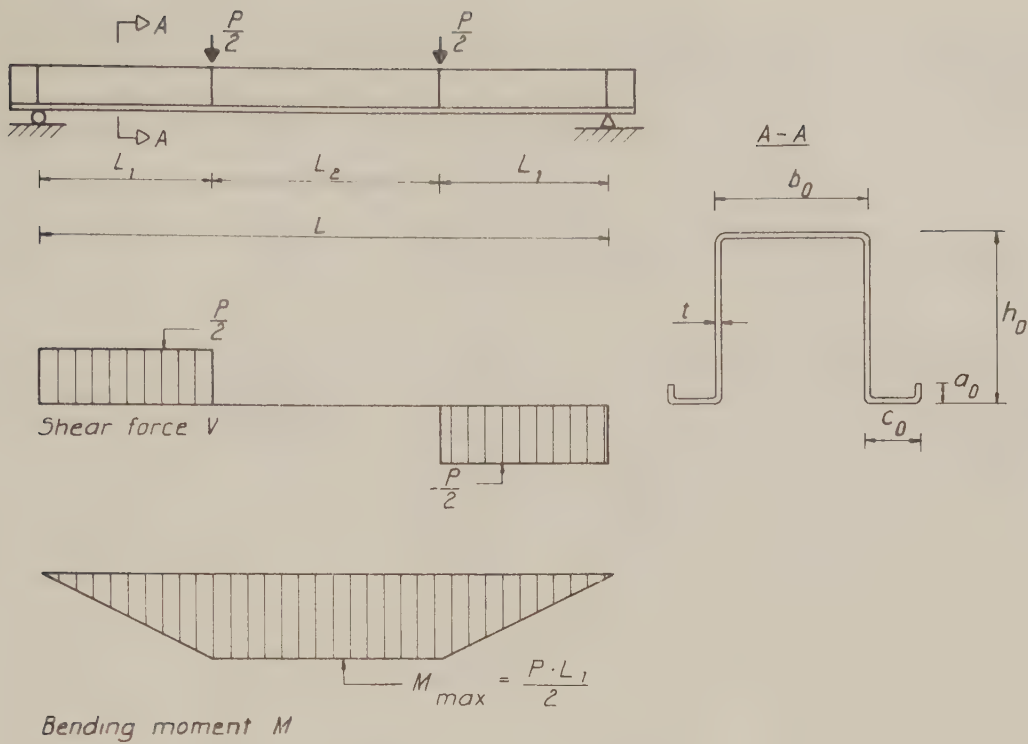


Fig. 6.2.1. Distribution of load, shear force and bending moment for test beams J1 and J2. Notations for cross section.

6.2.3 Loading and support arrangements.

The principal loading and support arrangements are shown in Fig. 6.2.2.

The support arrangement was designed to satisfy, as closely as possible, the following boundary conditions:

- 1) the test beams were simply supported with regard to vertical bending,
- 2) the webs and the free flanges were simply supported with regard to horizontal bending.

Condition 1) was realized by using roller and knife edge bearings at the respective supports. Condition 2) was not fully realized because the upper parts of the webs were clamped by screw joints where the dimensions of the outer flat bars were 10 x 80 mm. Their vertical extension was half the height of the webs. The

lower parts of the webs rested on half-round steel bars attached to the supports.

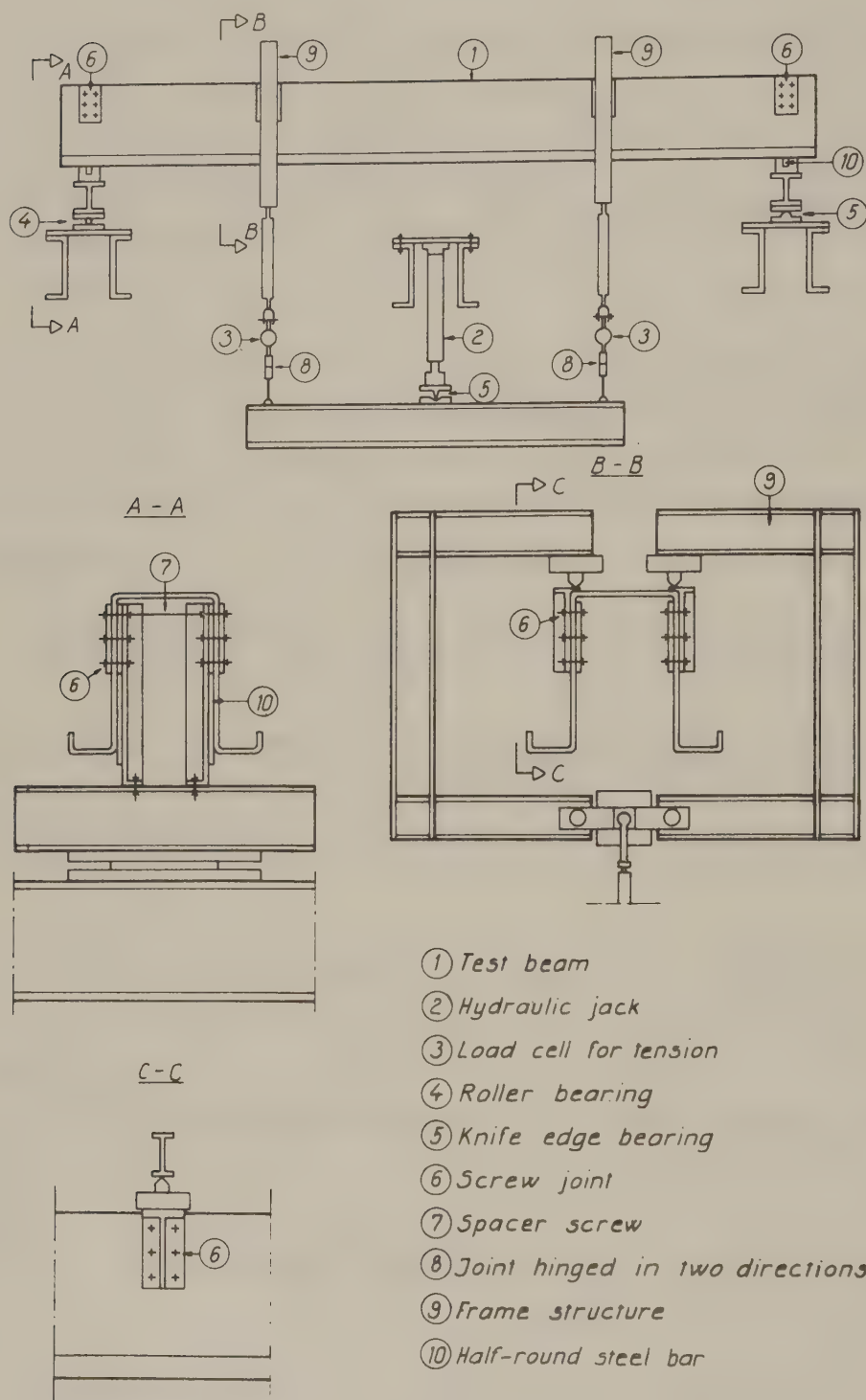


Fig. 6.2.2. Principal loading and support arrangements for test beams J1 and J2.

The load was generated using a hydraulic jack and was divided into two point loads by means of an I-beam. The point loads were further transferred by frame structures and knife-edge bearings to screw joints on the upper parts of the webs of the test beams. The dimensions of the screw joints were the same as those of the supports.

To ensure that the point loads acted in the plane of the webs, slots were milled in screw joints in the elongation of the webs. Further, to ensure that only vertical point loads acted on the webs, the frame structures were provided with hinges placed directly below the webs.

The main disadvantages of the screw joints at the point loads were that

- 1) vertical bending of the webs was partly prevented,
- 2) transverse bending of the webs was partly prevented,
- 3) local buckling could not occur.

The disturbances introduced by the screw joints have been neglected.

6.2.4. Dimensions. Material properties.

The test beams were made by brake forming of cold-rolled sheet. Since test beams J1-J2 and K1-K4 (cf. Section 6.3 for further details) were taken out from the same cold-rolled sheet, their material properties will be discussed together. The material consisted of cold-rolled sheet COR-TEN A. The general properties of the material were reported in Section 3.3.4.

Two tensile test pieces were cut from the compression flange of each test beam and tested as described in Appendix B. The results of the test are given in Table 6.2.2. A typical stress-strain curve is shown in Fig. 6.2.3. As there was very little difference in yield stress between the test pieces, the mean value of the yield stress was subsequently used for test beams J1-J2 and K1-K4.

Table 6.2.2. Results of tensile tests carried out on pieces from test beams J1, J2 and K1-K4.

Test beam	Tensile test piece	t mm	σ_y N/mm ²	σ_u N/mm ²	δ_5 %
J1	:1	1.54	337	502	41
	:2	1.54	335	502	41
J2	:1	1.57	340	508	40
	:2	1.57	339	505	39
K1	:1	1.58	340	506	42
	:2	1.58	334	503	39
K2	:1	1.55	333	498	37
	:2	1.55	332	497	37
K3	:1	1.60	336	500	41
	:2	1.59	339	506	38
K4	:1	1.52	338	505	34
	:2	1.52	334	505	38
Mean value :			336	503	39

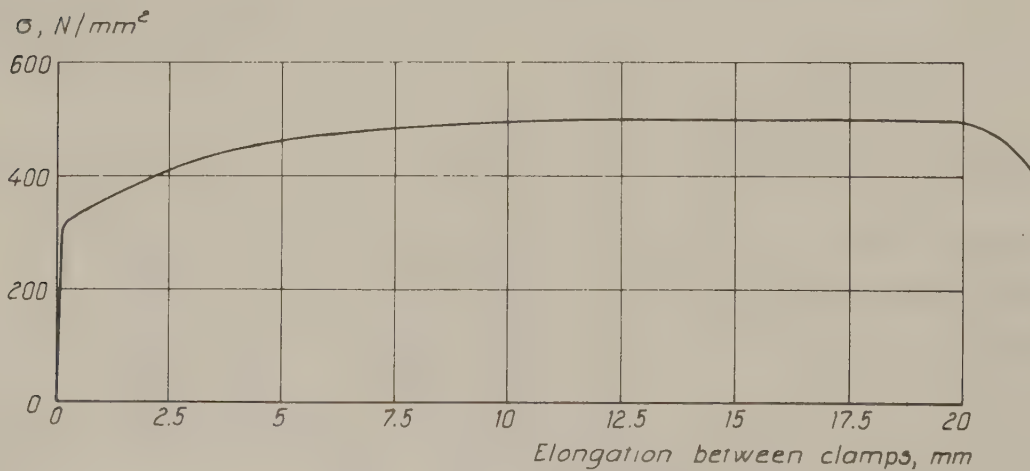


Fig. 6.2.3. Typical stress-strain curve. Tensile test piece J1:2.

One bending test piece was cut from each test beam in the same region as the tensile test pieces and tested as described in Appendix C. The results of the test are given in Table 6.2.3.

Table 6.2.3. Results of vibration tests carried out on bending test pieces from test beams J1, J2 and K1-K4.

Test beam	Bending test piece	t mm	b mm	L mm	m g	f Hz	$E \cdot 10^{-5}$ N/mm ²
J1	:b1	1.55	20.02	202.8	48.46	198.5	2.04
J2	:b1	1.58	20.01	202.8	49.62	202.9	2.05
K1	:b1	1.59	20.01	202.7	49.89	204.8	2.05
K2	:b1	1.55	20.02	202.8	48.79	199.6	2.02
K3	:b1	1.61	20.02	202.8	50.36	205.9	2.05
K4	:b1	1.53	20.02	202.8	47.92	196.0	2.03
Mean value							2.04

Measured mean values of the cross-sectional dimensions of test beams J1 and J2 are given in Table 6.2.4. Measured lengths of the beams were in agreement with the nominal values given in Table 6.2.1.

Table 6.2.4. Measured cross-sectional dimensions of test beams J1 and J2.

Test beam	b_o mm	h_o mm	c_o mm	a_o mm	t mm	R mm
J1	174.2	259.9	67.4	23.9	1.552	4
J2	174.6	172.1	64.8	25.0	1.544	4

Cross-sectional constants and critical stresses for test beams J1 and J2 are given in Table 6.2.5. The critical stress of the flange $\sigma_{cr,f}$ has been calculated with the assumption of hinges at the corners and an infinitely long plate. The critical stress σ_{cr} has been determined in the following way: The stress distribution in the central section of the hat beam has been calculated according to Appendix G. The lower parts of the webs have been assumed cut off in such a way that equivalent tensile and compressive stresses are obtained over the rest of the web heights. The channel section obtained is assumed to be provided with a lower flange so that a box section is obtained. The critical stress of the box beam is

determined according to the accurate method given in Section 2.4.1. The critical load corresponding to the critical stress σ_{cr} is indicated by P_{cr} .

Table 6.2.5. Cross-sectional constants and critical stresses for test beams J1 and J2.

Test beam	b/t	h/t	$\sigma_{cr,f}$ N/mm ²	σ_{cr} N/mm ²
J1	111.3	167.5	60.1	78.9
J2	112.1	110.5	59.3	79.7

6.2.5. Measuring devices and measurement procedure.

The measurements comprised the determination of applied loads, the measurement of vertical and horizontal deflections and the recording of strains. In the measurement of test beam J1 the computer-controlled data acquisition system described in Appendix D was used. In view of the lack of accuracy in the strain measurements of test beam J1, all measurements with test beam J2 were performed manually. To avoid creep effects the deformations were kept constant at every load level. In this case "deformation" means the vertical deflection at the central section of the test beam.

Load measurement.

The two Bofors KRG-4 load cells used were connected to each other and only the total applied load was recorded. The total applied load was also recorded as a continuous function of time.

Measurement of vertical and horizontal deflections.

For the determination of vertical deflections, potentiometers with a resolution of 0.03 mm were used for the automatic measurements, while dial gauges graduated in units of 0.01 mm were used for the manual measurements. Positions and numbering of dial gauges for test beam J2 are shown in Fig. 6.2.4.

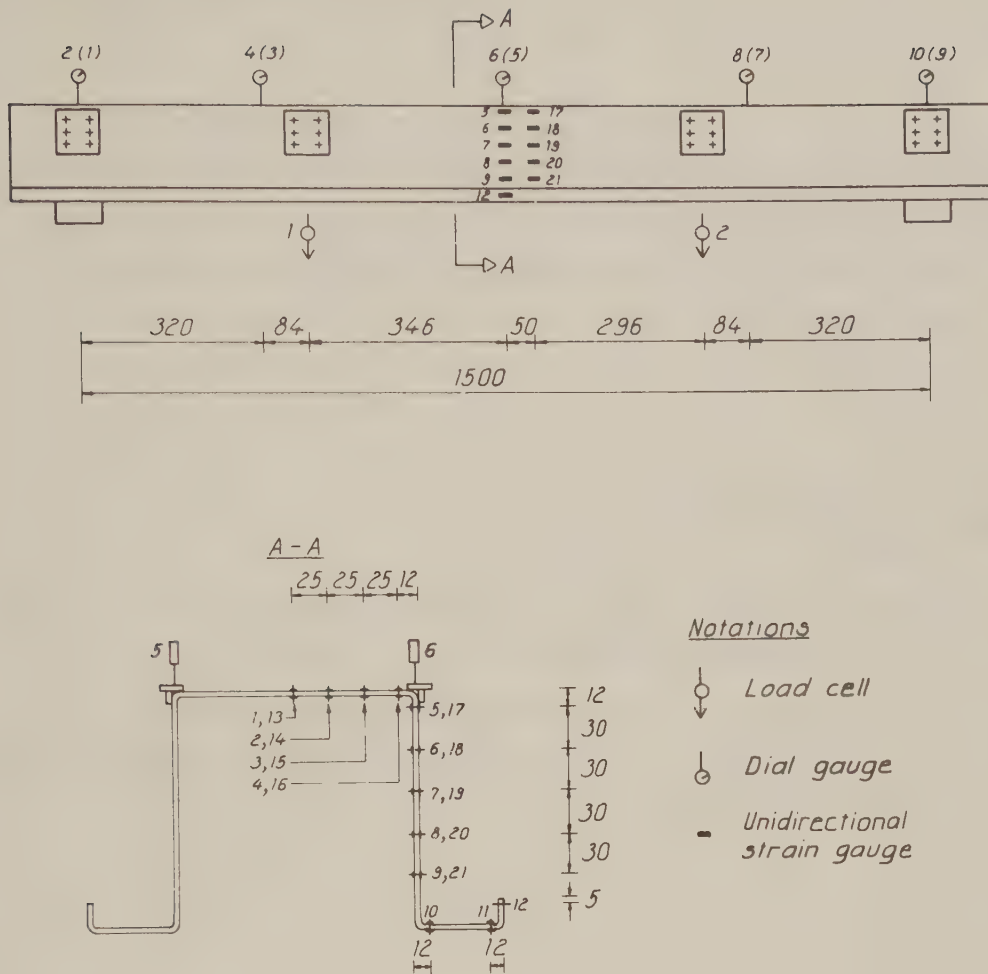


Fig. 6.2.4. Positions and numbering of load cells, dial gauges and strain gauges for test beam J2.

The horizontal deflection of the test beams is here defined as the change in distance between the two free flanges. In the automatic as well as in the manual measurements, the horizontal deflection was measured by means of a slide gauge graduated in units of 0.1 mm. The horizontal deflection was measured in the same sections of the test beams as the vertical deflection. These positions are shown for test beam J2 in Fig. 6.2.4.

Strain measurement.

The strain was measured by means of steel compensated wire strain gauges with a measuring length of 10 mm. The strain gauges were located opposite to each other on the outer and inner sides of the test beams. All strain gauges were fastened to the test beams

in the longitudinal direction. Positions and numbering of strain gauges for test beam J2 are shown in Fig. 6.2.4.

For test beam J1, the strain measurement was not achieved with satisfactory accuracy due to some defect in the scanner unit. The results from that test must thus be considered with caution. For test beam J2, the strains were measured manually with a strain gauge indicator Peekel B 105.

6.2.6. Results.

Vertical and horizontal deflections.

The relationship between the total applied load P and the vertical deflection $v_{c/L}$ for test beams J1 and J2 is shown in Figs. 6.2.5 and 6.2.6 respectively. The corresponding relationship for the horizontal deflection, i.e. the change in distance between the two flanges in tension, is shown in Figs. 6.2.7 and 6.2.8.

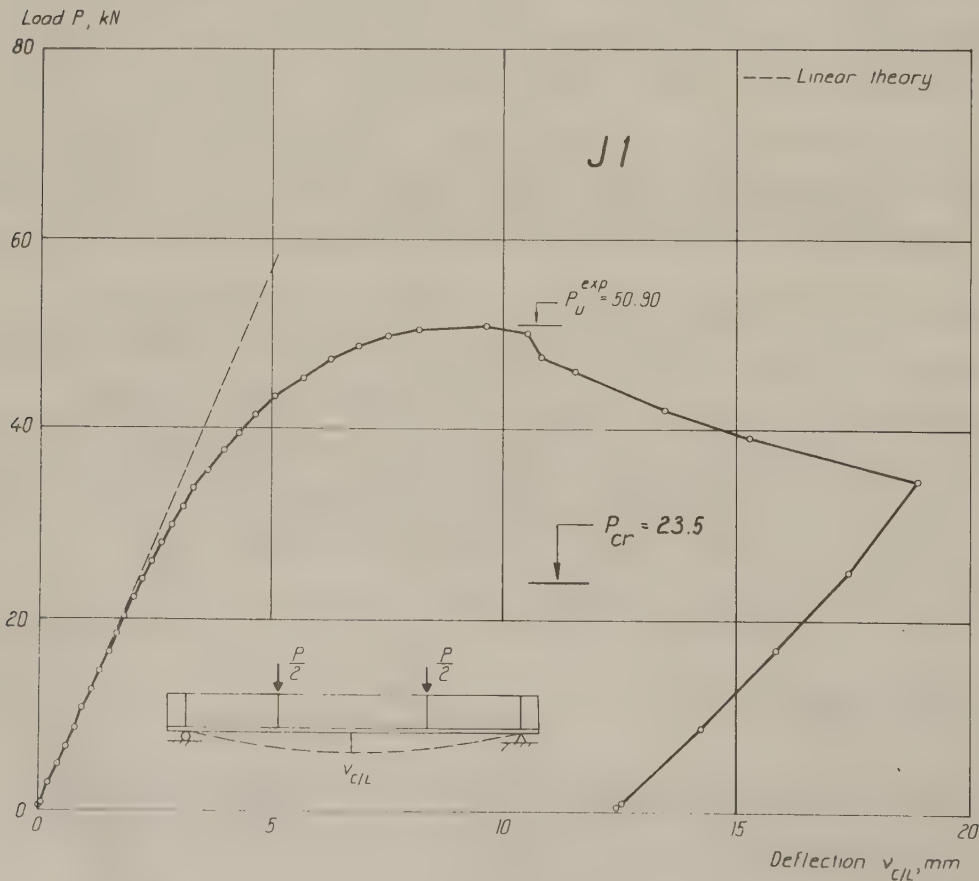


Fig. 6.2.5. Load-deflection curve for test beam J1.

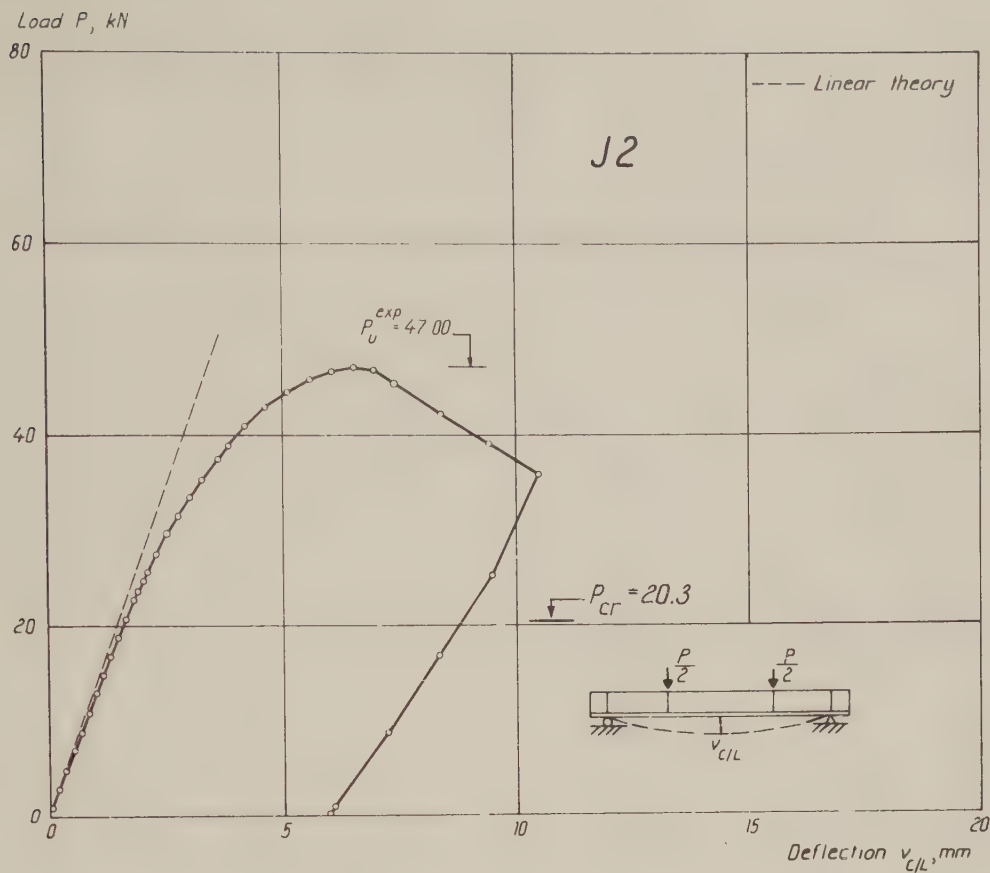


Fig. 6.2.6. Load-deflection curve for test beam J2.

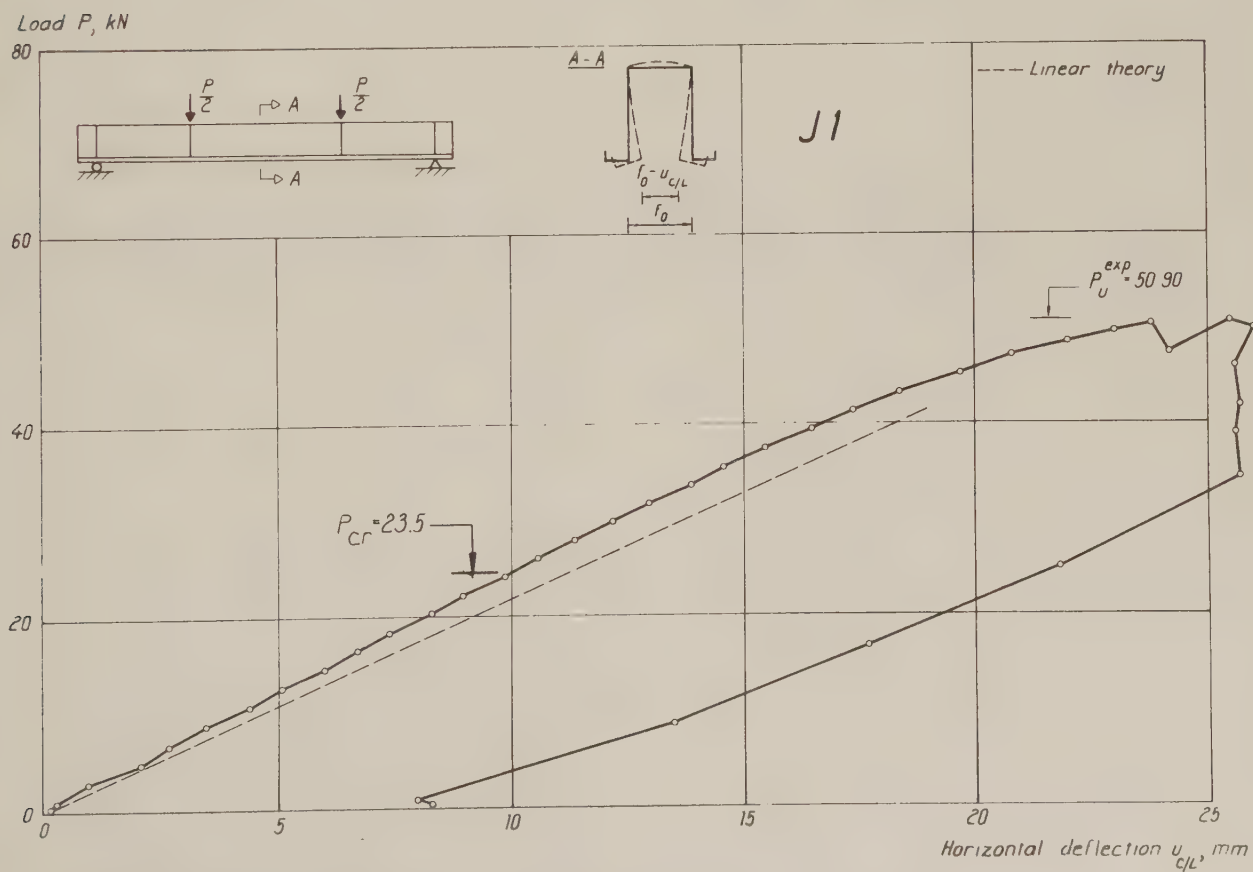


Fig. 6.2.7. Relationship between load and horizontal deflection of the free flanges for test beam J1.

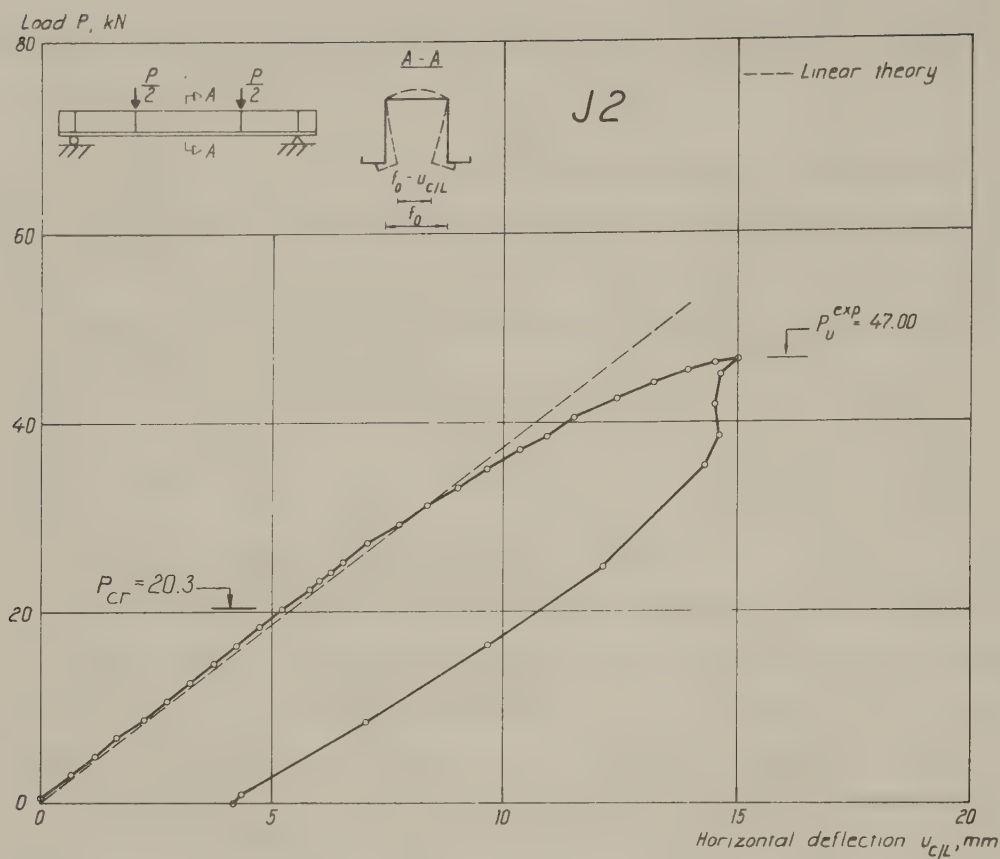
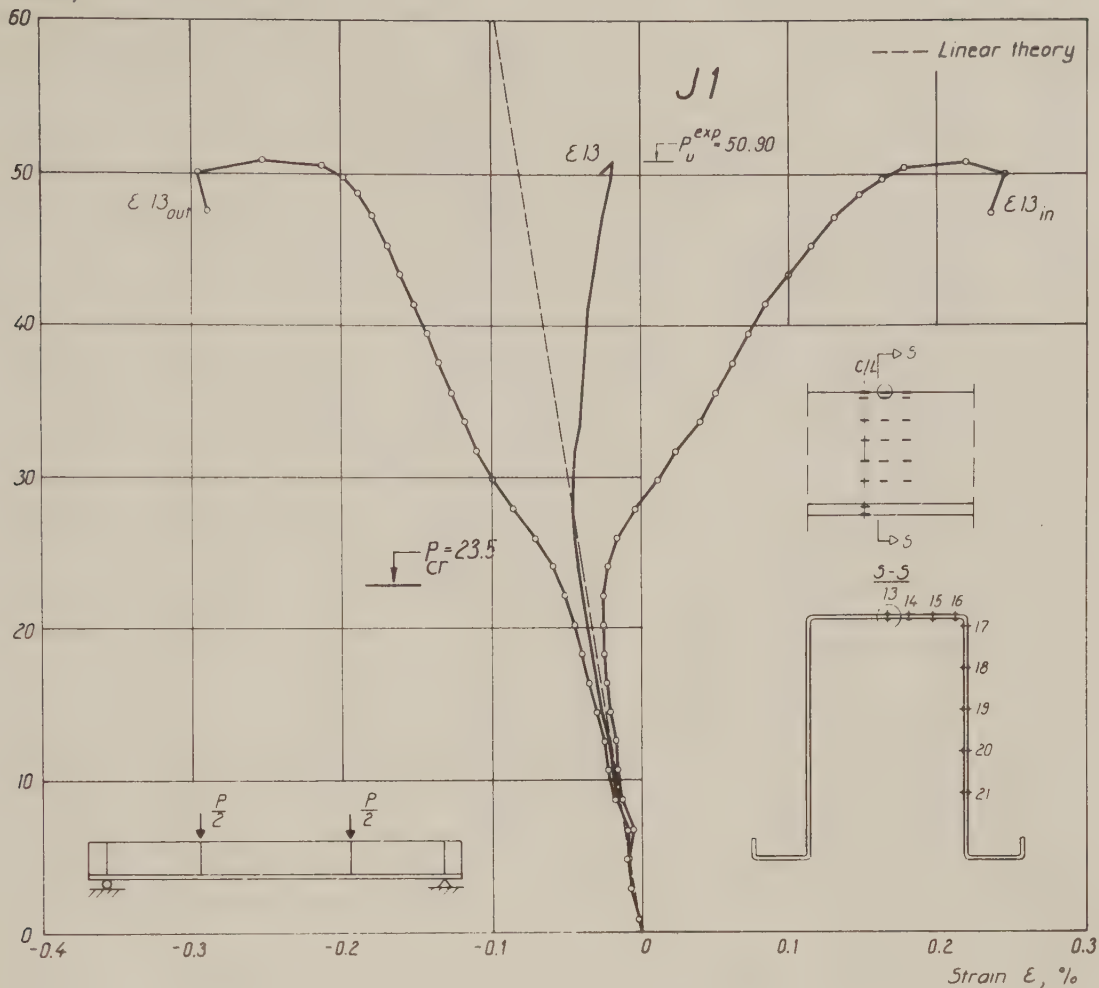


Fig. 6.2.8. Relationship between load and horizontal deflection of the free flanges for test beam J2.

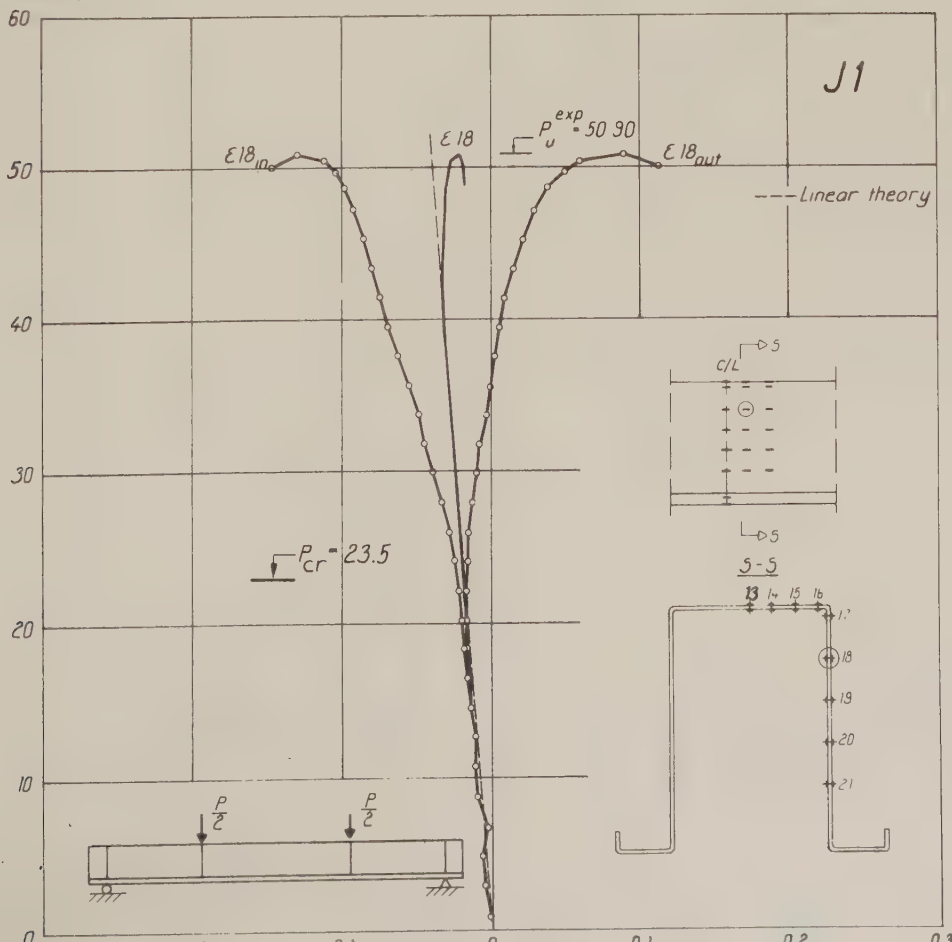
Strains.

In Figs. 6.2.9 and 6.2.10, the strains of some points on test beams J1 and J2 respectively are plotted versus the applied load P . The strain distributions of the central sections of the test beams are shown in Figs. 6.2.11 and 6.2.12 for three load levels.

a)

Load P , kN

b)

Load P , kN

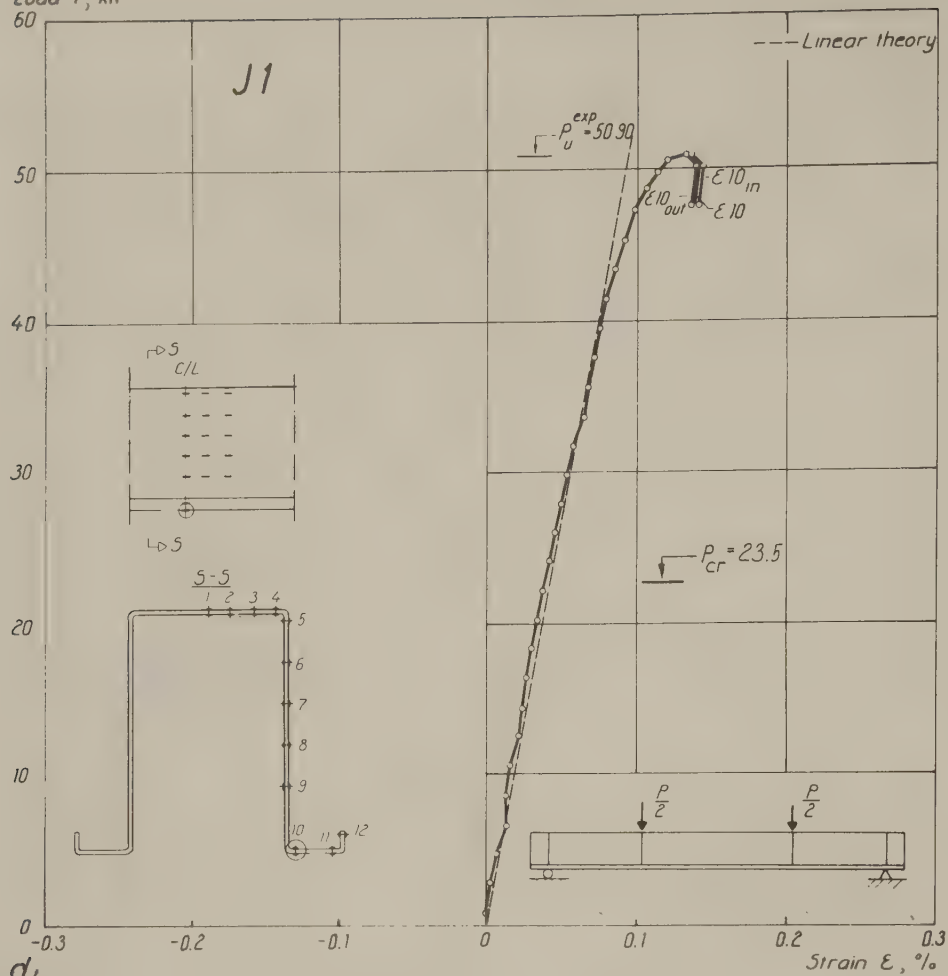
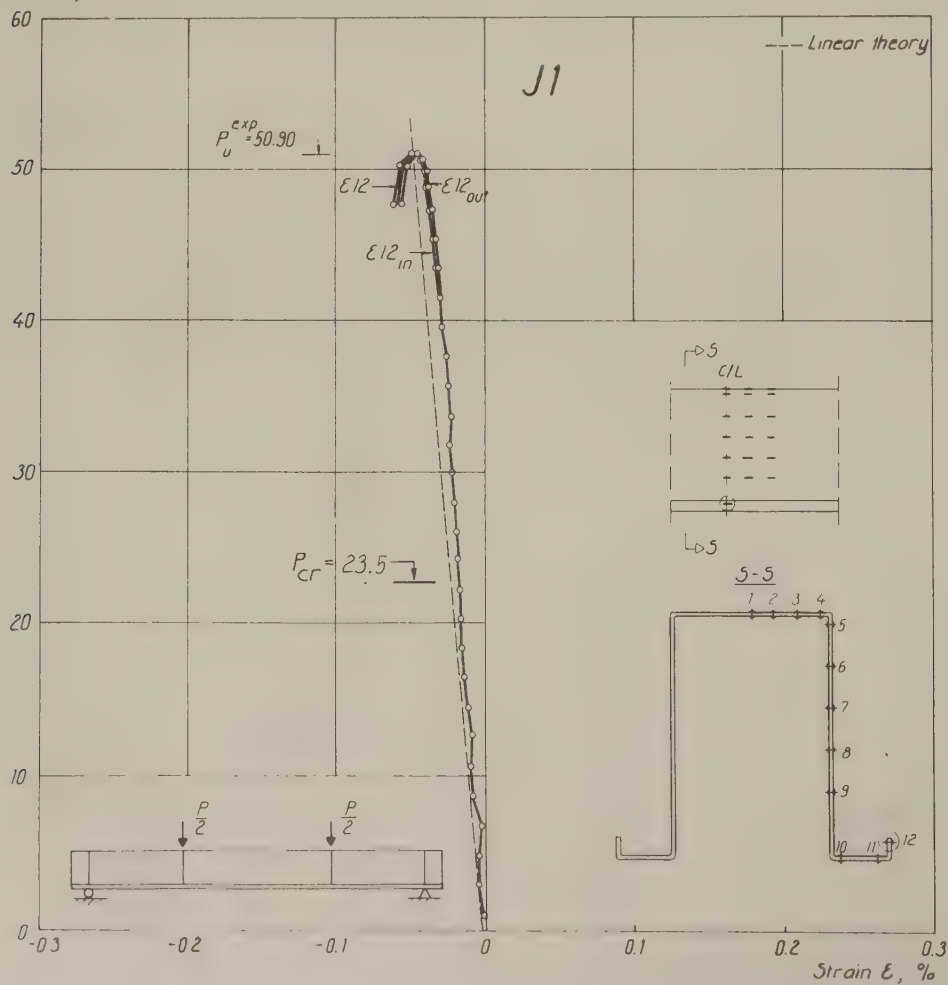
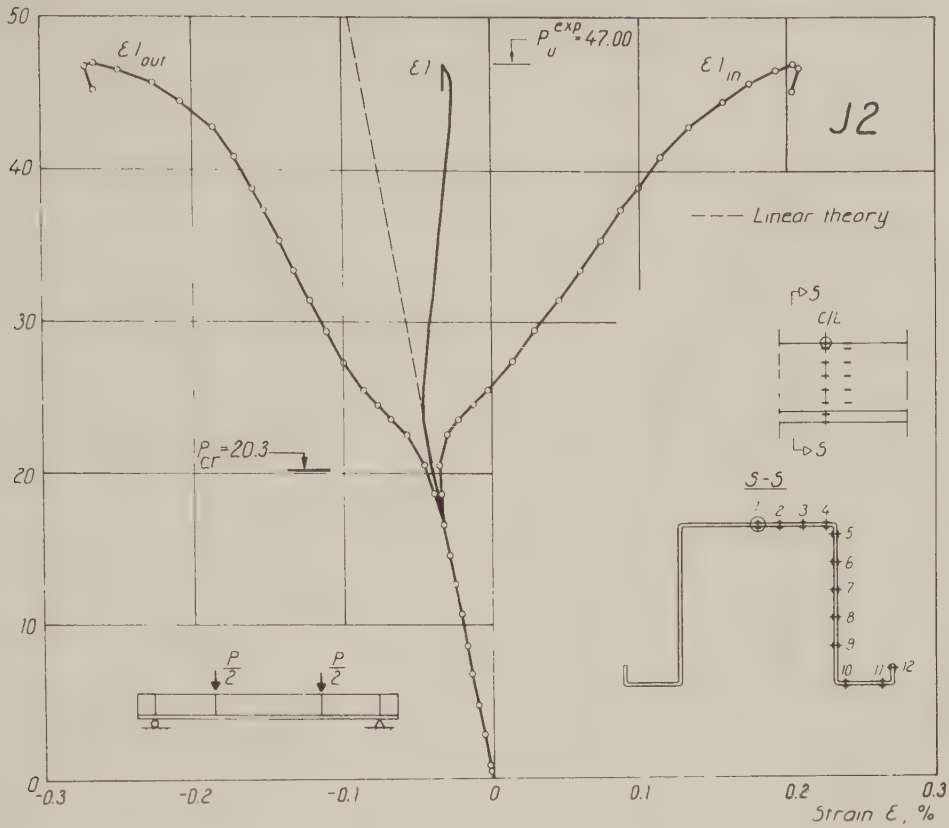
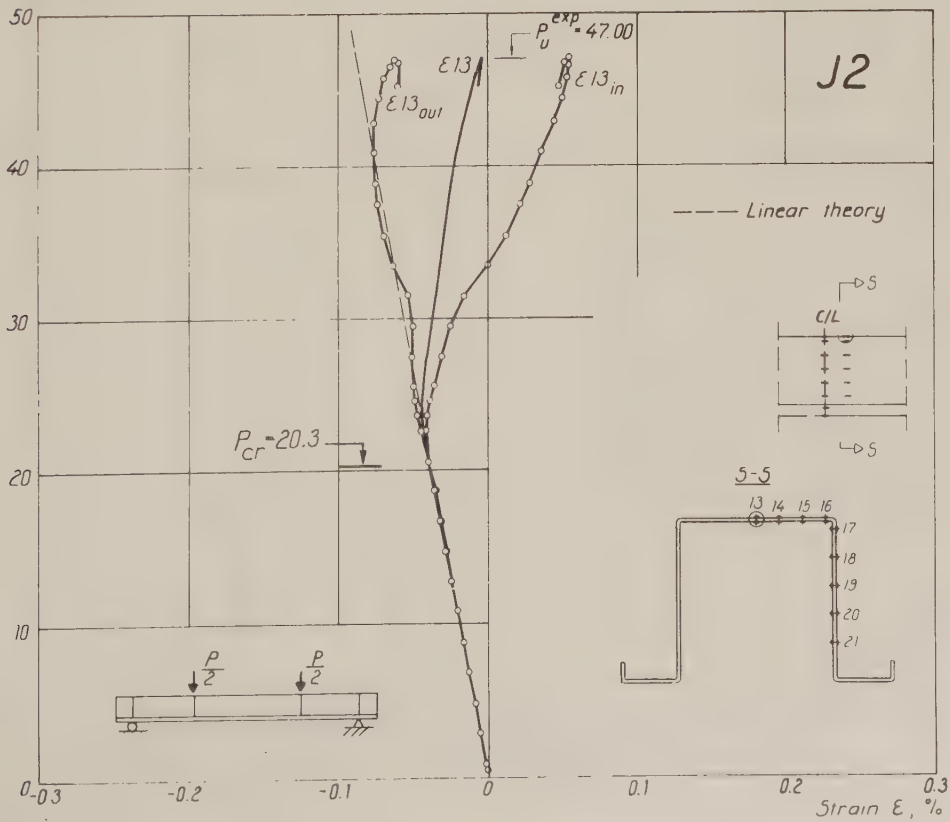
Load P , kN
60Load P , kN
60

Fig. 6.2.9 a-d. Longitudinal strains at points on the outer and inner surfaces of test beam J1.

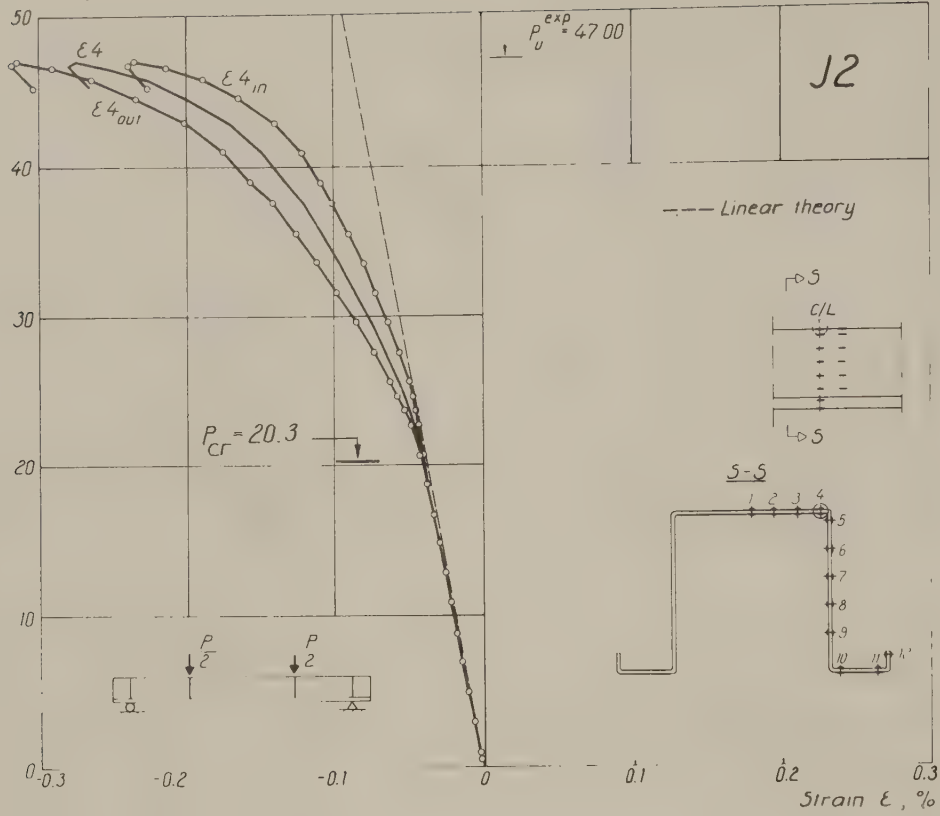
a,

Load P , kN

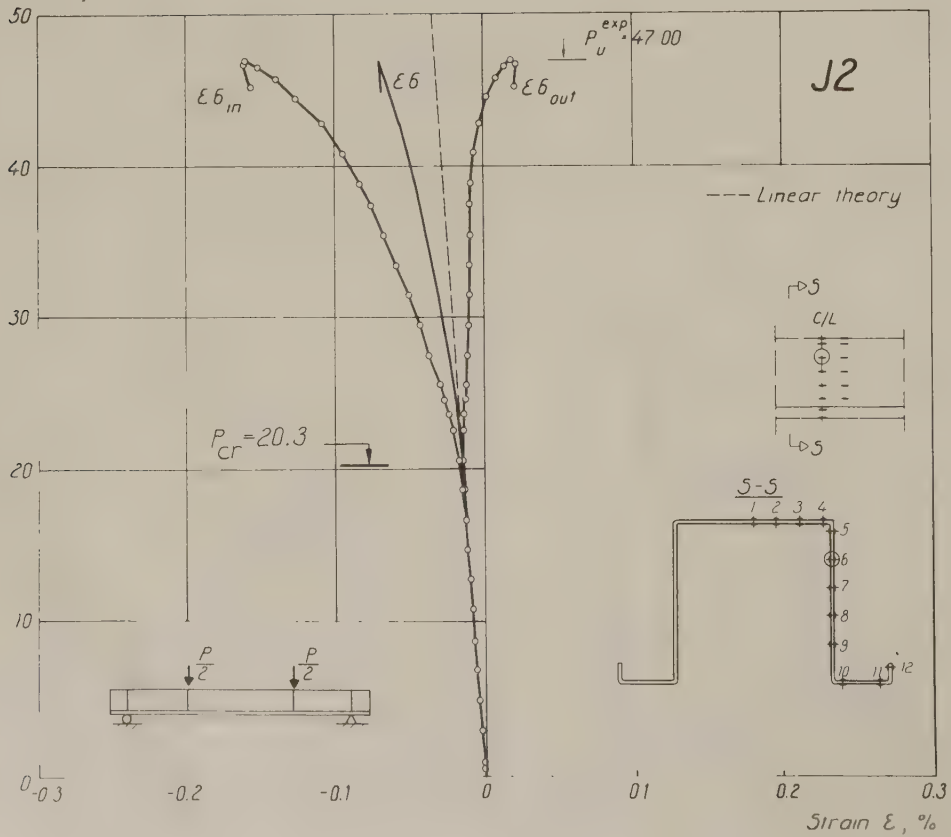
b,

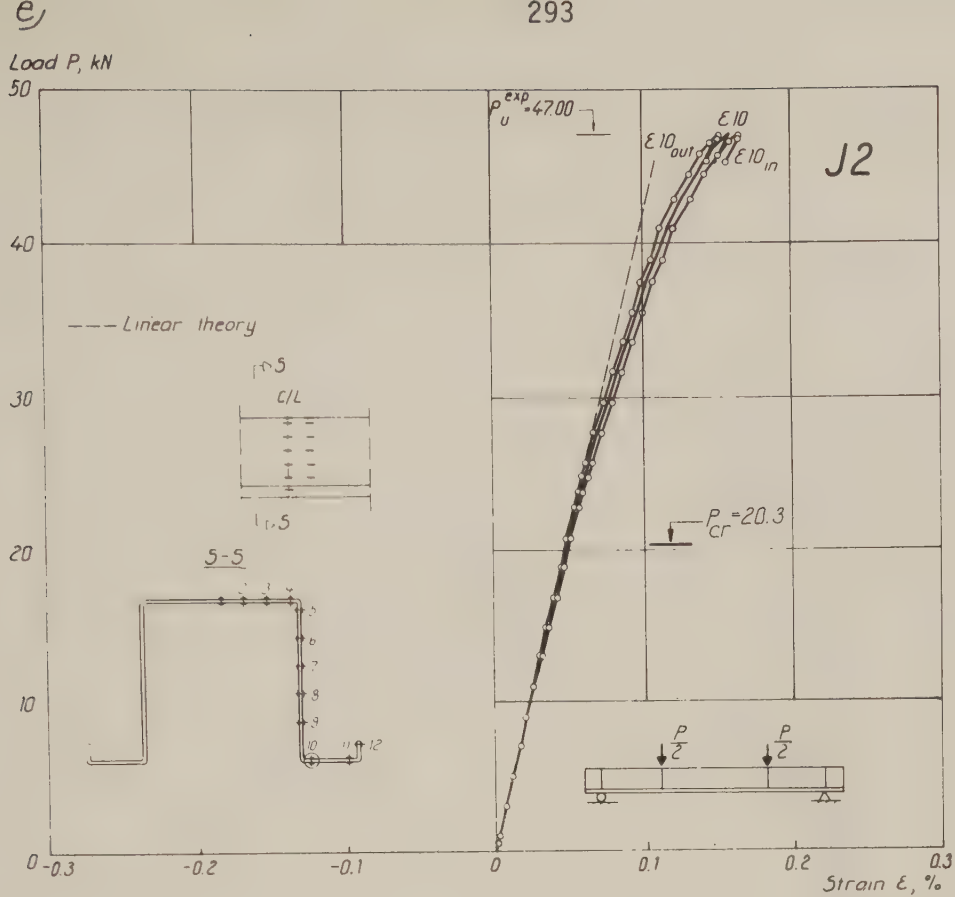
Load P , kN

c.

Load P , kN

d.

Load P , kN



f)

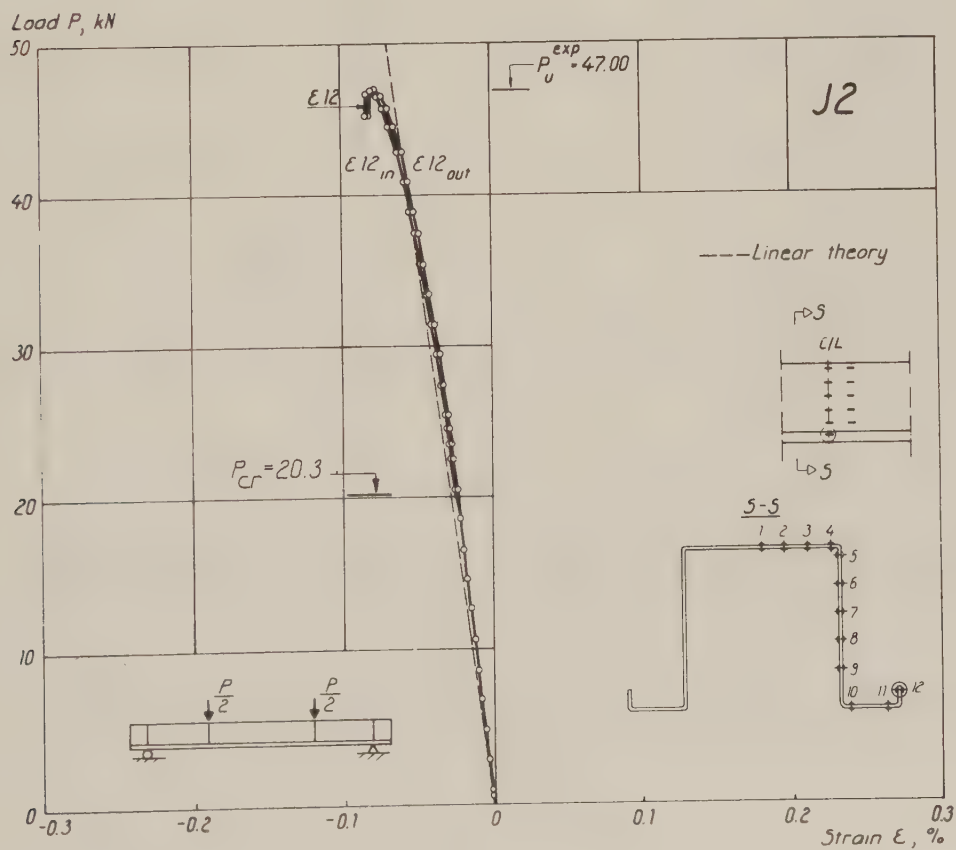


Fig. 6.2.10 a-f. Longitudinal strains at points on the outer and inner surfaces of test beam J2.

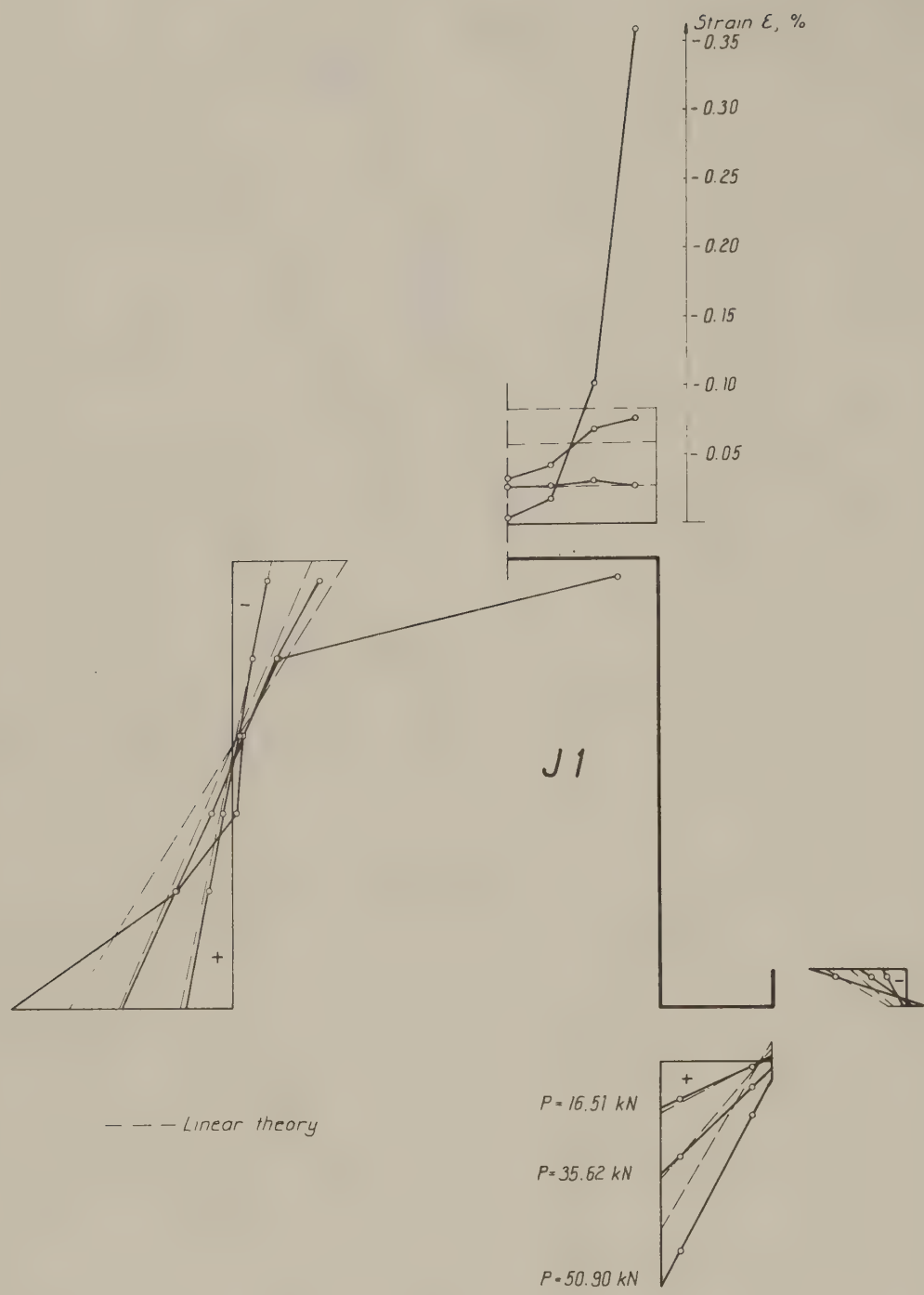


Fig. 6.2.11. Distribution of strains in midspan of test beam J1.

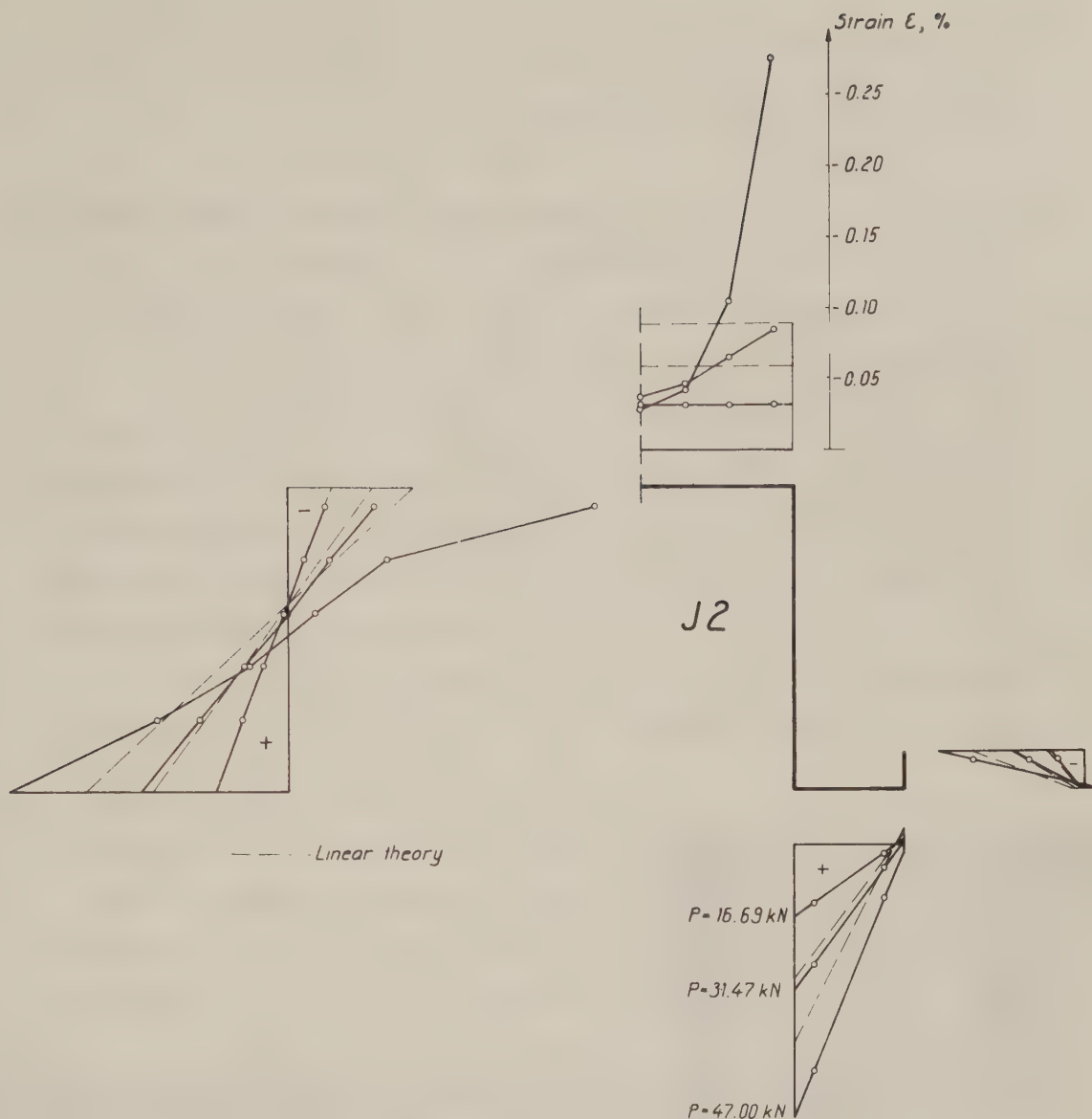


Fig. 6.2.12. Distribution of strains in midspan of test beam J2.

Ultimate loads.

Measured ultimate loads and ultimate moments are given in Table 6.2.6.

Table 6.2.6. Measured ultimate loads and ultimate bending moments of test beams J1 and J2.

Test beam	P_u^{exp} kN	M_u^{exp} kNm
J1	50.90	15.45
J2	47.00	9.49

6.2.7. Discussion of results.

Vertical and horizontal deflections.

The vertical and horizontal deflections of the test beams have been calculated according to the theory given in Appendix G. The results of the calculations are shown in Figs. 6.2.5-6.2.8. The agreement between measured and calculated vertical deflections is good at low loads and up to the critical load P_{cr} . The deviation at high loads depends on development of buckles in the compressed flange and in the compressed parts of the webs. The calculated horizontal deflections are in rather good agreement with the measured ones. The agreement in this case is also good for high loads, owing to the fact that no local buckles develop in the free flanges, and consequently the bending rigidity referring to horizontal deflections is maintained.

If the simplified theory, given in Appendix F, is used instead, the theoretical deflections of the central section will increase somewhat. Thus the vertical and horizontal deflections of test beams J1 and J2 will increase by about 1 and 5-10 per cent respectively. Further, if the elastic constant of the free flange is put equal to zero, i.e. if the transverse flexural rigidity of the webs and the compression flange is neglected, the corresponding increases will be about 3 and 10-15 per cent respectively.

Strains.

In Figs. 6.2.9 and 6.2.10, measured strains are compared with strains calculated according to the theory given in Appendix G. The agreement is good in the sub-critical range. As can be expected, the measured curves will deviate from the theoretical ones when the theoretically determined critical load P_{cr} is approached. The same conclusions can be drawn by studying Figs. 6.2.11 and 6.2.12.

A comparison between the results of the simplified theory (Appendix F) and of the accurate theory (Appendix G) shows that the strains are in very good agreement in the compression flange, in rather good agreement in the tension flange and entirely different in the free end of the "lip".

Character of failure.

Failure occurred as a local folding of the compressed flange and of the adjacent parts of the webs, in the same way as for the box beams previously tested (cf. Section 3.3.8). The failure develops rather slowly with a striking degree of plasticity. As can be seen in Figs. 6.2.5 and 6.2.6, there is a gradual decrease in the load-carrying capacity after failure.

An example of failure is shown in Fig. 6.2.13.

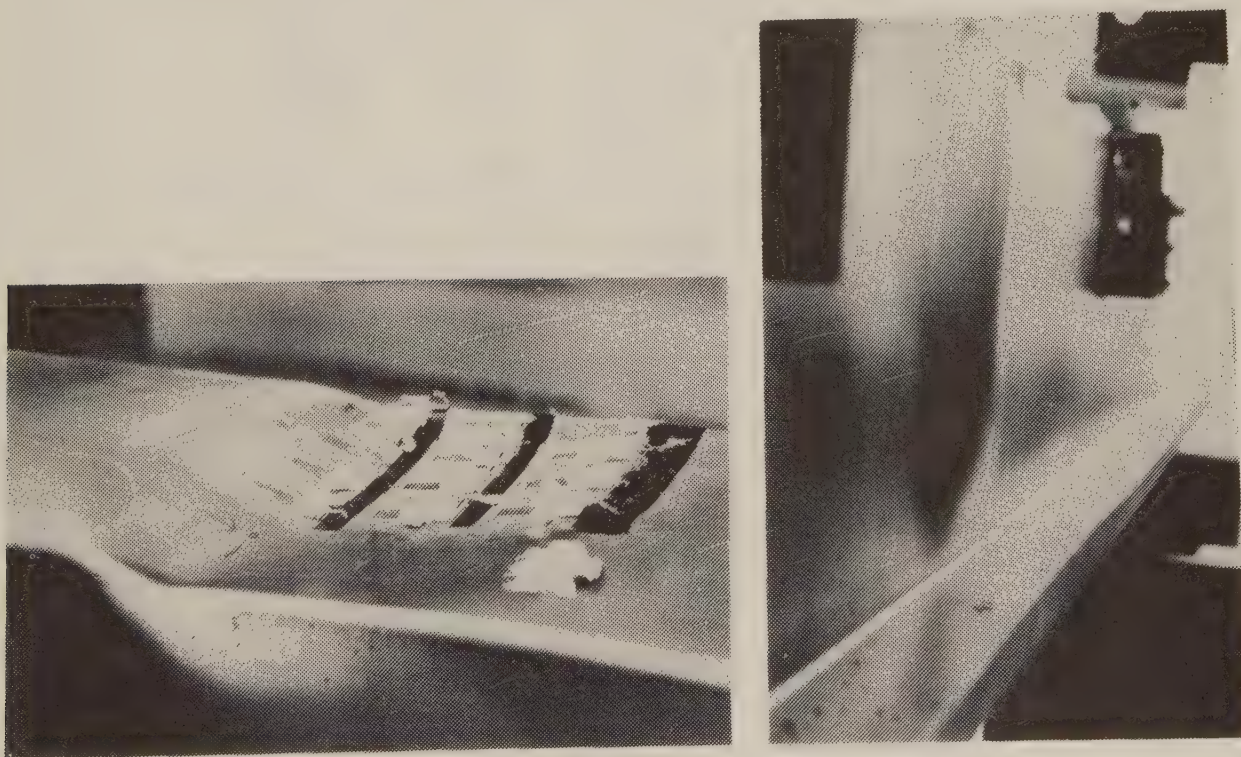


Fig. 6.2.13. Example of local failure.

Ultimate loads.

The ultimate bending moments of test beams J1 and J2 have been determined in accordance with the two methods described in

Section 6.1.3. The results are shown in Table 6.2.7 where the ratios of experimental to theoretical ultimate moments are also given.

It is interesting to note that the second method where the open hat beam section has been treated like a closed box beam section gives values that are only slightly on the unsafe side. One of the reasons is that the free flanges have a large curvature in the horizontal plane giving rise to second order stabilizing forces on account of the tension forces in the free flanges. For long beams where the curvature usually is less, therefore, this method ought to give poorer agreement between theoretical and experimental ultimate bending moments.

Table 6.2.7. Theoretical ultimate bending moments. Ratios of measured to theoretical ultimate moments.

Test beam	Reduced tension flanges		Box beam section	
	M_u^{th}	M_u^{exp}/M_u^{th}	M_u^{th}	M_u^{exp}/M_u^{th}
	kNm		kNm	
J1	14.51	1.06	15.97	0.97
J2	8.92	1.06	10.02	0.95

6.3. Single-span beam loaded with distributed load.

6.3.1. Purpose.

The main purpose of the experiments was to study the post-buckling behaviour of hat-section beams subjected to pure bending and to find out the influence of the compressed flange when it was free to move or restrained by a corrugated steel sheeting.

6.3.2. Scope.

The experiments comprised a test series of four beams, K1-K4. Distribution of load, shear force and bending moment are shown in Fig. 6.3.1. The compressed flanges of the test beams K1 and K2 were free to move during the tests, while the compressed flanges of the test beams K3 and K4 were restrained by corrugated sheeting. Nominal dimensions of the test beams are given in Table 6.3.1.

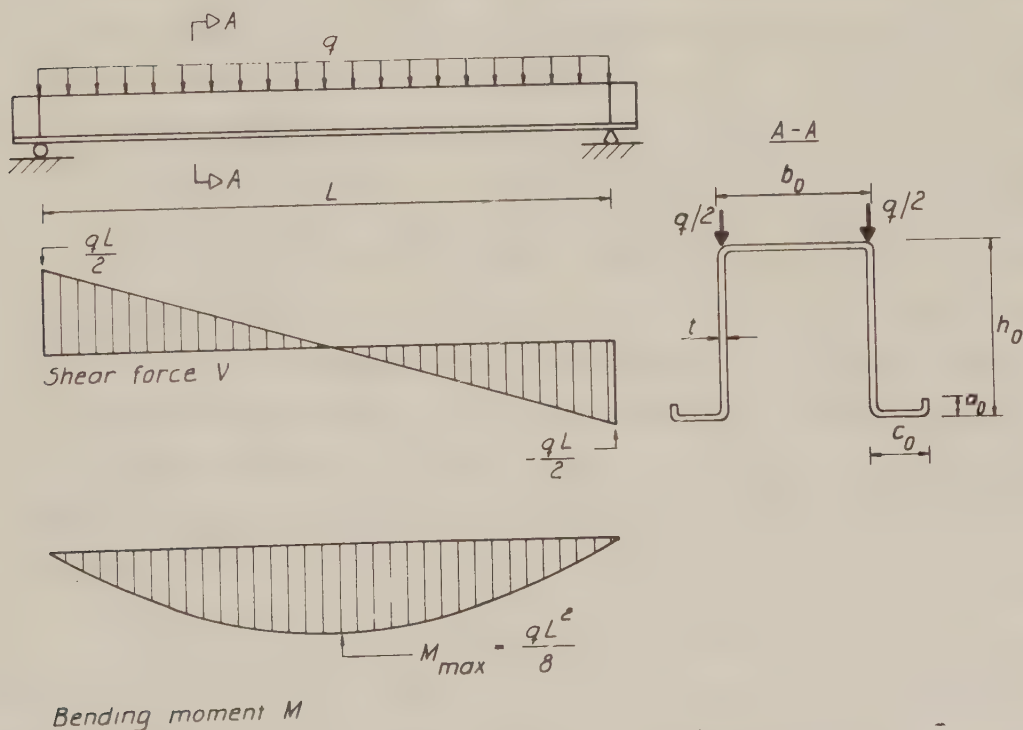


Fig. 6.3.1. Assumed distribution of load, shear force and bending moment for test beams K1-K4. Notations for cross section.

Table 6.3.1. Nominal dimensions of test beams K1-K4.

Test beam	L mm	b_o mm	h_o mm	c_o mm	a_o mm	t mm
K1, K3	2254	173	260	65	22	1.5
K2, K4	1501	173	173	65	22	1.5

6.3.3. Loading and support arrangements.

The principal loading and support arrangements for test beams K1 and K2 are shown in Fig. 6.3.2. For test beams K3 and K4 the loading arrangements were somewhat changed, as can be seen in Fig. 6.3.3. The support arrangements were the same as those described in Section 6.2.3.

To obtain a uniformly distributed load, a hydraulic jack and a number of load-distributing members were used. For test beams K1 and K2, the loads were transferred directly to the compression flanges by means of U-section members with rubber spacers. Eight pairs of U-section members were used on each test beam. The transverse distance between two U-sections was 150 mm. The thickness of the rubber spacers was 4 mm and their hardness 40 shore. With these loading arrangements, the load was transferred to a plane slightly inside the webs. A small transverse bending moment was thus applied to the test beams. For test beams K3 and K4, the load was transferred to the compression flanges via corrugated steel sheeting. To obtain a uniform load distribution on the corrugated steel sheeting, flat bars were placed on the top of the corrugations. The same loading arrangements as for test beams K1 and K2 were used above these flat bars. The corrugated steel sheeting was fastened to the test beams at 300 mm intervals in the middle of the compression flange by POP-rivets TAP-44 (diameter 3.2 mm length 5.8 mm). The dimensions of the corrugated steel sheeting, designated DO-TP 20, are given in Fig. 6.3.4. All the test beams K1-K4 were fitted with lateral stays.

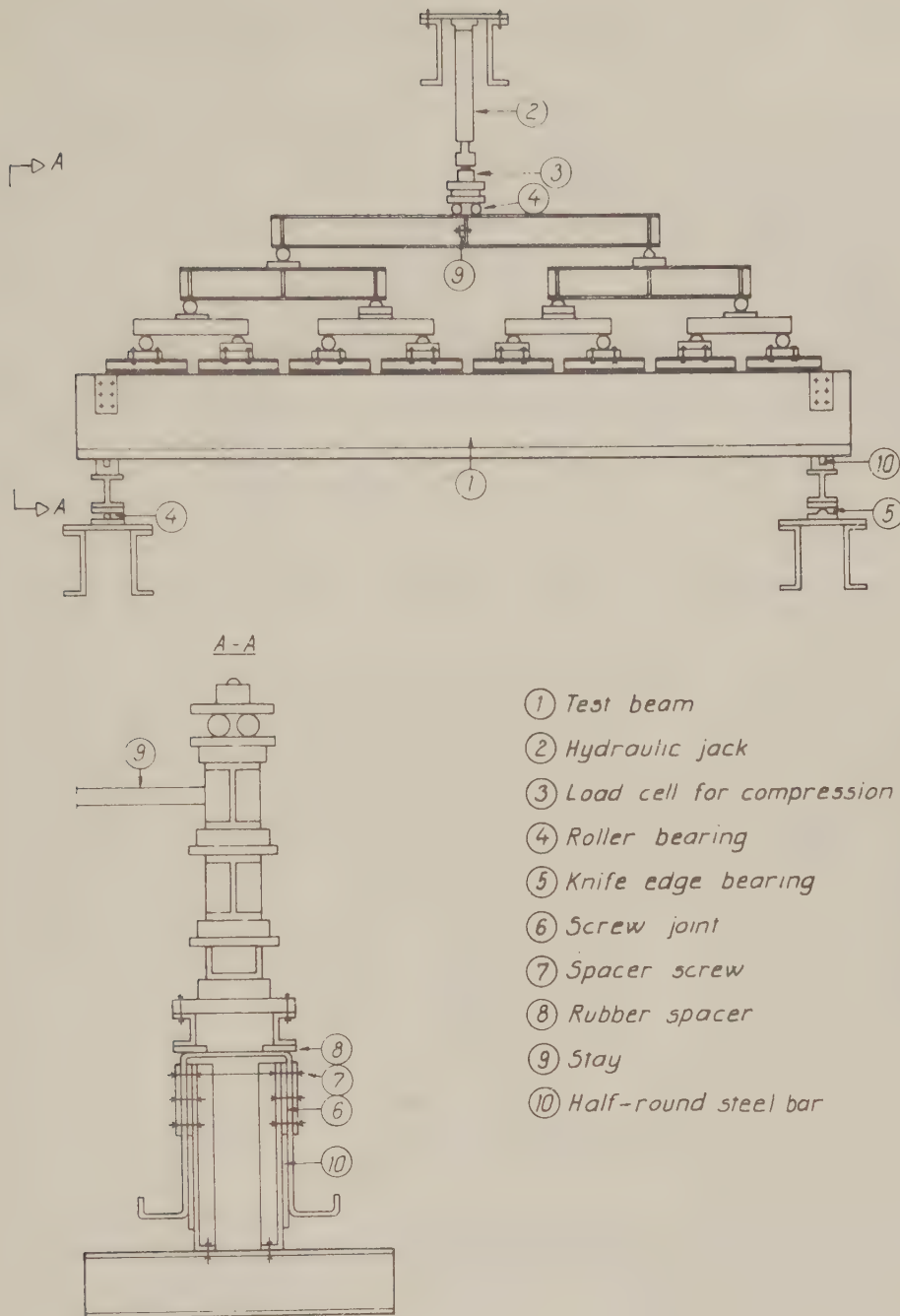


Fig. 6.3.2. Principal loading and support arrangements for test beams K1 and K2.

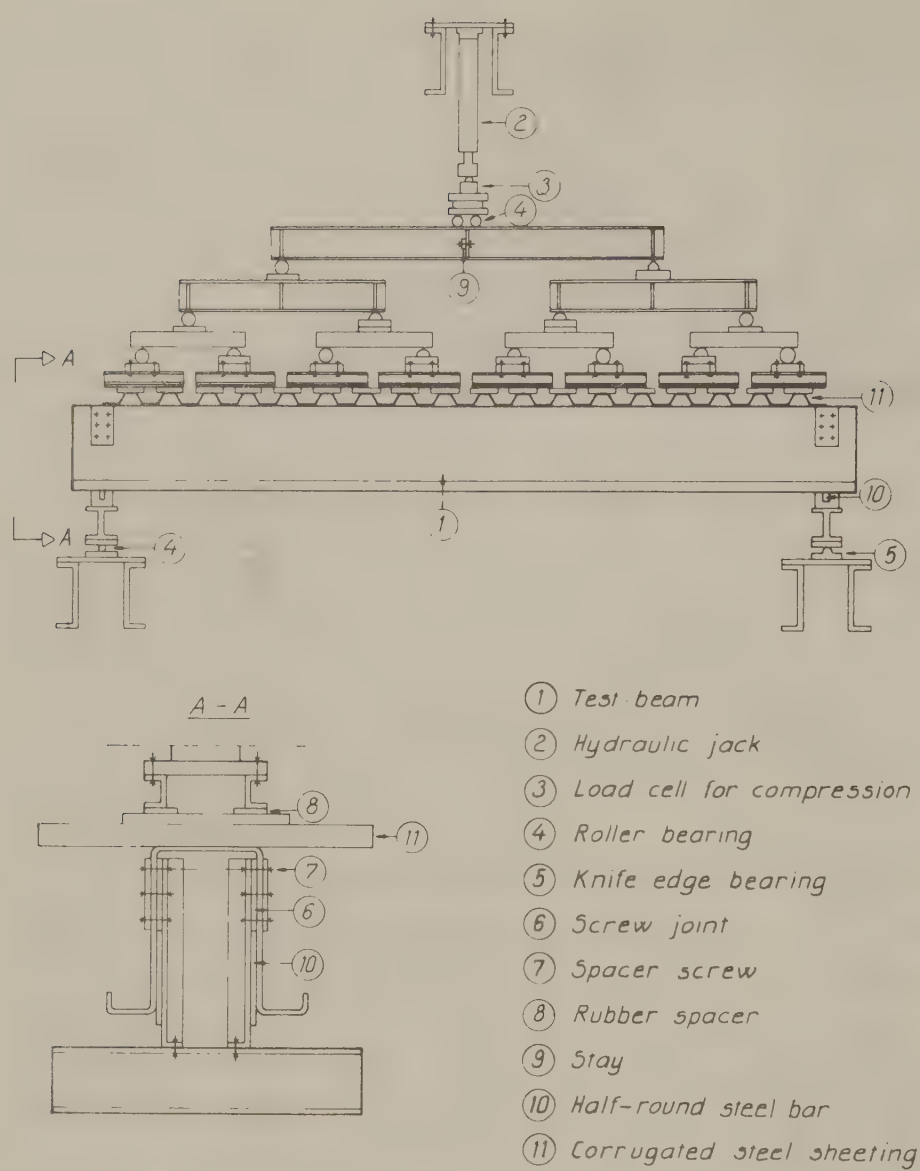


Fig. 6.3.3. Details from loading and support arrangements for test beams K3 and K4.

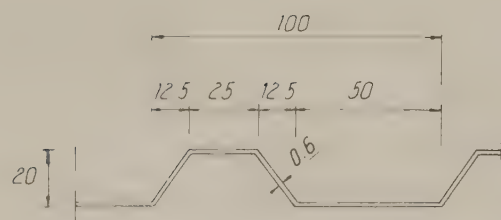


Fig. 6.3.4. Dimensions of steel sheeting used for test beams K3 and K4.

6.3.4. Dimensions. Material properties.

The manufacture and the material properties of test beams K1-K4 were described in Section 6.2.4. Mean values of measured cross-sectional dimensions of test beams K1-K4 are given in Table 6.3.2. Measured lengths of the beams were in agreement with the nominal lengths given in Table 6.3.1.

Four tensile test pieces were taken from the corrugated steel sheeting. The yield stress of the material was 26.3 N/mm^2 and the ultimate stress 36.3 N/mm^2 .

Table 6.3.2. Measured cross-sectional dimensions of test beams K1-K4.

Test beam	b_o mm	h_o mm	c_o mm	a_o mm	t mm	R mm
K1	173.8	257.8	66.6	26.3	1.548	4
K2	174.6	173.0	65.9	23.9	1.532	4
K3	173.9	258.1	66.5	26.8	1.581	4
K4	173.3	172.8	65.5	24.7	1.525	4

6.3.5. Measuring devices and measurement procedure.

All measurements were performed manually. The measurements comprised the determination of applied load and the recording of vertical and horizontal deflections. To avoid creep effects, the deformation was kept constant at each load level. In this case, "deformation" means the vertical deflection at the central section of the test beam.

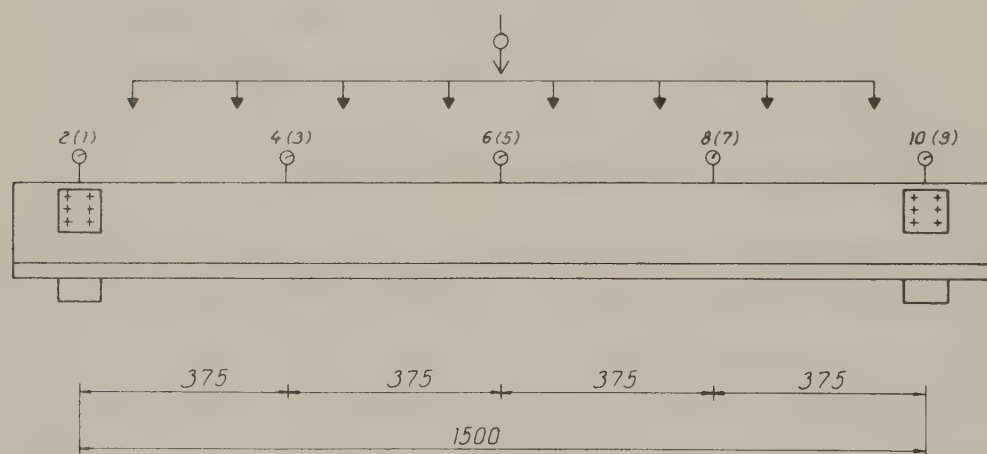
Load measurement.

The load cell used was of the Bofors LSK-2 type. The load was recorded by means of the digital voltmeter, HP 3450 A, described in Appendix C. The load was also recorded as a continuous function of time.

Measurement of vertical and horizontal deflections.

The vertical deflection of the test beams was measured by means of dial gauges graduated in units of 0.01 mm. For test beams K1 and K2, the dial gauges measured against pieces of perspex fastened to the webs (cf. Fig. 6.2.4 Section A-A). For test beams K3 and K4, where corrugated steel sheeting was fastened to the compression flange, small apertures were cut out in the sheeting to permit measurement of the deflection of the test beams. Thus the dial gauges numbered 1, 2, 9 and 10 (see Fig. 6.3.5) measured against pieces of perspex as for test beams K1 and K2. The dial gauges numbered 3, 4, 7 and 8 were connected by means of thin copper wires to the webs. The dial gauges numbered 5 and 6 measured against a trough of the corrugated sheeting.

The horizontal deflection, defined as the change in distance between the two free flanges, was measured by means of a slide gauge graduated in units of 0.1 mm. The horizontal deflection was measured at the same sections as the vertical deflection, see Fig. 6.3.5.



Notations

○ Load cell

○ Dial gauge

Fig. 6.3.5. Positions and numbering of load cells and dial gauges for test beams K2 and K4.

6.3.6. Results.

Vertical and horizontal deflections.

In Figs. 6.3.6-6.3.9 the relation between the total applied load Q and the deflection $v_{C/L}$ at the central section of the test beams is shown. The horizontal deflection $u_{C/L}$, i.e. the change in distance between the free flanges, at the central section is plotted versus Q in Figs. 6.3.10-6.3.13.

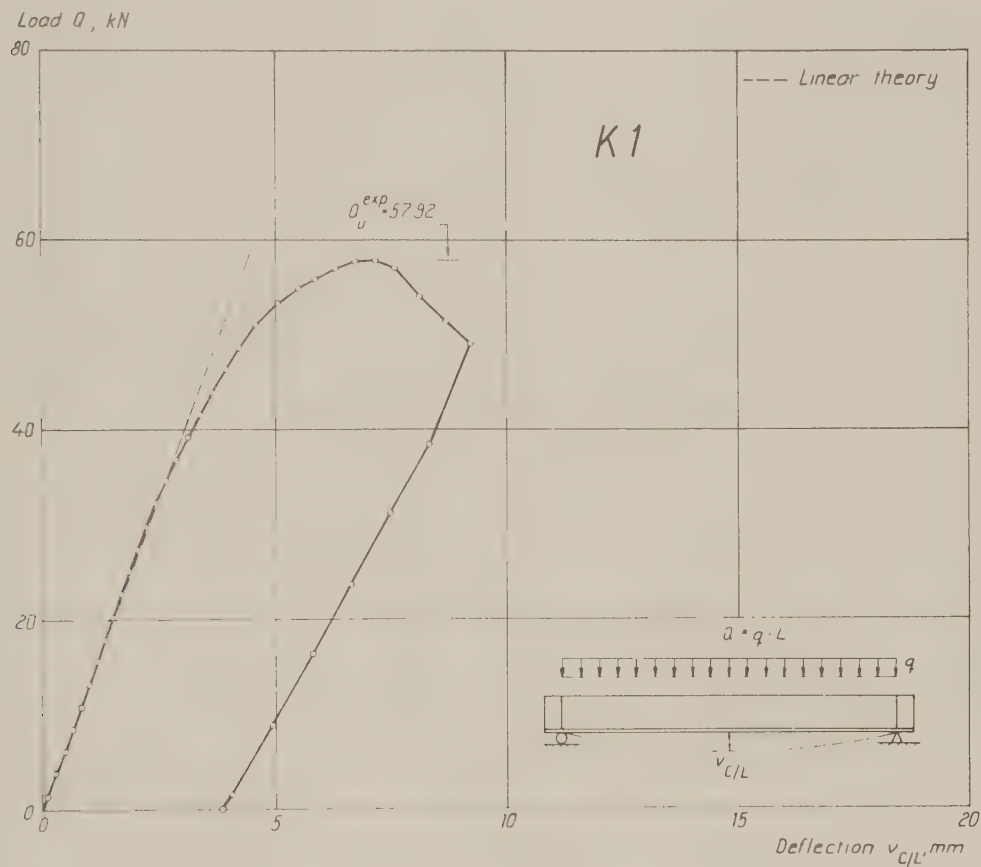


Fig. 6.3.6. Load-deflection curve for test beam K1.

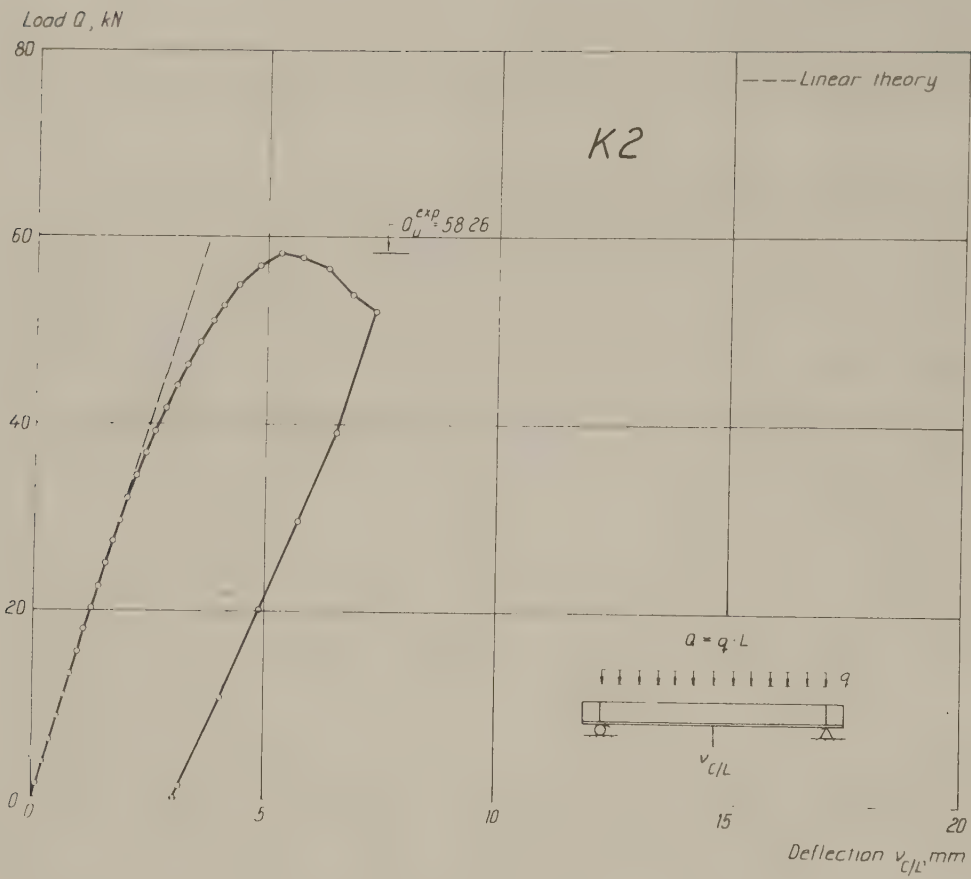


Fig. 6.3.7. Load-deflection curve for test beam K2.

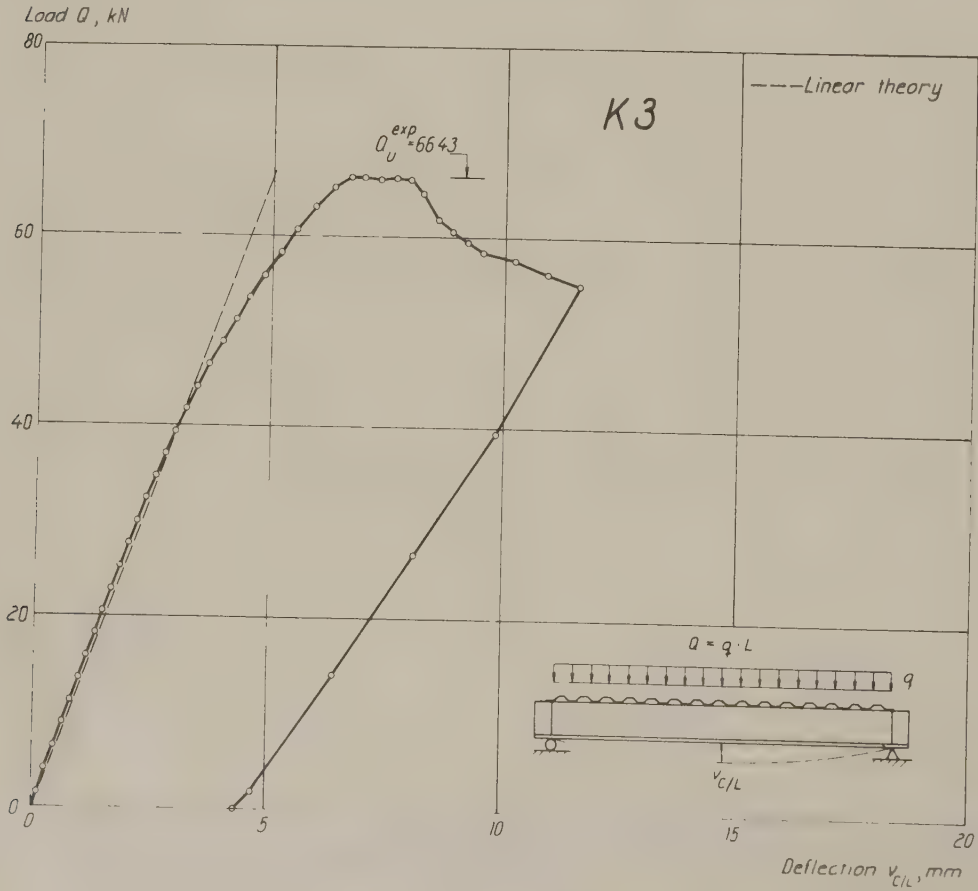


Fig. 6.3.8. Load-deflection curve for test beam K3.

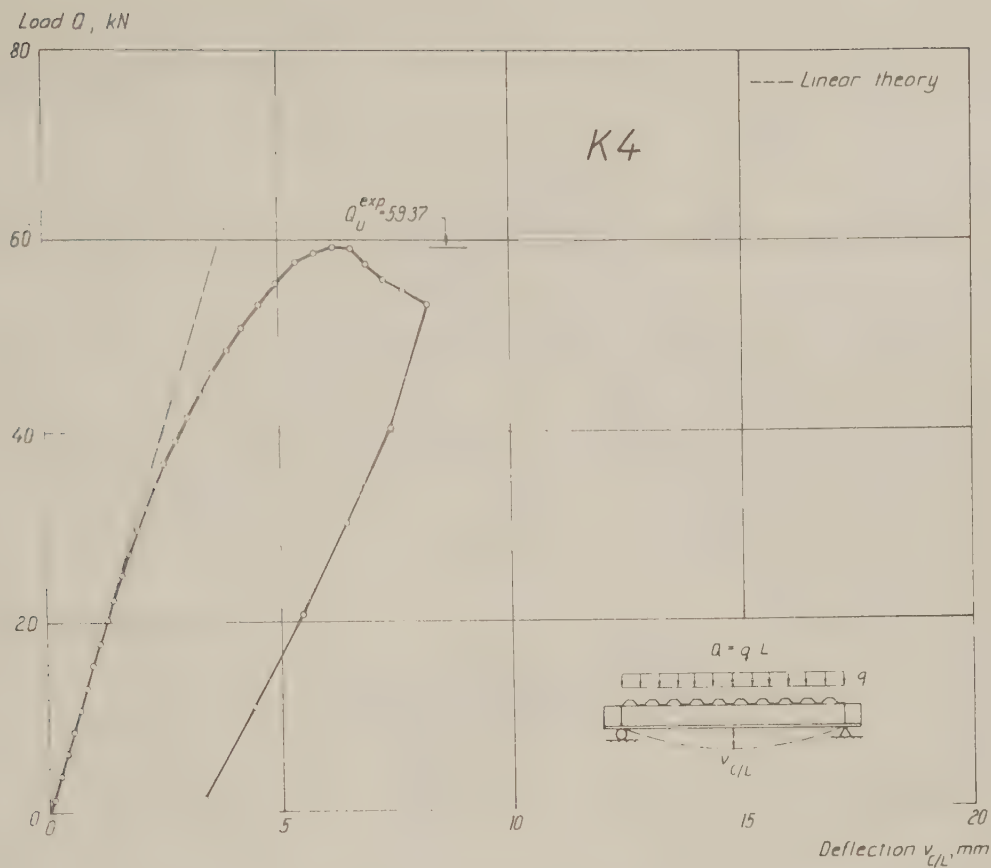


Fig. 6.3.9. Load-deflection curve for test beam K4.

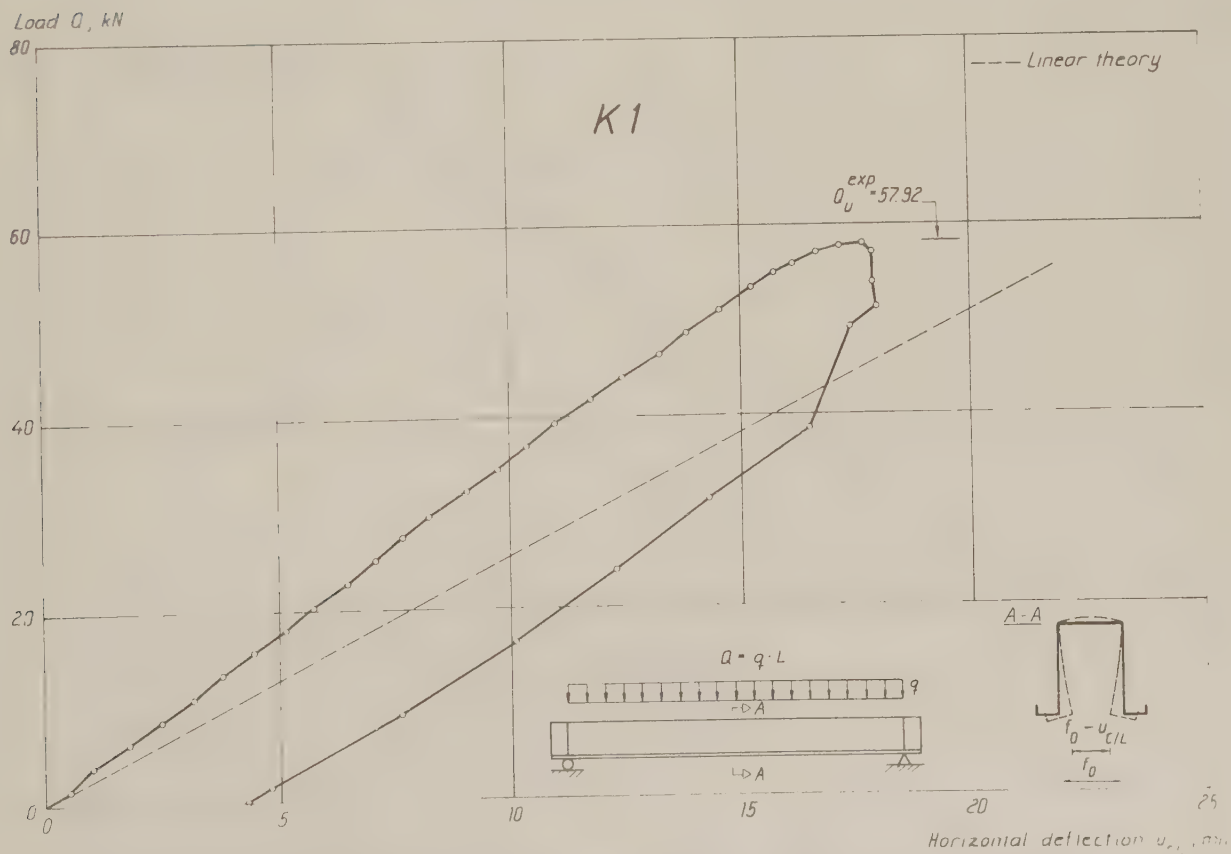


Fig. 6.3.10. Relationship between load and horizontal deflection of the free flanges for test beam K1.

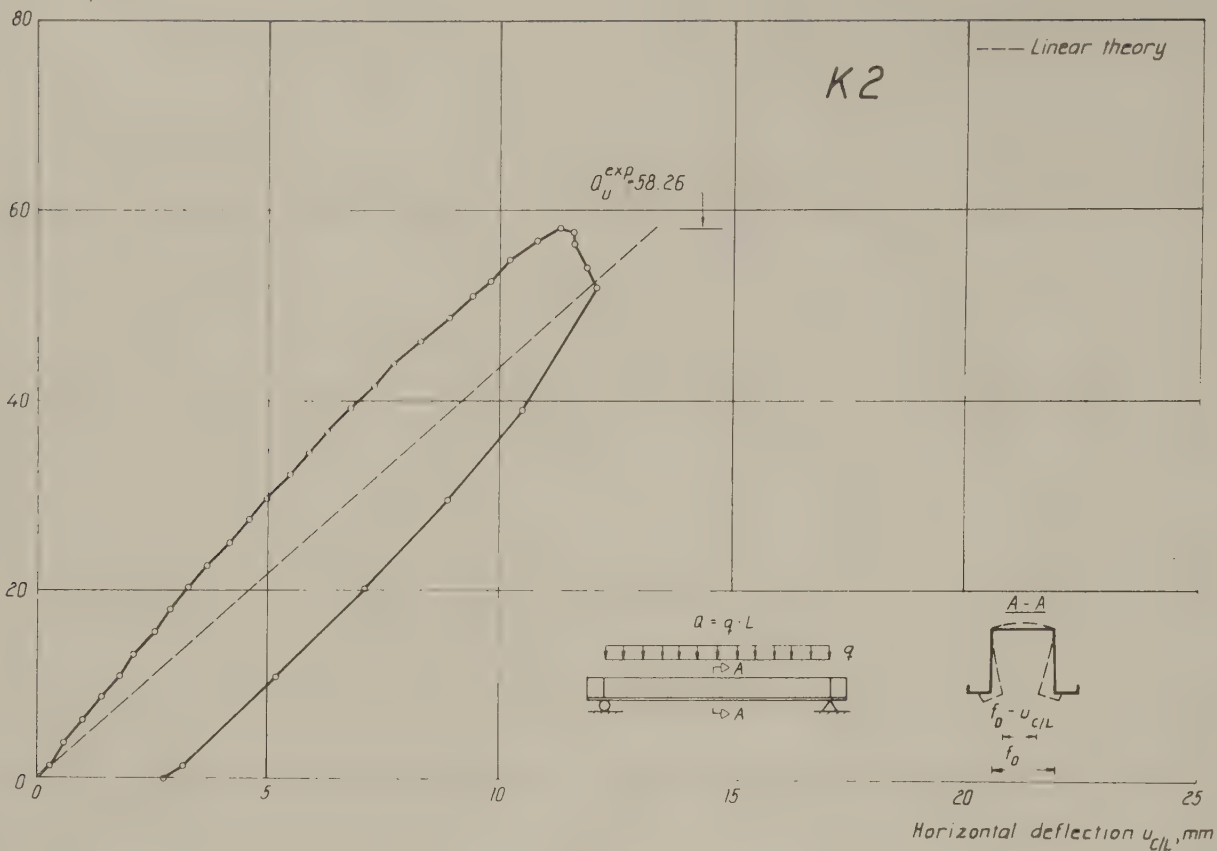
Load Q , kN

Fig. 6.3.11. Relationship between load and horizontal deflection of the free flanges for test beam K2.

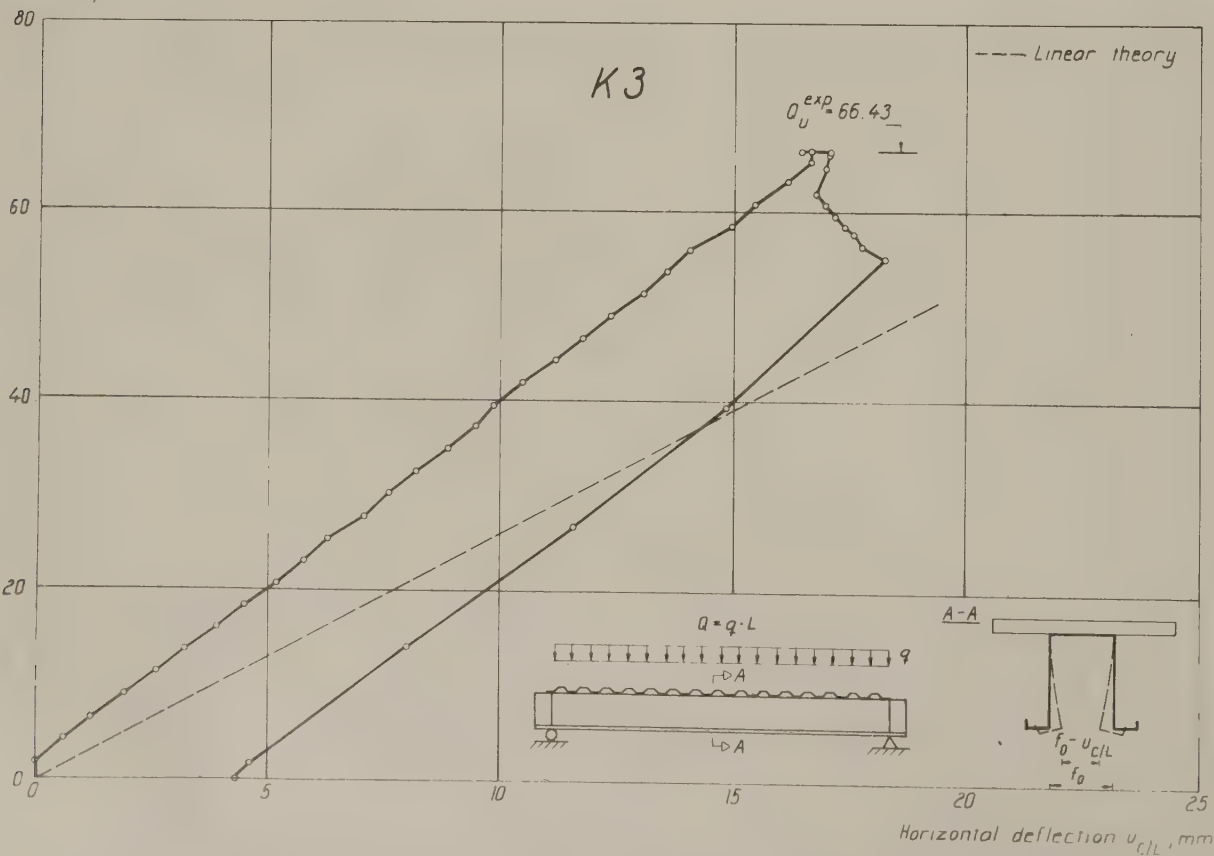
Load Q , kN

Fig. 6.3.12. Relationship between load and horizontal deflection of the free flanges for test beam K3.

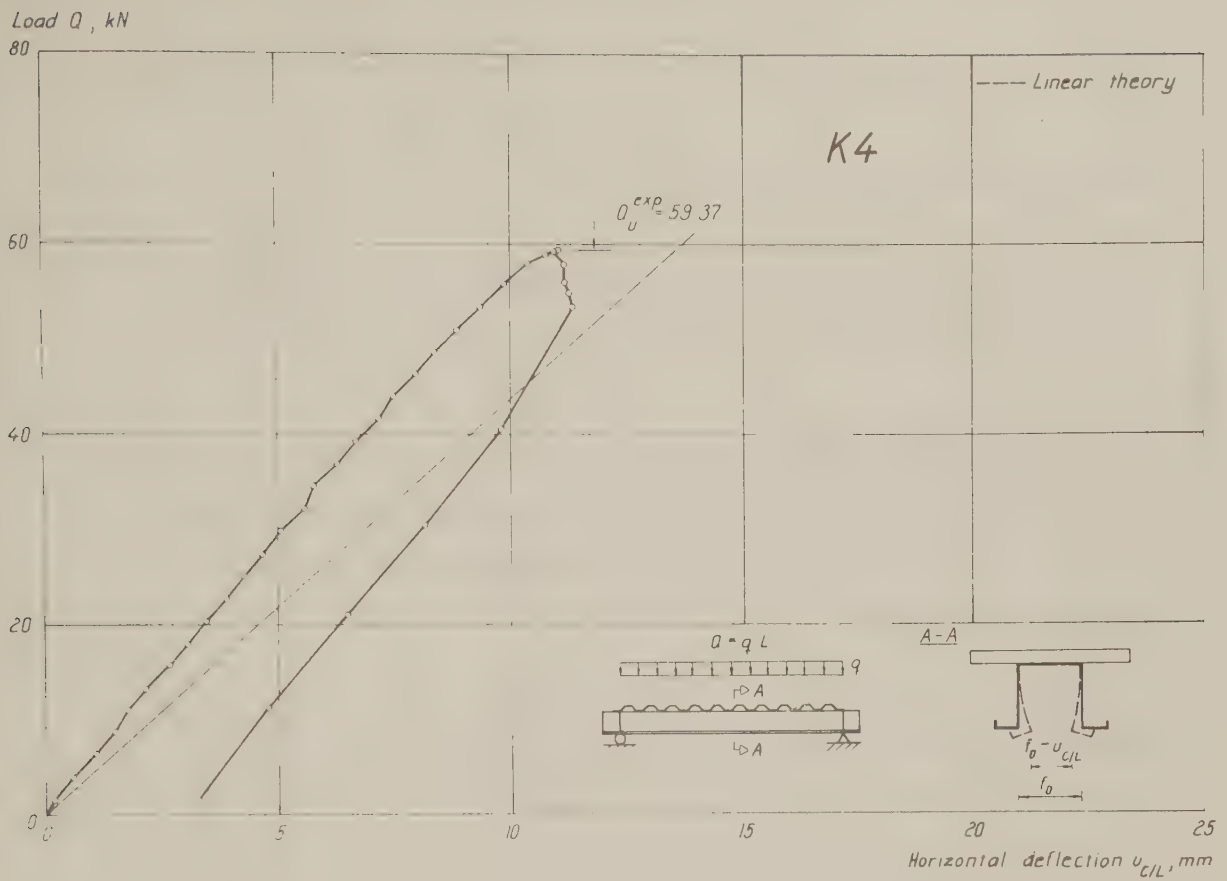


Fig. 6.3.13. Relationship between load and horizontal deflection of the free flanges for test beam K4.

Ultimate loads.

Measured ultimate loads and ultimate moments are given in Table 6.3.3.

Table 6.3.3. Measured ultimate loads and ultimate moments for test beams K1-K4.

Test beam	Q_u^{exp} kN	M_u^{exp} kNm
K1	57.92	16.32
K2	58.26	10.93
K3	66.43	18.72
K4	59.37	11.14

6.3.7. Discussion of results.

Vertical and horizontal deflections.

Vertical and horizontal deflections have been calculated for test beams K1-K4 by application of the theory given in Appendix G. The results of the calculations are shown in Figs. 6.3.6-6.3.13.

The measured vertical deflections are in very good agreement with the theoretical deflections up to about half the ultimate load. There is no clear influence of the corrugated sheeting on the vertical deflection.

The measured horizontal deflections are about 25-30 per cent less than the theoretical deflections in the linear range. This is probably an effect of the external load transmission. From Appendices F and G it can be seen that the horizontal deflections are dependent on the size of the horizontal restraining forces in the free flanges which appear when the beam is subjected to vertical bending as a rigid body. The calculations in Appendices F and G are based on the assumption that the external forces act in the elongation of the webs. However, in the testing of beams K1-K4 the loads were transferred somewhat within the plane of the webs. Thus, additional horizontal restraining forces q_H will arise in the free flanges (see Fig. 6.3.14). These can be derived to

$$q_H = \frac{3b_1(b - b_1)}{2b \cdot h\left(\frac{2h}{b} + 3\right)} \cdot q$$

and are in the direction opposite to that of the restraining forces of Appendices F and G. To obtain a reduction of 25-30 per cent of the theoretical horizontal deflections, it can be calculated that the load eccentricity b_1 must be ≈ 15 mm. For test beams K1 and K2 this distance seems somewhat too high, but there is no doubt that the load eccentricity plays an important role in explaining the phenomenon. The corrugated sheeting shows a slight tendency to diminish the horizontal deflection.

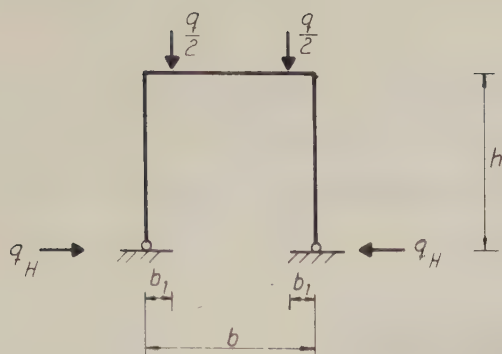


Fig. 6.3.14. Additional horizontal restraining forces.

Character of failure.

For test beams K1 and K2, the behaviour at failure was the same as for test beams J1 and J2, cf. Section 6.2.7.

The character of failure of test beams K3 and K4 was largely the same. There was, however, one typical difference, namely that the local folding of the compressed flange appeared quite symmetrically in one of the upward waves of the corrugated sheeting. The local failure of test beam K3 is shown in Fig. 6.3.15. For test beams J1, J2, K1 and K2, where the compressed flange was free, the local folding was always unsymmetrical with respect to the longitudinal direction. Thus, it can be said that the corrugated sheeting forced the buckle pattern of the compressed flange to another failure mode.

Ultimate loads.

Ultimate bending moments calculated according to the two methods given in Section 6.1.3 are given in Table 6.3.4. Ratios of measured to calculated ultimate moments are also shown. The influence of the corrugated sheeting on the ultimate loads is not obvious, but there is a slight tendency for the ultimate load to increase when corrugated sheeting is used. A comparison between the results presented in Tables 6.2.7 and 6.3.4 shows that the ratios of measured to calculated ultimate bending moments are somewhat higher for the test beams from serie K than those from serie J. The main reasons were that the load was transferred in a

plane somewhat eccentric to the webs (see previous discussion in this Section) and that the compressed flange could not buckle freely on account of external restraints in the form of rubber spacers and corrugated sheeting. From this discussion it is clear that the size of the ultimate bending moment seems to be not insensitive to the method of load transfer.



Fig. 6.3.15. Local failure of test beam K3.

Table 6.3.4. Theoretical ultimate bending moments. Ratios of measured to calculated ultimate moments.

Test beam	Reduced tension flanges		Box beam section	
	M_u^{th}	M_u^{exp}/M_u^{th}	M_u^{th}	M_u^{exp}/M_u^{th}
	kNm		kNm	
K1	14.31	1.14	15.75	1.04
K2	8.86	1.23	9.94	1.10
K3	14.87	1.26	16.38	1.14
K4	8.78	1.27	9.84	1.13

7 DEVELOPMENT PROSPECTS. FUTURE RESEARCH

In this investigation, the post-buckling behaviour of box beams built up of flat plates, has been studied. It has here been established that for slender beams, local buckles develop even at rather moderate loads. The development of local buckles means a reduced bending and torsional stiffness and a lowered load-carrying capacity of the box beams. On account of their appearance, the buckles may also not be acceptable in certain visible structures. A simple way to prevent local buckles from developing is to roll in longitudinal folds in the webs and flanges during the manufacture of the beams. It should also be possible to apply other types of initial patterns to the beams with advantage.

Another way of preventing the development of local buckles is to make the flat plate interact with some type of stiffening structural element. This interacting element does not need to be made of steel but may consist of another material e.g. a wood-based one.

The proper way of development in connection with the design of thin-walled beams is not only dependent on the load-carrying capacity of the beams but is also a question of the appropriate method of production and a suitable technique of erection. It ought here to be particularly stressed that connections at supports and at regions of concentrated loads, if any, require special attention.

In the investigation a number of hat-section beams were also tested, where the load was transferred via a trapezoidal corrugated sheeting. The beams were placed in such a manner that the free flanges faced away from the corrugated sheeting (see Section 6.3). In order to utilize hat-section beams more effectively they ought to be turned over so that the free flanges are fastened to the corrugated sheeting so that the initial cross-sectional shape is better retained. As for the box beams, the aim should be to prevent local buckles from developing.

As a continuation of this research on thin-walled structures at the Department of Structural Engineering in the Lund Institute of

Technology, a project concerning "Problems in connection with the utilization of high strength material in steel structures" has been started. The project is intended to lead to a development of the design of steel structures where high strength steel can be used in a more economic way.

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APPENDIX A

Definition of cross-sectional dimensions and critical stresses for box beams.

Notations for cross-sectional dimensions of a box beam are given in Fig. A1. The width b and the height h are defined as the centre distances between the webs and the flanges respectively. The notations b_0 and h_0 refer to the outer dimensions and are only used when the measured dimensions of the test beams are referred to. The corner radius R refers to the inside radius.

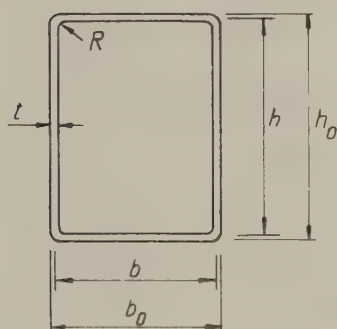


Fig. A1. Notations for cross-sectional dimensions of a box beam.

In the calculation of the moment of inertia I of the beam section, the real cross section is replaced by an assumed section with the inner corner radius R equal to zero. The moment of inertia when the whole beam section is effective is denoted by I_0 . The assumption that the corner radius R is equal to zero means that the theoretical moment of inertia will be somewhat too high. For test beams B1-B7 and C1-C7, the deviation of I_0 is always less than 2.2 %.

The torsional constant I_T defined by Eq. (4.1.5) is calculated by assuming the inner corner radius equal to zero. The deviation from the true value is negligible.

The theoretical critical stress σ_{cr} of a box beam section subjected to bending can be determined according to the procedure given in Section 2.4.1, where the transmission of transverse bending moment between webs and flanges is taken into account. As was mentioned in Section 2.4.1, it is often satisfactory to

calculate the critical stress of each plate under the assumption of hinged longitudinal edges. In that case the critical stresses can be derived for the compression flange to

$$\sigma_{cr,f} = 4 \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (A1)$$

and for the webs to

$$\sigma_{cr,w} = 23.9 \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{h}\right)^2 \quad (A2)$$

In the case of pure torsion of a box beam, the critical stresses under the same assumption can be derived for the flanges to

$$\tau_{cr,f} = 5.34 \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (A3)$$

and for the webs to

$$\tau_{cr,w} = 5.34 \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{h}\right)^2 \quad (A4)$$

The above formulae have been derived under the assumption that the plates are of infinite length.

APPENDIX B

Tensile tests.

The tensile tests were conducted in an Alwetron F 5000 electronic universal testing machine. The change in distance between the clamps was registered on the recorder of the testing machine. The dimensions of the tensile test pieces were those prescribed by SIS 11 21 16 (Swedish standard), see Fig. B1. The graphs of the tensile stress versus the elongation between the clamps presented in this report have been corrected for slipping in the clamp devices.

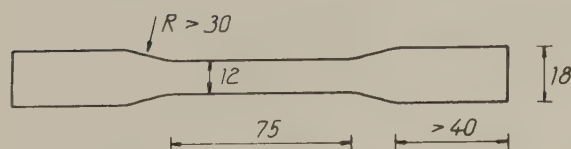


Fig. B1. Dimensions of a tensile test piece.

APPENDIX C

Determination of dynamic modulus of elasticity.

The dynamic modulus of elasticity was determined by testing simply supported test pieces in bending according to Fig. C1. The test pieces were subjected to transverse vibrations by an electro-magnetic transducer and the amplitude was recorded by a magnetic head. The frequency of the forced vibration was determined by a frequency counter. The natural frequency of the test piece is obtained when the frequency of the forced vibration causes maximum amplitude.

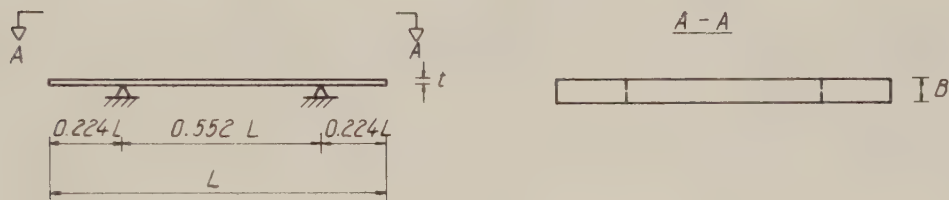


Fig. C1. Bending test piece.

For the fundamental frequency the modulus of elasticity E can be derived from the expression

$$E = \frac{48 \pi^2 m L^3}{c^4 B t} f^2 \quad (C1)$$

where m = mass

c = constant ($c = 4.730$ for the fundamental frequency)

f = natural frequency

B , L and t are defined in Fig. C1.

APPENDIX D

Computer-controlled data acquisition system.

The principal layout of the computer-controlled data acquisition system is shown in Fig. D1. The system comprises three separate parts namely the mobile computer unit, the mobile data acquisition system unit and the connection unit.

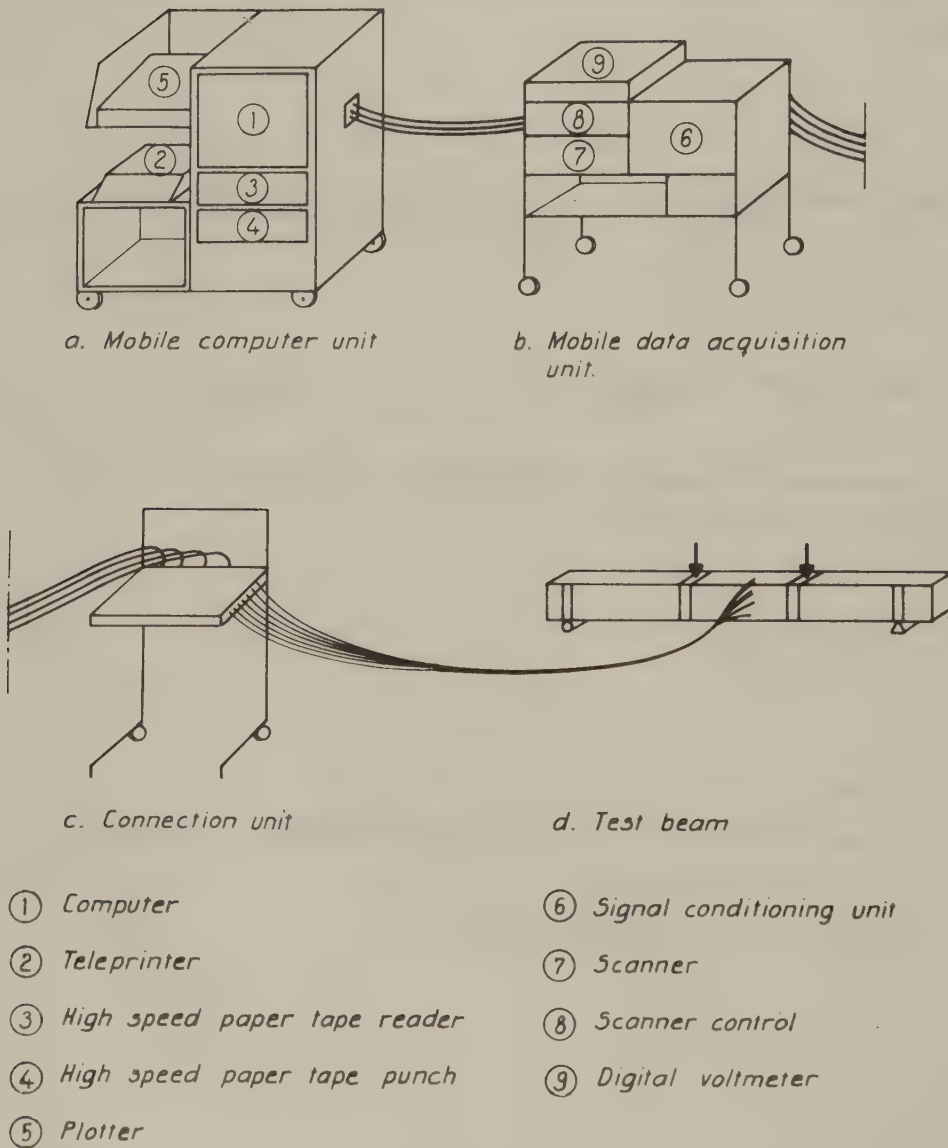


Fig. D1. Principal layout of computer-controlled data acquisition system.

The mobile computer unit consists of

- 1) a computer, HP 2116 C (HP = Hewlett Packard), with a 16 K core memory. The word size is 16 bits and the memory cycle time 1.6 microseconds.
- 2) a teleprinter, Teletype ASR 33, with a typing speed of 10 signs/second. It reads and punches 8-channel paper tape at the same speed.
- 3) a high-speed paper tape reader, HP 2748 A, which reads an 8-channel paper tape at a speed of 415 signs/second.
- 4) a high-speed paper tape punch, FACIT 4070, which punches an 8-channel paper tape at a speed of 70 signs/second.
- 5) a plotter, HP 7210 A, with an accuracy better than 1 mm/400 mm.

The mobile data acquisition system unit comprises four parts, namely

- 6) the signal conditions unit, which makes it possible to choose different types of transducers by means of electric switches.
- 7) the scanner, HP 2911 A, consisting of 200 channels, each with 3 poles. The maximum scanning speed is more than 8 channels/second.
- 8) the scanner control, HP 2911 B.
- 9) the digital voltmeter, HP 3450 A, which has 5 full decades + 20 % overrange. The resolution of the most sensitive range is 1 microvolt.

The different types of transducers giving information about the test beams were connected to the signal conditioning unit via connection units.

It was also possible to use a simple form of process control by means of three reserved channels in the scanner.

APPENDIX E

Initial normal deflections.

The initial normal deflections of test beams F1-F6, G1-G4 and H1-H5 were determined according to Fig. E1. Thus the normal deflections δ_l , δ_r , δ_{up} , δ_{low} are defined as being positive when directed outwards from the cross section. Further, the normal deflections are presented in different sections along the beams assuming the zero section to be located at the left-hand support in the figure.

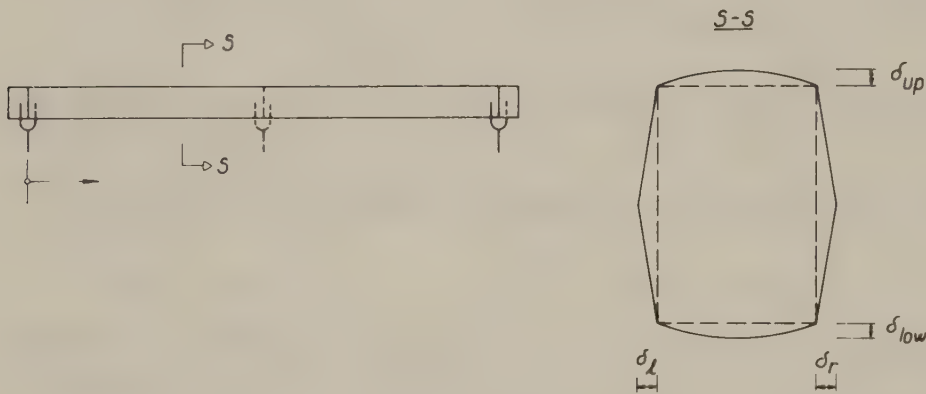


Fig. E1. Notations for measured initial normal deflections.

Measured normal deflections of the test beams are given in Tables E1-E8. When the studied sections happen to coincide with supports or with sections where external forces are applied, the normal deflections have been measured 100 mm from the edge of the actual clamp device.

Table E1. Initial normal deflections of test beams F1 and F2.

Section mm	Test beam F1				Test beam F2			
	δ_{ℓ} mm	δ_r mm	δ_{up} mm	δ_{low} mm	δ_{ℓ} mm	δ_r mm	δ_{up} mm	δ_{low} mm
0	-2.5	-2.5	-1	-0.5	-1	-1	0.5	0
800	-6.5	-7	1.5	1.5	-6	-6	0.5	1.5
1600	-5	-7	1.5	1	0	-1	-1	-1
2400	-4	-5	0	0.5	-5	-7.5	1	1.5
3200	-2.5	-5.5	0.5	0	-5	-8	2	1.5
4000	-5.5	-5.5	1.5	1	-5	-5.5	2	1.5
4800	-2	-2	0.5	0	-2.5	-2	-1	-1

Table E2. Initial normal deflections of test beams F3 and F4.

Section mm	Test beam F3				Test beam F4			
	δ_{ℓ} mm	δ_r mm	δ_{up} mm	δ_{low} mm	δ_{ℓ} mm	δ_r mm	δ_{up} mm	δ_{low} mm
0	-3.5	-3	-1	-1.5	0	0	-0.5	0
800	-5.5	-5	1	1	-2	-2	-1	-1
1600	-3.5	-7	0.5	1	-5	-5	0.5	1
2400	-6	-8.5	1	2	-5	-5	2	1.5
3200	-7	-8	1.5	2	-7	-7	1.5	2
4000	-7.5	-7	2	1.5	-6	-6	1.5	1.5
4800	-1.5	-2.5	-1	0	-2	-2	0	-1

Table E3. Initial normal deflections of test beams F5 and F6.

Section	Test beam F5				Test beam F6			
	δ_l mm	δ_r mm	δ_{up} mm	δ_{low} mm	δ_l mm	δ_r mm	δ_{up} mm	δ_{low} mm
0	-3	-2	0	-0.5	-1	-0.5	0	0
0.2 L	-5	-5	0.5	0.5	-2.5	-1	0	0
0.4 L	-1	-4	-0.5	-0.5	-5	-4	0	0.5
0.6 L	-5	-6	0.5	1	-6	-3.5	1	1
0.8 L	-3.5	-6	0.5	1	-6.5	-5	0.5	0.5
1.0 L	-2	-3	0	-1	-2.5	-1	0	-0.5

Table E4. Initial normal deflections of test beams G1 and G2.

Section mm	Test beam G1				Test beam G2			
	δ_l mm	δ_r mm	δ_{up} mm	δ_{low} mm	δ_l mm	δ_r mm	δ_{up} mm	δ_{low} mm
0	-1.5	-1.5	-1	-0.5	-3	-1.5	0.5	-1
800	-5	-2	0.5	0.5	-5	-4.5	0.5	1.5
1600	-4	-4	1.5	0	-5.5	-5	0	1.5
2400	-5.5	-5	2	1	-8	-6	1.5	1
3200	-6	-5	1	0.5	-8	-6	1.5	0.5
4000	-4.5	-2.5	1.5	1	-7	-6.5	2	-1
4800	-1	-2	-1	-0.5	-4.5	-3.5	-0.5	-1

Table E5. Initial normal deflections of test beams G3 and G4.

Section mm	Test beam G3				Test beam G4			
	δ_l mm	δ_r mm	δ_{up} mm	δ_{low} mm	δ_l mm	δ_r mm	δ_{up} mm	δ_{low} mm
0	-1.5	-2	-0.5	0	-3	-2.5	-0.5	-0.5
800	-2	-4	1	1	-7	-5	1	2
1600	-4	-7	0.5	0	-6.5	-5.5	0	-0.5
2400	-7	-7	1.5	1	-9	-6.5	1.5	1.5
3200	-5	-6	0	1	-8	-7	1.5	1.5
4000	-7.5	-6.5	1	1.5	-7.5	-7	1	1.5
4800	-3	-2.5	-1	-0.5	-2	-2.5	-1	0

Table E6. Initial normal deflections of test beams H1 and H2.

Section mm	Test beam H1				Test beam H2			
	δ_l mm	δ_r mm	δ_{up} mm	δ_{low} mm	δ_l mm	δ_r mm	δ_{up} mm	δ_{low} mm
0	-2	-2.5	0	-0.5	-1.5	-3	-0.5	-0.5
0.2 L	-4	-6	0	0.5	-5	-6	1.5	1
0.4 L	-3	-6.5	0.5	0.5	-5.5	-6.5	1	1
0.6 L	-2.5	-6.5	1	1	-6	-7	1	1
0.8 L	-2	-6	0	1	-4	-6	1	1
1.0 L	-1.5	-1.5	0	-0.5	-2.5	-2	0	-0.5
1.2 L	-4	-7.5	0	1.5	-5	-6	1	1
1.4 L	-2	-6.5	0	1	-3.5	-5	0.5	1
1.6 L	-0.5	-3	-0.5	0	-2.5	-2	-0.5	0.5
1.8 L	-1.5	-6.5	0	0.5	-6.5	-5.5	0.5	1
2.0 L	-1	-4	-0.5	-0.5	-3	-2	-1	-1

Table E7. Initial normal deflections of test beam H3.

Section mm	Test beam H3			
	δ_{ℓ} mm	δ_r mm	δ_{up} mm	δ_{low} mm
0	-2	-1.5	0	-1
0.2 L	-5	-4.5	0.5	0.5
0.4 L	-5.5	-4	0.5	0.5
0.6 L	-6	-6	1	0.5
0.8 L	-5.5	-4	1	0.5
1.0 L	-2	-1.5	0	-0.5
1.2 L	-4.5	-5.5	0	0
1.4 L	-4	-5.5	0.5	0
1.6 L	-1.5	-2	-0.5	-0.5
1.8 L	-5	-6	0	0
2.0 L	-1.5	-1	-0.5	-0.5

Table E8. Initial normal deflections of test beams H4 and H5.

Section mm	Test beam H4				Test beam H5			
	δ_{ℓ} mm	δ_r mm	δ_{up} mm	δ_{low} mm	δ_{ℓ} mm	δ_r mm	δ_{up} mm	δ_{low} mm
1/6 L	-5	-5	0	0.5	-5	-5.5	0	0
3/6 L	-5.5	-5	1	1	-3	-5.5	1	1
5/6 L	-4.5	-5	0	1	-5	-6	0	0.5
7/6 L	-5	-5.5	0	0.5	-3.5	-4	0.5	0
9/6 L	-3.5	-5	0	0.5	-3	-5	1	1
11/6L	-4	-3.5	0	0	-4	-5	0	1

APPENDIX F

Determination of deflections and stresses for a simply supported hat-section beam subjected to pure bending. Simplified theory.

The calculations are simplified by assuming that the areas of the lips are concentrated at points as shown in Fig. F1. Further, it is assumed that the basic assumptions made in Appendix G are valid.

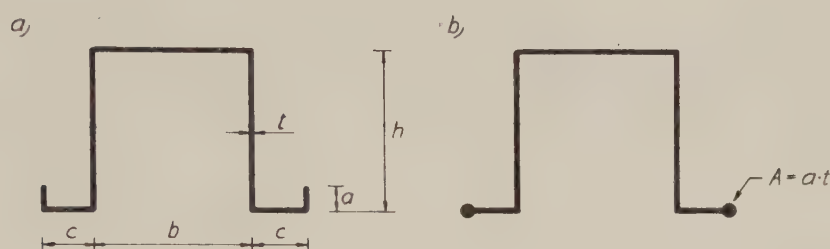


Fig. F1. a) Real cross section.

b) Simplified cross section used in the calculations.

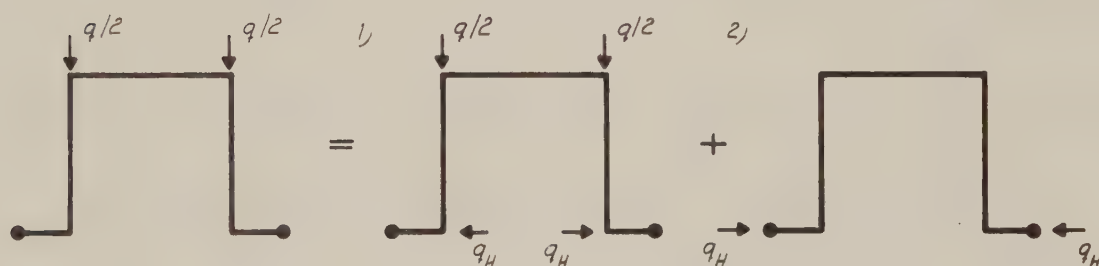


Fig. F2. The real load case is divided into load cases 1 and 2.

A beam element is subjected to a distributed load q according to Fig. F2. To simplify the calculations, the real load case is divided into the two load cases presented in Fig. F2. Load case 1 can here be treated according to the ordinary theory of beams, i.e. plane sections remain plane. Load case 2 is analysed by means of the technique of a beam supported on an elastic foundation.

The distributed load q_H , necessary to maintain the cross-sectional shape in load case 1, can be derived from the condition that the shear stresses in the free flanges are to be in equilibrium with q_H .

On integration of the shear stresses, the following expression for q_H is obtained

$$q_H = \frac{ct}{I_1} \left[a(y_1 - \frac{a}{2}) + \frac{cy_1}{2} \right] q \quad (F1)$$

where I_1 = ordinary moment of inertia

y_1 = vertical distance from the free flanges to the neutral axis.

To calculate the influence of the distributed load q_H in load case 2, we consider the differential equation for the horizontal deflection curve of one of the free flanges (cf. Fig. 6.1.2)

$$EI_2 \frac{d^4 z}{dx^4} + k_2 z = q_H \quad (F2)$$

Thus, before the equation can be used, it is evident that the flexural rigidity EI_2 and the elastic constant k_2 have to be determined.

The flexural rigidity of the free flange is calculated by introducing assumed links at each corner of the beam section. Further, the stresses in the longitudinal direction caused by q_H are assumed to have a linear distribution in each plate, see Fig. F3. From the condition that no resultant external normal

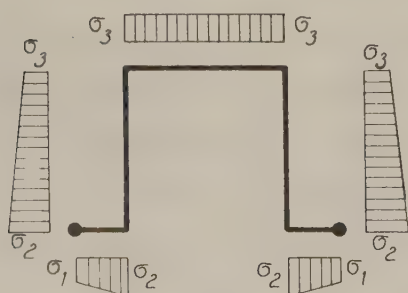


Fig. F3. Assumed distribution of longitudinal stresses for load case 2.

forces and bending moments act on the beam i.e.

$$N = 0$$

$$M_z = 0$$

we can calculate a fictitious moment of inertia for the free flange (Fig. F4) that will give the same stress distribution in the free flange as for the simplified beam section shown in Fig. Flb.

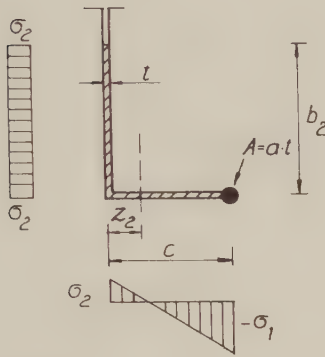


Fig. F4. Fictitious beam section for the determination of the longitudinal stress distribution for load case 2.

The fictitious moment of inertia I_2 will thus be:

$$I_2 = t \left[\frac{c^3}{12} + c \left(\frac{c}{2} - z_2 \right)^2 + a (c - z_2)^2 + b_2 z_2^2 \right] \quad (F3)$$

where

$$b_2 = \frac{\frac{c^2}{2} + ac - z_2(a + c)}{z_2}$$

$$z_2 = \frac{c}{c_3 + 1}$$

$$c_3 = \frac{(c + h) c_2 - \frac{h^2}{3}(b + h)}{(2a + c) c_2}$$

$$c_2 = \frac{2}{3} h^2 + bh$$

The elastic constant k_2 is determined from the change in shape of the cross section, see Fig. F5.

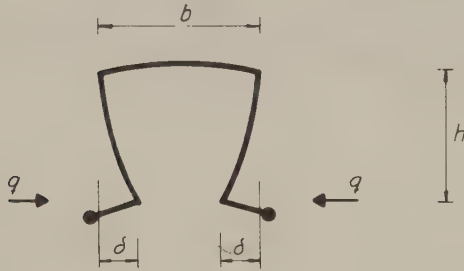


Fig. F5. Change of shape of cross section for load case 2.

Simple calculations give

$$k_2 = \frac{q}{\delta} = \frac{6D}{h(3b^2 + 2h^2)} \quad (F4)$$

where

$$D = \frac{Et^3}{12(1 - \nu^2)}$$

If now the boundary conditions of the free flange are introduced, the deflection curve and thus the stresses can be calculated from the differential equation.

For the actual test beams, the following boundary conditions were used:

- a) the beams are simply supported with respect to vertical bending
- b) the free flanges are simply supported with respect to horizontal bending.

The final distribution of deflections and stresses will be obtained by superposition of load cases 1 and 2.

APPENDIX G

Determination of deflections and stresses for a simply supported hat-section beam subjected to pure bending. Accurate theory.

This appendix describes a more advanced method than that presented in Appendix F, for the determination of deflections and stresses.

The cross-sectional deformations of a hat-section beam subjected to pure bending can be characterized by three independent components, namely :

- 1) vertical translation of the whole cross section as a rigid body,
- 2) horizontal translation of the free flanges,
- 3) vertical translation of the "lips".

These three elementary deformations have been used in a computer programme in which external and restraining forces have gradually been reduced by successively releasing and locking the deformations.

The influence of different boundary conditions will be discussed.

Assumptions.

The following basic assumptions are made:

- a) The material is characterized by linear elastic properties.
- b) The beam section is required to be symmetrical with respect to the x-y plane, (cf. Fig. 6.1.2).
- c) The cross-sectional dimensions do not vary along the beam.
- d) The plate thickness t is small in comparison with other cross-sectional dimensions.

- e) The deformations are small in comparison with the dimensions of the beam (the plate thickness excluded).
- f) The influence of shear deformations is neglected.
- g) The longitudinal stresses vary linearly in every plate over the cross section.
- h) The external loads act in the plane of the webs.

Vertical translation of the whole cross section as a rigid body.

This deformation case can be treated as plane bending of a beam according to the hypothesis of Navier. To maintain the cross-sectional shape, restraining forces q_{V1} and q_{H1} as in Fig. G1 must be introduced. These restraining forces can be determined by

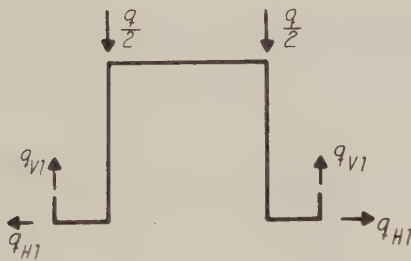


Fig. G1. Forces acting on a beam element subjected to vertical translation as for a rigid body.

integration of the shearstresses, which gives the following expressions:

$$q_{H1} = \frac{ct}{I_1} \left[a(y_1 - \frac{a}{2}) + \frac{1}{2} cy_1 \right] q \quad (G1)$$

$$q_{V1} = \frac{a^2 t}{I_1} (\frac{1}{2} y_1 - \frac{1}{3} a) q \quad (G2)$$

where

I_1 = moment of inertia of the total beam section

y_1 = vertical distance from the neutral axis to the free flanges.

Horizontal translation of the free flanges.

To determine the influence of a horizontal translation of the free flanges, the theory of a beam on an elastic foundation is used (cf. Appendix F). For this purpose the transverse load q_H may be divided in two parts, namely

1) q_H' which is taken up by longitudinal normal stresses (Fig. G2a)

and

2) q_H'' which is taken up by transverse bending stresses (Fig. G2b).

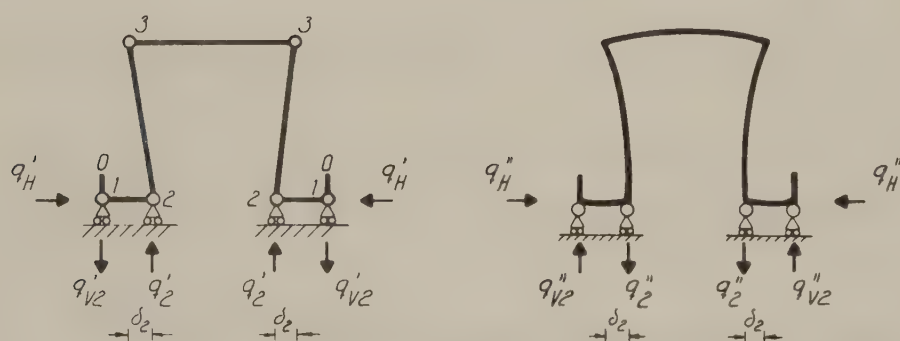


Fig. G2. External and restraining forces acting on a beam element subjected to horizontal translation of the free flanges.

a) The load is taken up by longitudinal stresses.

b) The load is taken up by transverse bending stresses.

In a horizontal translation of the free flanges according to Fig. G2a, plates 0-1, 2-3 and 3-3 will not develop any curvature. Thus, the longitudinal stresses will be constant in the transverse direction for these plates in every section, see Fig. G3. From the condition that no external longitudinal forces act on the beam, i.e. $N = 0$, the fictitious moment of inertia with respect to transverse bending of the free flanges can be derived. The dimensions of the fictitious beam are given in Fig. G4.

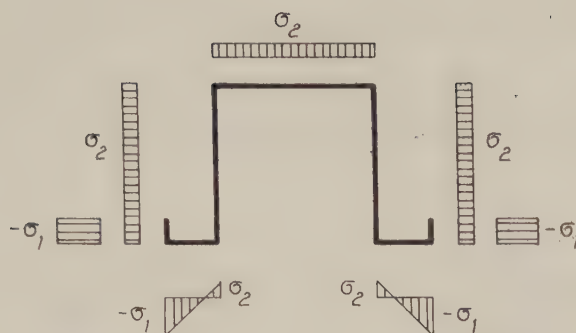


Fig. G3. Assumed stress distribution for the determination of the flexural rigidity of the free flanges.

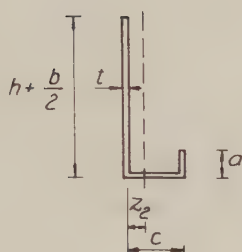


Fig. G4. Fictitious beam section for the determination of the flexural rigidity of the free flanges.

The position of the neutral axis z_2 and the moment of inertia I_2 are given by:

$$z_2 = \frac{c(2a + c)}{2c + 2h + 2a + b} \quad (G3)$$

$$I_2 = t \left[a(c - z_2)^2 + \left(h + \frac{b}{2}\right) z_2^2 + \frac{1}{12} c^3 + c\left(\frac{c}{2} - z_2\right)^2 \right] \quad (G4)$$

The longitudinal stresses give rise to shear stresses which can be determined from the condition of force equilibrium i.e.

$$\frac{\partial \sigma}{\partial x} + \frac{\partial \tau}{\partial s} = 0 \quad (G5)$$

where

x = longitudinal coordinate

s = transverse coordinate

The restraining forces q_2' and q_{V2}' are obtained by integration of the shear stresses:

$$q_2' = \frac{c - z_2}{I_2} \operatorname{th} \frac{(b + h)(2a + c)}{2(b + c + 2h)} q_H' \quad (G6)$$

$$q_{V2}' = \frac{c - z_2}{I_2} t \frac{a^2}{2} q_H' \quad (G7)$$

The elastic constant and the restraining forces can be derived from a study of the deformations of the beam element according to Fig. G2b:

$$k_2 = \frac{q_H''}{\delta_2} = \frac{6D(3b + 2c + 6h)}{h^2(6bh + 3h^2 + 6bc + 4ch)} \quad (G8)$$

and

$$q_2'' = q_{V2}'' = \frac{6D + k_2 h^3}{ch(2c + 3h)} \delta \quad (G9)$$

For a definite distribution of external horizontal forces acting on the free flanges, the transverse deflection curve, the longitudinal stresses and the restraining forces now can be determined.

Vertical translation of the "lips".

The influence of a vertical translation of the "lips" may be analysed in the same way as that of the horizontal translation of the free flanges.

Thus, the load may be divided into two parts q_V' and q_V'' , where q_V' is taken up by longitudinal normal stresses (Fig. G5a) and q_V'' by transverse bending stresses (Fig. G5b).

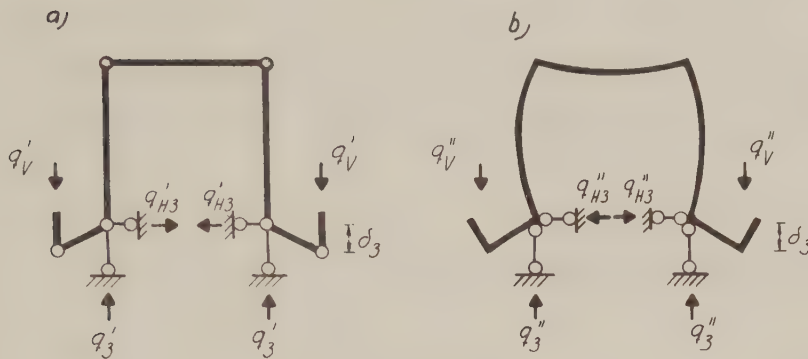


Fig. G5. External and restraining forces acting on a beam element subjected to vertical translation of the "lips".

a) The load is taken up by longitudinal stresses.

b) The load is taken up by transverse bending stresses.

The longitudinal stresses caused by the load q_V' will be distributed according to Fig. G6. The corresponding fictitious beam section is shown in Fig. G7.

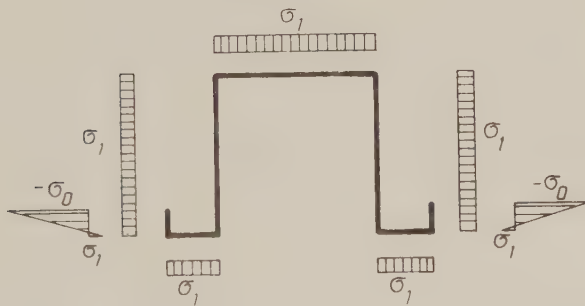


Fig. G6. Assumed stress distribution for the determination of the flexural rigidity of the "lips".

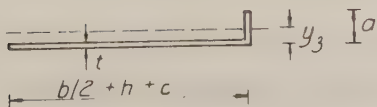


Fig. G7. Fictitious beam section for the determination of the flexural rigidity of the "lip".

The position of the neutral axis y_3 and the moment of inertia I_3 are given by

$$y_3 = \frac{a^2}{2a + b + 2c + 2h} \quad (G10)$$

$$I_3 = t \left[\frac{1}{12} a^3 + a \left(\frac{a}{2} - y_3 \right)^2 + \left(\frac{b}{2} + h + c \right) y_3^2 \right] \quad (G11)$$

The restraining forces can be derived to

$$q'_3 = t \frac{y_3}{I_3} \frac{h}{2} (b + h) q'_V \quad (G12)$$

$$q'_{H3} = t \frac{y_3}{I_3} \frac{c}{2} (b + c + 2h) q'_V \quad (G13)$$

With the deformation picture of Fig. G5b, the elastic constant and the restraining forces are given by

$$k_3 = \frac{q''_V}{\delta_3} \frac{6D}{c^2 \left(2h - \frac{h^2}{3b + 2h} + 2c \right)} \quad (G14)$$

$$q''_3 = q''_V \quad (G15)$$

$$q''_{H3} = k_3 c \left(\frac{1}{3b + 2h} + \frac{1}{h} \right) \delta_3 \quad (G16)$$

All the quantities necessary for the determination of the vertical deflection curve of the "lip", the longitudinal stresses and the restraining forces have thus been derived.

Method of solution. Boundary conditions.

Starting from the three elementary deformation cases just studied, a computer programme for analysis of deflections and stresses of a simply supported hat-section beam was worked out. The solution was obtained by gradually decreasing the external and restraining forces by successively releasing and locking the deformations. A flow chart of the computer programme is shown in Fig. G8.

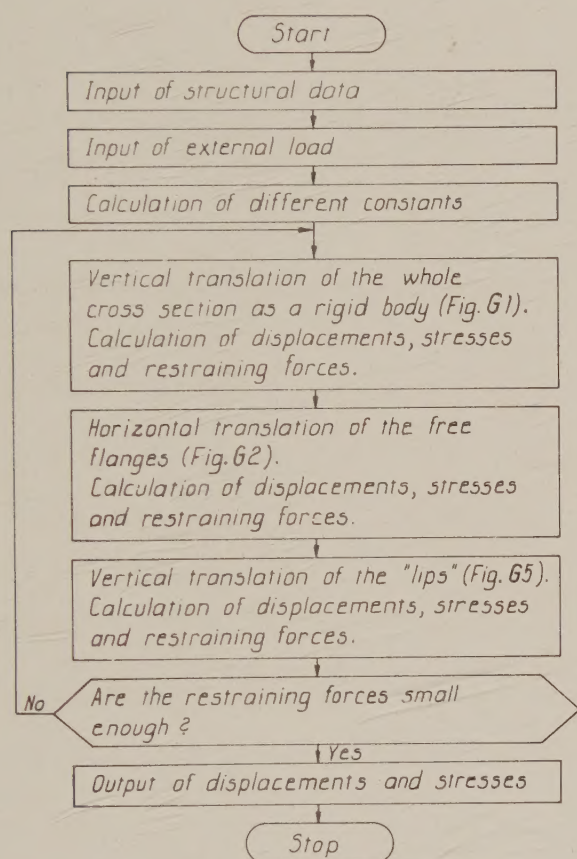


Fig. G8. Schematic flow chart of the computer programme.

To compare the results with performed tests the following boundary conditions were assumed:

- 1) The beams are simply supported with regard to vertical bending.
- 2) The free flanges are simply supported with regard to horizontal bending.
- 3) The lips are simply supported with regard to vertical bending.

The third boundary condition may seem to be somewhat curious as the lips were free from external restraints. However, the test beams are somewhat longer than the theoretical span length, so that in reality the lips will be under the influence of shear forces as well as bending moments at the theoretical supports. As a fairly reasonable boundary condition, it has therefore been assumed that the lips are simply supported with regard to vertical bending.

